Reciprocity Between Pick-up and Kicker Structures Including the Far-Field Zone

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Abstract

An analytical calculation is presented describing the interaction of a moving charged particle with the field of a time-harmonic electric or magnetic dipole. This solution completely agrees with the reciprocity theorem. It is shown that for the pick-up case and $\gamma \gg 1$, useful pick-up sensitivities can be achieved for distances of many wavelengths from the particle path, which are considered here to be the far-field zone. The theoretical results are supported by measurements carried out in the near field of a proton beam in the CERN Antiproton Collector. For the same condition $\gamma \gg 1$, longitudinal and transverse kickers can be implemented.

1 INTRODUCTION

The interaction of a homogeneous plane wave of frequency ν with a charged particle of rest mass m_0 is referred to as Thomson scattering (when $h\nu < m_0c^2$) and Compton scattering (when $h\nu > m_0c^2$) respectively [1] (p. 679). It has been shown that the interaction (or scattering cross-section) for low and medium intensities of the electromagnetic field is very small indeed (radiation pressure), in particular for Thomson scattering. In these scattering mechanisms the modification of the particle motion due to the wave is taken into account. However, in the following the motion of the particle shall be considered unaffected by the field, which is a first approximation for high γ-values. The vast majority of structures used to interact with charged particles are near-field devices, i.e. their transverse dimensions are usually below or in the order of one free space wavelength. Using a field description in terms of homogeneous and inhomogeneous (evanescent) plane waves [2] shows that the contribution of these evanescent waves decays very rapidly beyond one wavelength from the structure surface, except for those evanescent waves with a phase velocity slightly less than c which can be synchronous with a relativistic particle [3].

2 FIELD OF A CHARGE IN UNIFORM MOTION

We assume that a charge Q moves with a velocity v along the z-axis (see Fig. 1). The equation of the charge motion is

$$t = t_0 + \frac{z}{v}, \tag{1}$$

where t_0 is the time when the particle passes at $z = 0$. In cylindrical coordinates the electromagnetic field produced by the moving charge is given in the frequency domain by:

$$\begin{aligned} E_z(\omega) &= \frac{Q\,\mathrm{sgn}(v) jk}{2\pi\epsilon_0 c\beta^2\gamma^2} K_0(\kappa r) e^{j\omega(t-t_0-\frac{z}{v})} \\ E_r(\omega) &= \frac{Q}{2\pi\epsilon_0 |v|}\kappa K_1(\kappa r) e^{j\omega(t-t_0-\frac{z}{v})} \end{aligned} \tag{2}$$

where $\kappa = \left|\frac{k}{\beta\gamma}\right|$, $k = \frac{\omega}{c}$.

$\beta = v/c$ and $\gamma = (1-\beta^2)^{-1/2}$ are the usual relativistic factors; $K_\nu(x)$ is the modified Bessel function of the second kind, of order ν. With $\zeta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}$,

$$H_\varphi(\omega) = \frac{\beta}{\zeta_0} E_r(\omega) = \frac{Q\,\mathrm{sgn}(v)}{2\pi}\kappa K_1(\kappa r) e^{j\omega(t-t_0-\frac{z}{v})}. \tag{3}$$

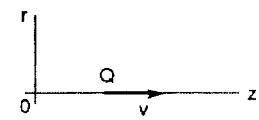

Figure 1: Charge in uniform motion.

3 MAGNETIC DIPOLE

3.1 *Field of a magnetic dipole oriented along the y-axis*

Consider a magnetic dipole with moment $\vec{m}$ parallel to the y-axis, placed at point $(-a, 0, 0)$ (see Fig. 2). The components of the electromagnetic field acting on the charge Q are, with $R = \sqrt{a^2+z^2}$ [4] (p. 437):

$$E_\varphi = -\frac{\zeta_0}{4\pi} m_y \frac{jk}{R^2}(1+jkR)e^{j\omega t - jkR}$$

hence

$$\begin{cases} E_z = -E_\varphi \sin\varphi = -E_\varphi \dfrac{a}{R} \\ E_x = E_\varphi \cos\varphi = E_\varphi \dfrac{z}{R} \end{cases} \tag{4}$$

$$H_y = -H_\theta = -\frac{m_y}{4\pi R^3}(1+jkR-k^2R^2)e^{j\omega t - jkR}. \tag{5}$$

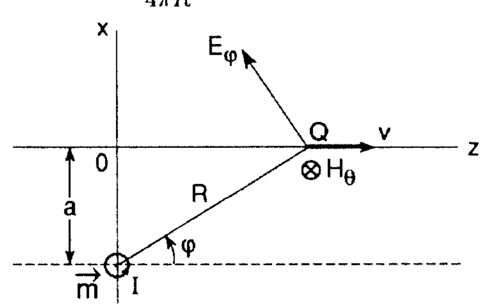

Figure 2: Charge in the field of a magnetic dipole.

