Higher Order Modes Interaction with Multi-bunch Trains in Accelerating Structures

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Abstract

Time domain analysis of interaction of not fully relativistic multi-bunch beams with the accelerating and the higher order modes of a cavity is presented. This fundamental problem in the design of RF guns or FEL and Collider injector cavities is studied by a simple model that couples Newton and Maxwell equations, taking into account also space charge, RF focusing, beam loading and build-up effects of higher order modes under beam-tube cutoff frequency. It uses a current density description of the beam and a slowly varying envelope approximation (SVEA) for the time evolution of the modes amplitude. A fast running code (HOMDYN), based on this model has been developed: examples of application are illustrated.

1. INTRODUCTION

The mathematical model of the interaction of high charge, not fully relativistic bunch trains with accelerating structures, that we are going to illustrate, has already been introduced [1].

With respect to the former work we have added the field excitation term due to a localized antenna in order to study propagation effects inside the cavity during the filling and re-filling from bunch to bunch passage. Moreover the code has been tested in comparison with the results given by the reference PIC code ITACA [2].

The code HOMDYN now allows to follow the evolution of both the longitudinal and transverse dimensions of each bunch in a train and the energy spread of each bunch and of the whole train, taking into account space charge fields and the interaction with the fundamental and higher order modes of the accelerating structure. We recall that the model uses a current density description of the beam and supposes the envelope of the cavity modes to be varying slowly with respect to their period. This approximation leads to only first order equations for the field amplitude, thus reducing the numerical computing time. Motion and field equations are coupled together through the driving current term.

The present version deals only with TM monopole modes: an extension to dipole modes is under development.

This code can be advantageously applied to the treatment of RF guns and capture sections where the beam is not fully relativistic.

2. THE FIELD EQUATIONS

Expressing the longitudinal component of the electric field on the cavity axis [3]

$$E_z(z,t) = \sum_n A_n(t)\,\hat{e}_n(z)\,\sin(\omega_n t + \phi_n(t))$$

$$= \sum_n \left(\alpha_n(t)\,\hat{\eta}_n(z)\,e^{i\omega_n t} + \alpha_n^*(t)\,\hat{\eta}_n^*(z)\,e^{-i\omega_n t} \right)$$

as a sum of normal orthogonal modes $\hat{\eta}_n(z)=\hat{e}_n(z)/i$ where $\hat{e}_n(z)=\hat{e}_n(r=0,z)$, the equation for the electric field complex amplitude $\alpha_n(t)=(A_n/2)e^{i\phi_n}$ driven by a beam current distribution $J(z,t)$ in the cavity is :

$$\frac{d\alpha_n}{dt} + \left(1+\frac{i}{2Q_n}\right)\frac{\omega_n}{2Q_n}\alpha_n =$$

$$= -\frac{e^{-i\omega_n t}}{2\omega_n \varepsilon}\left(1+\frac{i}{2Q_n}\right)\int_v \left(\frac{\partial J}{\partial t}\bullet\hat{e}_n\right)dv \; +$$

$$-\frac{1}{\omega_n}\left(1+\frac{i}{2Q_n}\right)K_n\,e^{i(\Omega_{1,n}t+\psi_n)}$$

where we neglect the second order derivative according to the SVEA approximation hypotheses. The last term accounts for a feeding sinusoidal current representing a RF generator at a specific location z_g on the cavity axis which drives all the resonant modes with an incident amplitude $|\alpha_1|^g$, phase ψ_n, detuning shift $\Omega_{1,n}=(\omega_1-\omega_n)$ and coupling proportional to the ratio of any form factor $\hat{e}_n(z_g)$ to the fundamental (n=1) form factor $\hat{e}_1(z_g)$ on the coupler position:

$$K_n(z_g) = \frac{\omega_1^2}{2Q_1}\left|\alpha_1\right|^g\frac{\hat{e}_n(z_g)}{\hat{e}_1(z_g)}$$

The beam current density term can be written as follows:

$$\int_v \left(\frac{\partial J}{\partial t}\bullet\hat{e}_n\right)dv = \frac{q\beta_b c}{L}\left(\hat{e}_n(z_h)\frac{dz_h}{dt} - \hat{e}_n(z_t)\frac{dz_t}{dt}\right) +$$

$$+\frac{qc}{2}\left(\hat{e}_n(z_h)+\hat{e}_n(z_t)\right)\frac{d\beta_b}{dt}$$

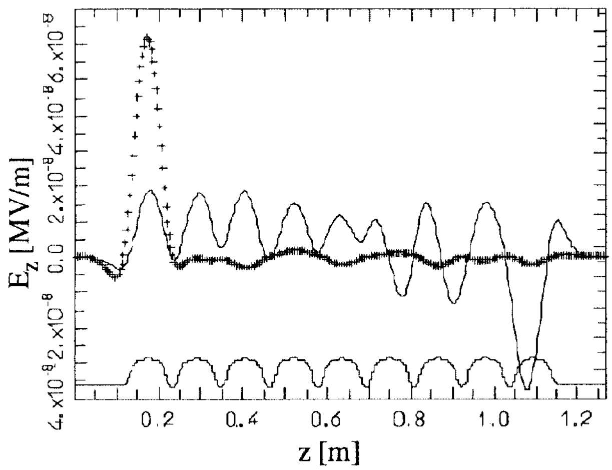

Figure 1. Field seen by two bunches propagating in the cavity. Immediately after the shock excitation has been turned off (+++ line) and after 130 nsec (solid line).

