# A 3rd Harmonic Cavity for $DA\Phi NE$

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#### Abstract

In order to control bunch length in the DAΦNE main rings [1] an active 3rd harmonic RF cavity has been proposed. The energy acceptance, bunch length and Landau damping of the multibunch instabilities with the cavity are discussed. Alternative cavity shapes optimized with the aim of reducing the number of dangerous HOMs and increasing the fundamental mode shunt resistance are presented.

### 1. INTRODUCTION

The longitudinal size of a single bunch in storage rings is of great importance for reaching high luminosity. In order to avoid geometrical luminosity reductions the bunch length has to be shorter than  $\beta_y/1.5$  [2], where  $\beta_y$  is the vertical betatron function at the interaction point. On the other hand, a too short bunch produces strong parasitic losses, fast multibunch instabilities, and short Touschek lifetime. For DA $\Phi$ NE, a bunch length of 3 cm has been chosen as a reasonable compromise.

Preliminary calculations on the bunch lengthening have shown that the final bunch length in DA $\Phi$ NE is less than 3 cm; therefore additional lengthening is necessary [3]. A third harmonic cavity is planned to be installed in the DA $\Phi$ NE storage rings for this purpose.

In this paper we investigate the influence of the third harmonic cavity on the beam dynamics. The bunch lengthening (shortening) in the double RF system is computed taking into account the longitudinal broad-band impedance calculated elsewhere [3]. A simple analytical formula to estimate the enhancement of Landau damping of multibunch instability due to the cavity is derived. The last section is devoted to the discussion of possible cavity shapes with particular attention to the minimization of both the dissipated RF power and the number of dangerous HOMs.

## 2. BUNCH LENGTH

The potential seen by a particle in a double RF system, consisting of a main RF cavity and a  $n^{th}$  harmonic cavity, is given by [4, 5, 6]:

$$\varphi(\tau) = \frac{\alpha_c}{(E/e)T_o} \frac{V_g}{\omega} \left[ \sin(\varphi_{so}) \pm \frac{k}{n} - \sin(\varphi_{so} - \omega\tau) \mp \right]$$

$$\mp \frac{k}{n} \cos(n\omega\tau) - (U_o/e)\tau$$
(1)

with  $\alpha_c$  the momentum compaction, *E* the nominal energy of the particle,  $T_o$  the revolution period,  $\hat{V}_g$  peak voltage in the main RF cavity,  $\mathbf{k} \, \hat{V}_g$  the peak voltage in the harmonic cavity,  $U_o$  the energy loss per turn. In order to provide phase stability the synchronous phase  $\varphi_{so}$  must satisfy the condition:

$$\mp \frac{kn}{\sin\varphi_{so}} < 1 \tag{2}$$

The potential (1) is written for the case when the harmonic cavity does not accelerate synchronous particles and has a maximum voltage slope at  $\varphi = \varphi_{s0}$  to have the most effective bunch length control. The upper sign in (1) and in the following refers to the shortening regime, while the lower one is used for the bunch lengthening case.

Applying the general theory of RF acceleration [7], we find that the relative change of the momentum acceptance due to the third harmonic cavity in a storage ring ( $\varphi_{SO} \sim \pi/2$ ):

$$\frac{\Delta E_{double}}{\Delta E_{main}} = \sqrt{1 \pm \frac{k}{3}} \tag{3}$$

does not exceed  $\pm 6\%$  if k < 1/3. Here  $\Delta E_{double}$  and  $\Delta E_{main}$  are the heights of the bucket in the double RF system and in the single one, respectively.

The "natural" bunch length in the double RF system is computed by solving Haissinski's equation [8] for the potential (1) with zero broad-band impedance. In the linear approximation and expressing the voltage of the harmonic cavity in terms of the supplied power P and the shunt impedance of the fundamental mode  $R_S$  the rms bunch length is:

$$\sigma = \sigma_o / \sqrt{1 \pm \frac{n\sqrt{2R_sP}}{\hat{V}_g \sin(\varphi_{so})}} \tag{4}$$

In Fig. 1 we plot the normalized bunch length versus k in the case of a third harmonic cavity at different values of  $\sigma_0$ . As obtained in the linear approximation, for short bunches the ratio  $\sigma/\sigma_0$  does not depend on  $\sigma_0$ , while for longer bunches  $\sigma/\sigma_0$  is rather sensitive to  $\sigma_0$ . Nevertheless, in the range of interest defined by (2) the approximation is quite accurate.