3.2 Longitudinal voltage seen by the particle

The particle is assumed to be ultra-relativistic, so that its velocity v is considered to be constant from $z = -\infty$ to $+\infty$. This is the only approximation used here.

The total longitudinal voltage seen by the particle is, with Eq. (4):

$$\begin{aligned} V_{\|}(\omega) &= \int_{-\infty}^{+\infty} E_z\, dz \\ &= \frac{\zeta_0}{4\pi} m_y\, jka \int_{-\infty}^{+\infty} \frac{1+jkR}{R^3} e^{j\omega(t_0+\frac{z}{v})-jkR} dz \\ &= \frac{\zeta_0}{4\pi} m_y\, jka\, e^{j\omega t_0} \int_{-\infty}^{+\infty} \frac{1+jkR}{R^3} e^{j\frac{\omega}{v}z-jkR} dz\,. \end{aligned} \quad (6)$$

With $\omega/v = k/\beta$, it can be shown [5] that

$$\int_{-\infty}^{+\infty} \frac{1+jkR}{R^3} e^{j\frac{k}{\beta}z-jkR} dz = \frac{2}{a}\kappa K_1(\kappa a)\,,\quad \beta^2 < 1 \quad (7)$$

hence

$$V_{\|}(\omega) = \frac{\mu_0}{2\pi} m_y\, j\omega\, \kappa K_1(\kappa a) e^{j\omega t_0}\,. \quad (8)$$

The physical voltage seen by the particle is Re $(V_{\|})$. As long as $\kappa a = |ka/\beta\gamma| < 1$, $V_{\|}$ behaves as a^{-1} and can be significant even at high frequencies and large distances, provided $\beta\gamma$ is large enough. On the other hand, when κa goes to infinity, $V_{\|}$ vanishes exponentially.

The magnetic dipole may be represented by a closed filament carrying a current I embracing an area S [4] (p. 235):

$$m_y = I(\omega)\cdot S\,.$$

Let us call $\vec{J}_1$ the current distribution in the dipole; the beam current $\vec{J}_2$ is given by

$$I_b(z) = Q\,\mathrm{sgn}(v)\, e^{j\omega(t-t_0-\frac{z}{v})}\,. \quad (9)$$

Therefore, with Eq. (4):

$$\begin{aligned} \int_{\text{all space}} \vec{E}_1\, \vec{J}_2\, dV &= \int_{-\infty}^{+\infty} E_z\, Q\,\mathrm{sgn}(v) e^{j\omega(t-t_0-\frac{z}{v})} dz \\ &= Q\,\mathrm{sgn}(v) e^{2j\omega t}\times \\ &\int_{-\infty}^{+\infty} \frac{\zeta_0}{4\pi} m_y\, jka \frac{1+jkR}{R^3} e^{-j\omega(t_0+\frac{z}{v})-jkR} dz \end{aligned}$$

With Eq. (7) where the sign of β is changed this becomes

$$\int_{\text{all space}} \vec{E}_1\, \vec{J}_2 dV = Q\,\mathrm{sgn}(v) \frac{1}{2\pi} m_y\, j\omega\mu_0\, \kappa\, K_1(\kappa a) e^{-j\omega t_0} e^{2j\omega t}\,. \quad (10)$$

3.3 Magnetic field seen by the magnetic dipole

From Eq. (3) where $r = a$, $z = 0$ this field is given by

$$H_y = -H_\varphi = -\frac{Q\,\mathrm{sgn}(v)}{2\pi} \kappa\, K_1(\kappa a) e^{j\omega(t-t_0)}. \quad (11)$$

In the closed filament of the magnetic dipole, this field induces an electromotive force

$$\begin{aligned} \mathcal{E} &= -j\omega\mu_0 H_y S \\ &= j\omega\mu_0 \frac{Q\,\mathrm{sgn}(v)}{2\pi} S\, \kappa\, K_1(\kappa a) e^{j\omega(t-t_0)} \end{aligned} \quad (12)$$

which is produced by the beam current (9).

From Eq. (12)

$$\begin{aligned} \int_{\text{all space}} \vec{E}_2\, \vec{J}_1\, dV &= \mathcal{E} I e^{j\omega t} \\ &= j\omega\mu_0 \frac{Q\,\mathrm{sgn}(v)}{2\pi} m_y\, \kappa\, K_1(\kappa a) e^{-j\omega t_0} e^{2j\omega t} \end{aligned} \quad (13)$$

which is identical to Eq. (10). This identity is a particular case of the Lorentz reciprocity theorem [6] (p. 64), [7] in reciprocal media, and confirms a previous statement by Goldberg and Lambertson [8]: the loop as a magnetic dipole can be used as a pick-up or as a kicker.