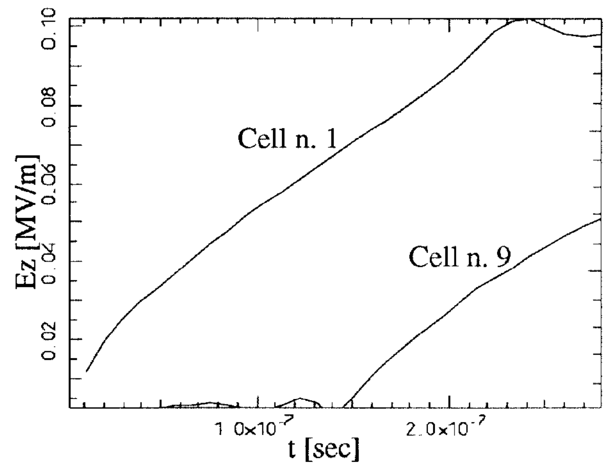

Figure 2. Peak field in cell n.1 and n.9 versus time

where the indexes b,h,t refer to bunch barycenter, head and tail position respectively.

The evolution of the field amplitude during the bunch to bunch interval is given by the analytical solution of the equation driven by the generator only, which connects successive numerical integrations applied during any bunch transit.

Taking into account the superposition of the resonant normal modes it is possible to simulate the transient behavior of a RF pulse which propagates from the generator location through a multi-cell cavity, before reaching a steady state corresponding to a standing wave pattern. An example of a shock excitation of a cavity (TESLA design) and the consequent pulse propagation towards the last cell is shown in figure 1 while the filling delay of the last cell during a continuos filling from the first cell is shown in figure 2. One can notice in the second case that the time delay is Δt=110 ns corresponding to a wave front traveling time in the cavity with the group velocity of the middle pass band mode ($\pi/2$), see ref. [4].

3. THE BEAM EQUATIONS

The basic approximation in the description of beam dynamics lays in the assumption that each bunch is represented as a uniform charged cylinder, whose length and radius can vary under a self-similar time evolution, i.e. keeping anyway uniform the charge distribution inside the bunch. The present choice of a uniform distribution is dictated just by sake of simplicity in the calculation of space charge and HOM contributions to the beam dynamics. The longitudinal space charge field at a distance d from the bunch tail, see figure 6, is given by [5]:

$$E_z^{sc}(d)=\frac{q}{2\pi\varepsilon_o\gamma_b R^2 L}\left(\sqrt{\gamma_b^2(L-d)^2+R^2}+\right.$$

$$\left.+\gamma_b(2d-L)-\sqrt{(\gamma_b d)^2+R^2}\right)$$

A further improvement of the model to include gaussian distributed bunches is under way. According to this assumption, and to the general hypothesis that the space charge and HOM effects on beam dynamics are perturbative, we can write, under a paraxial approximation, the equations for the longitudinal motion of the bunch barycenter:

$$\frac{dz_b}{dt}=\beta_b c \qquad \frac{d\beta_b}{dt}=\frac{e}{m_o c\gamma_b^3}E_z(z_b,t)$$

The evolution of the bunch radius R is described according to the envelope equation [6], including RF-focusing (first term), space charge effects (second), thermal emittance (third), damping due to acceleration (fourth) and solenoidal lens (fifth), transformed into the time-domain:

$$\frac{d^2R}{dt^2}=-\frac{e}{2\gamma_b m_o c}\left[c\frac{\partial E_z(0,z,t)}{\partial z}+\beta_b\frac{\partial E_z(0,z,t)}{\partial t}\right]+$$

$$+\frac{2Ic^2}{I_A\beta_b\gamma_b^3 R}+\frac{(\varepsilon_n c)^2}{\gamma_b^2 R^3}-\beta_b\gamma_b^2\frac{d\beta_b}{dt}\frac{dR}{dt}-\left[\frac{eB_z(0,z)}{2m_o\gamma_b}\right]^2 R$$

where I is the peak current, I_A is the Alfven current (17 kA), ε_n the rms normalized beam emittance and the RF focusing force $F^{rf}=e[E_r(r,z,t)-\beta c B_\theta(r,z,t)]$ has been expressed through the linear expansion off-axis of the $E_z(o,z,t)$ field.