Figure 1. Bunch length versus k.

The bunch length (4) has been used as starting point to evaluate the bunch lengthening with the longitudinal impedance estimated for DA $\Phi$ NE. Above the turbulent threshold the bunch length was calculated by solving Haissinski's equation and applying Boussard's criterion [9] as a scaling law as described in [10]. Figure 3 shows the bunch length versus power for different values of the shunt impedance.



Figure 2. Bunch length versus dissipated power.

#### 3. LANDAU DAMPING

The higher harmonic cavity introduces some additional non-linearities on the restoring force, which increase the synchrotron frequency spread, thus stabilizing the multibunch coherent oscillations through the Landau damping mechanism.

In order to estimate this effect, we use the Vlasov's equation linearized with respect to small perturbations of the stationary distribution [11,12] and get the dispersion integral:

$$1 = \frac{jm\alpha_c I_b}{(E/e)\omega_s(0)} \frac{Z(p)}{p} \int_o^\infty \frac{\partial g(\hat{\tau})}{\partial \hat{\tau}} \frac{J_m^2[p\omega_o \hat{\tau}]}{[\omega_{cm} - m\omega_s(\hat{\tau})]} d\hat{\tau} \quad (5)$$

where *m* is the mode number of oscillation, I<sub>b</sub> the average beam current,  $\omega_s(\hat{\tau})$  the incoherent synchrotron frequency,  $\hat{\tau}$ the amplitude of the synchrotron oscillation, Z(p) the longitudinal HOM shunt impedance at the frequency  $p\omega_o + m\omega_{cm}$ , *p* an integer number,  $\omega_o$  the revolution angular frequency,  $\omega_{cm}$  the coherent synchrotron frequency,  $g(\hat{\tau})$  the stationary distribution in phase space, and  $J_m$  the Bessel function of the first kind of the mth order.

The dependence  $\omega_{s}(\hat{\tau})$  is found by solving the equation of motion of a single particle in the potential (1). Using a perturbation method [13] and limiting ourselves to the first approximation which is suitable for the DA $\Phi$ NE parameters we obtain:

$$\omega_{s}(\hat{\tau}) = \omega_{so}\sqrt{1 \pm \frac{kn}{\sin\varphi_{so}}} \left\{ 1 - \frac{0.0625(\omega\hat{\tau})^{2}}{\left(1 \pm \frac{kn}{\sin\varphi_{so}}\right)} \left(1 \pm \frac{kn^{3}}{\sin\varphi_{so}}\right) \right\}$$

By substituting  $\omega_s(\hat{\tau})$  in the dispersion integral we get for the Gaussian stationary distribution:

$$\frac{Z(p)}{p} = F \frac{E\pi\omega_{so}^2 \sigma^2 \omega^2}{4eI_b \alpha_c} \left\{ \frac{\pm \pi e^{-y} J_m^2 \left[ p\omega_o \sigma \sqrt{2y} \right] + j \mathfrak{I}_{pv}}{\mathfrak{I}_{pv}^2 + \left( \pi e^{-y} J_m^2 \left[ p\omega_o \sigma \sqrt{2y} \right] \right)^2} \right\}$$
(6)

where  $\Im_{pv}$  is the principal value of the integral:

$$\mathfrak{I}_{pv} = PV \int_0^\infty \frac{e^{-x} J_m^2 \left[ p \omega_0 \sigma \sqrt{2x} \right]}{x - y} dx \text{ , and } F = 1 \pm \frac{kn^3}{\sin \varphi_{so}}$$

and y a function depending on  $\omega_{cm}$ . The sign + or - in the double bracket depends on whether F is positive or negative respectively. Examining eq.(6) we find that, in presence of a higher harmonic cavity, there is an enhancement of the Landau damping by a factor F, whose maximum value in the lengthening regime is  $n^2 - 1$ .

# 4. CAVITY DESIGN

We describe two basic alternative designs for the higher harmonic cavity, which were explored to fulfill different requirements, i. e.:

1) 'Single-mode' cavity, to have minimum contribution to the HOM impedance in DA $\Phi$ NE.

2) High shunt impedance cavity, either of the 'nosecone' or of the 'rounded' type, to keep RF power to a minimum.

We remind that the beam pipe aperture is quite large (a radius of 4.3 cm), and does not allow us to push the  $R_s$  of the fundamental mode to a very big value at the third harmonic frequency. In this work we used the well-known codes URMEL [14] and TBCI [15].