3.4 Transverse voltage seen by the particle

Again the transverse displacement of the particle is neglected, so that the total transverse momentum given to the particle is obtained by integrating the transverse force along the z-axis.The transverse force per unit charge along x is

$$F_x = E_x - v\mu_0 H_y\,.$$

The transverse voltage seen by the particle is defined as

$$V_x = \int_{-\infty}^{+\infty} F_x \frac{dz}{\beta}\,. \quad (14)$$

Using Eqs. (4), (5) and (8) it can be shown [5] that

$$V_x = -\frac{1}{jk}\frac{\partial V_{\|}}{\partial a} = -\frac{1}{jk}\frac{\partial V_{\|}}{\partial x} \quad (15)$$

which is simply the Panofsky–Wenzel theorem [8].

3.5 General orientation of the particle trajectory with respect to the dipole

One can keep the geometry of Fig. 2 and decompose $\vec{m}$ as

$$\vec{m} = m_x \vec{1}_x + m_y \vec{1}_y + m_z \vec{1}_z\,,$$

where $\vec{1}_x$ represents a unit vector along the x-axis.

As was just shown, m_y produces a longitudinal and transverse effect (along x) on the particle. It can also be shown that m_x produces only a transverse effect (along y), and that m_z does not produce any effect, neither longitudinal nor transverse.

4 ELECTRIC DIPOLE

We now consider an electric dipole of moment

$$\vec{p} = p_x \vec{1}_x + p_y \vec{1}_y + p_z \vec{1}_z$$

replacing $\vec{m}$ in Fig. 2.

4.1 *Longitudinal voltage seen by the particle*

The analogue [5] of Eq. (8) is, for the contribution of p_x:

$$V_{||}(\omega) = \frac{p_x}{2\pi\epsilon_0}\frac{j\omega}{v}\kappa K_1(\kappa a)e^{j\omega t_0} \quad (16)$$

and for the contribution of p_z:

$$V_{||}(\omega) = -\frac{p_z}{2\pi\epsilon_0}\kappa^2 K_0(\kappa a)e^{j\omega t_0} \quad (17)$$

whereas p_y does not contribute to $V_{||}$.
As long as $\kappa a < 1$, expression (16) behaves as a^{-1} and expression (17) as κ^2 ln $(0.89\ \kappa a)$.

Again, the Lorentz reciprocity theorem is fully verified.

4.2 *Transverse voltage seen by the particle*

This voltage can either be computed directly or by using the Panofsky–Wenzel theorem: p_x and p_z produce a transverse kick along x, whereas p_y produces a transverse kick along y.

5 POSSIBLE APPLICATIONS

So far, most structures used to interact with a charged particle beam are designed in such a way that the part of the surface to interact stays within a wavelength of the maximum frequency of interest. Examples are striplines (e.g. pick-ups for stochastic cooling), button beam-position monitors, and cavities in general. A somewhat different case is the free electron laser, where the wavelength of the radiation emitted amounts often to only a small fraction of the aperture. As there is a general tendency to proceed towards higher frequencies for accelerator applications, it usually implies small beam-pipe diameters with rather high transverse coupling impedances (e.g. CLIC). Also systems for bunched-beam stochastic cooling in the range above 10 GHz have been mentioned in discussions. The development of broadband pick-up and kicker structures up to 30 GHz should not pose any basic technological problem and appears to be of particular interest for bunched-beam stochastic cooling [9].

For high γ-beams one may consider the use of very small button pick-ups for beam diagnostics with bandwidth exceeding 50 GHz in a beam pipe of several centimetre diameter. From a technological point of view, there is no major obstacle to this proposal as all the technological ingredients already exist (very wideband hermetic feedthroughs, coaxial connectors from DC to >50 GHz, cables and signal treatment equipment like sampling scopes in the range 0–50 GHz).

The confocal resonator has been proposed [10] as a resonant beam-position and intensity monitor. The axis of this resonator would be orthogonal to the particle beam and one may consider both fundamental and higher-order-mode microwave beams. The R/Q of such an open cavity used as a microwave pick-up would be rather small ($\ll 1\Omega$) and hence also the loss factor. However, coupling impedances in the order of a few Ω may be obtained owing to the high Q-value of the device.

6 CONCLUSION

The theorem of reciprocity is applicable to the interaction of a charged particle beam with generalized structures also beyond a distance of a few wavelengths. In order to obtain a sizeable interaction between a surface impedance and a particle moving parallel to this surface at a distance $a > \lambda$, high $\beta\gamma$ values are required so that the voltage seen by the particle only decreases as a^{-1}. This may be visualized in the time domain by looking at the shape of the field 'slice' of a relativistic particle. The field 'slice' represents approximately a locally homogeneous, radially polarized, pulse-shaped electromagnetic wave. The interaction of this kind of electromagnetic field with various structures is well known from other fields of research, namely the nuclear electromagnetic pulse [11], where the ground equipment acts as a huge pick-up. On the other hand, some short electromagnetic pulses called 'electromagnetic missiles' [12] are possible examples of kickers which could act at long distances. A homogeneous plane wave may also have a net effect on a particle if it is limited in time.

Acknowledgements

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