The energy spread inside the bunch is derived by specifying the energy associated with a slice located at a position z_d and adding to the first order component coming from fundamental and HOM modes, the space charge effects:

$$\frac{d\gamma_d}{dt}=\frac{e}{m_o c}\beta_d\left[E_z(z_d,t)+E_z^{sc}(d,t)\right]$$

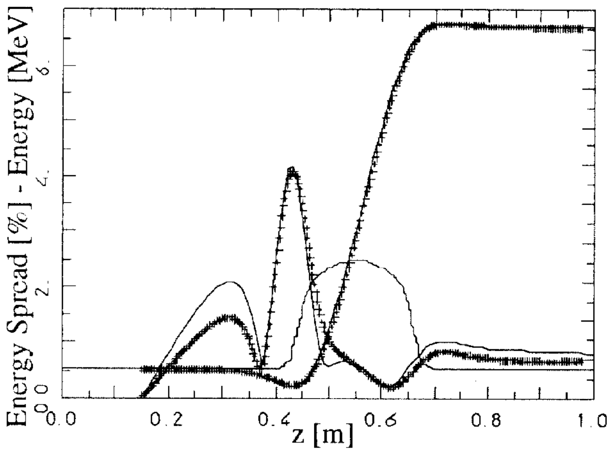

Figure 3. Bunch Energy and Energy spread

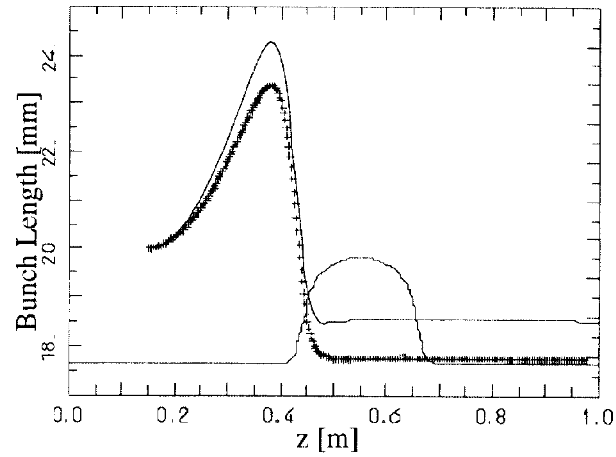

Figure 4. Bunch Length

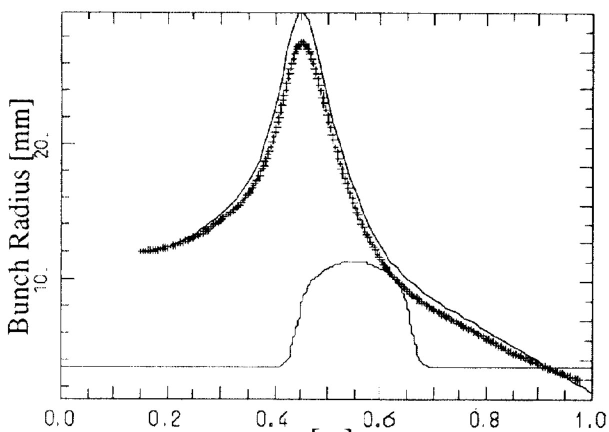

Figure 5. Bunch Radius

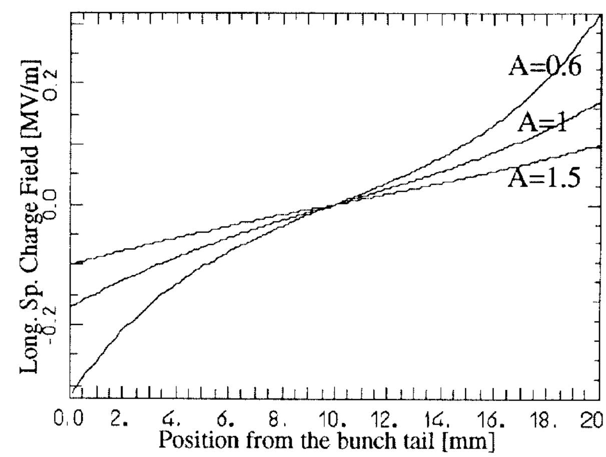

Figure 6. Longitudinal Space Charge Field in the bunch

The bunch lengthening is simply given by the head-tail velocity difference:

$$\frac{dL}{dt} = c\left(\beta_h - \beta_t\right)$$

To test the validity of our model we have considered the case of a 500 MHz cell resonator, computing the mode frequencies and field distributions by SUPERFISH code. The results obtained by HOMDYN (solid lines in the figures) are compared with the PIC code ITACA (+++ lines) for a single bunch passage, see figures 3-5. Multi-bunch computations are reported in reference [1]. The simulation parameters are the following: accelerating field 6.5 MV/m, bunch starting energy 500 KeV with no energy spread, bunch charge 10 nC, initial bunch length 20 mm and radius 12 mm. Notice that in the figures the cavity shape is also indicated. In figure 6 the longitudinal space charge field intensity is plotted along the bunch length for different value of the aspect ratio A=R/L with L=20 mm; one can notice that for A<1 the space charge effect becomes non linear far from the bunch barycenter.

4. REFERENCES

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