# 4.1. Nosecone cavity

A 'nosecone' design was considered as a possible solution to have a true single mode cavity (see Fig. 3).



Figure 3. Nosecone 'single-mode' cavity.

We studied the cell shape for several gap values, keeping the frequency of the fundamental mode at 1104 MHz. The cell is set directly on the pipe and the elliptical profile helps to shift the two first HOMs above cutoff, while the nose radius was chosen large enough not to have a too high surface electric field. In this way something is lost on the  $R_5/Q$  but all the monopole modes look free to propagate down the vacuum chamber. It can be generally said that only the first dipole mode TM<sub>110</sub> is left in such a cavity. The other dipole mode TE<sub>111</sub> practically disappears (see Table 1).

Construction and cooling of such a cavity may be difficult, due to its rather complicated geometry and reduced size. Tolerances for a construction error of  $\pm 0.1$  mm have been calculated by simulation for the most critical positions [16]. The cavity does not appear too sensitive from this point of view. The maximum tuner sensitivity calculated assuming a cylindrical tuner of 1.5 cm radius is given in Table 1.

Table 1 - Summary resu	ilts for the cavities
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	Nosecone	Nosecone	Rounded
	(gap=15mm)	High Rs	High Rs
Frequency (MHz)	1104.57	1104.59	1104.71
$R_{s}/O(\Omega)$	33.38	58.73	65.36
$R_{s}(M\Omega)$	0.335	1.96	1.85
$V_{gap}(kV)$	75	75	75
W (mJoule)	12.13	6.2	6.9
$E_s \max{(MV/m)}$	9.5	2.8	1.97
B <sub>s</sub> max (Gauss)	114	29	30.6
Tuner sensitivity (mm/MHz)	0.6	4.7	4.8
k [V/pC]	0.0699	0.171	0.157
k <sub>pm</sub> [V/pC]	0.000	0.0453	0.017
TM011 mode:			
Frequency (MHz)	propagating	1839.61	1916.26
$R_{s}(k\Omega)$		612.0	672.6
TM <sub>020</sub> mode:			
Frequency (MHz)	propagating	2429.	2342.
$R_{s}(k\Omega)$		4.7	7.6
TM <sub>110</sub> mode:		· .	
Frequency (MHz)	1601.	1644.	1604.
$R'_{s}(k\Omega)(at 4.3cm)$	266.	366.	299.
TE <sub>111</sub> mode:			
Frequency (MHz)	2020.	1453.7	1412.6
$R'_{s}(k\Omega)(at 4.3cm)$	0.416	84.6	150.3

#### 4.2. Rounded and Nosecone High $R_s$ cells

To minimize RF power, high  $R_s$  structures were considered too. First, a rounded profile (Fig. 4) was chosen with the optimum value  $a/h \sim 1$  [17], obtaining  $R_s = 1.85 \text{ M}\Omega$  and 4 (monopole + dipole) trapped modes, which have to be damped anyway.



Figure 4. Rounded high Rs cell.

Second, a quite optimized nosecone cell (Fig. 5) was accurately studied by re-adjusting the gap, to get maximum  $R_S/Q$  and simultaneously not to decrease Q. The resulting  $R_S$  is 1.96 M $\Omega$  at 1104 MHz. Also in this case 4 parasitic modes are left in the cavity. The cavities are less sensitive to construction errors than the nosecone cavity described in 4.1.



Figure 5. Nosecone high  $R_s$  cell.

All the described cavities comply (within at least a factor 10) with Kilpatrick's criterion on the maximum surface electric field [18]. A summary of the most important results is reported in Table 1.

### 5. CONCLUSIONS

Bunch lengthening in DA $\Phi$ NE with the double RF system has been studied. We have shown that a shunt impedance of  $R_S = 0.3+2$  M $\Omega$  allows to keep the bunch length in DA $\Phi$ NE under control with reasonable RF power (1+7 kW) while variations of the momentum acceptance do not exceed  $\pm 6\%$ . All the cavities shown have the necessary shunt impedance. The final choice will be made after careful analysis of the means to damp undesired HOMs. We also found that a high harmonic cavity gives an enhancement of the Landau damping effect by a factor  $(1 \pm kn^3 / \sin \varphi_{so})$ , beneficial for damping the multibunch instability.

### 6. REFERENCES

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