Theory of Multi-Bunch Resistive Wall Instability Damping using Feedback System with a Digital Filter

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Abstract

Theory of multi-bunch resistive wall instability damping using a feedback system with a digital filter is developed. The system of coupling equations is obtained for description of bunched beam motion including the wake fields for beam and the residual currents in feedback devices. To solve equations, the Z-transform method and special coordinates transformation are used. The general solutions and eigen frequencies are found. The influence of feedback frequency band on the tune shift is discussed.

1 INTRODUCTION

Transverse feedback systems (TFS) are used in synchrotrons to damp the coherent transverse beam oscillations. In these systems the kicker (DK) corrects the beam angle according to the beam deviation from the closed orbit in the pick-up (PU) location at each turn. A classical TFS consists of one PU and one DK per plane. These systems have been widely used and provide an amplitude decrease up to 25% per revolution [1]. In order to suppress fast resistive wall instability in UNK-1 (Protvino, Russia) [2], a more effective system is studied and developed [3]. It consists of two PUs and two DKs per plane connected by a feedback circuit with a digital filter and delay. This paper is based on studies of TFS for UNK-1 [3] and LHC [4], and the influence of the kicker parameters on multi-bunch instability damping is analysed.

2 THEORY

The description is based on the theory [5] of multi-bunch resistive wall instability damping where a standard model [7] is used for a wake field dependent only on the distance between bunches.

2.1 Basic Equations

In accordance with [5], the bunch dynamic equation of the transverse coherent motion for the deviation from the closed orbit $x_k[n,s]$ at the n-th turn for the k-th bunch can be written as

$$\left[\frac{d^2}{ds^2}+K(s)\right]x_k[n,s] = = -\frac{2Q_0}{R^2}f_k[n,s]+\Delta x'_k[n]\delta(s-s_K), \quad (1)$$

where $K(s)$ is a focusing strength, Q_0 is a machine tune, $R=C_0/2\pi$, and C_0 is a circumference of the closed orbit. The wake field force [5] is

$$f_k[n+1,s] = \Delta Q_{0k}x_k[n+1,s] + + \sum_{m=0}^{n+1}\sum_{j=1}^{k-1}\Delta Q_{kj}[m,\Delta s_{kj}]x_j[n+1-m,s] + + \sum_{m=0}^{n}\sum_{j=k}^{M}\Delta Q_{kj}[m,\Delta s_{kj}]x_j[n-m,s], \quad (2)$$

where M is a number of bunches. Coefficients ΔQ_{kj} in accordance with [7] are

$$\Delta Q_{kj}[n,\Delta s_{kj}] = -2\nu_b\sum_{m=1}^{\infty}h_m\exp\left(-\left(n+\frac{\Delta s_{kj}}{C_0}\right)\alpha_m\right), \quad (3)$$

where α_m and h_m are defined in [7, 5]; Δs_{kj} is a distance between bunches. The parameter ν_b is

$$\nu_b = \frac{r_pN_b}{2\pi\gamma_0}\frac{\beta_c}{b^2}; \qquad r_p=\frac{Z^2e^2}{4\pi\epsilon_0Amc^2},$$

where N_b is a number of particles per bunch; b is a vacuum tube radius; β_c is a value of average β-function. Because $0\le s\le C_0$, then due to the periodicity in the circular accelerator the following equation can be written

$$\widehat{X}_k[n,s+C_0] = \widehat{X}_k[n+1,s], \quad (4)$$

where $\widehat{X}_k$ is a column matrix in which the first element is $x_k[n,s]$ and the second one is $x'_k[n,s]$.

Equation (2) has a general form for a wake field force. A simpler model will be used further. It will be assumed that the distances between bunches are equal. In this case

$$\frac{\Delta s_{kj}}{C_0} = \frac{k-j}{M}, \qquad j<k,$$
$$\frac{\Delta s_{kj}}{C_0} = \frac{M+k-j}{M}, \qquad j\ge k. \quad (5)$$

2.2 Feedback

A feedback path consists of a pick-up, a digital filter and amplifiers with a gain $\mathbf{K}_a$, and a kicker with inductance L_K and resistance R_K. The voltage after the digital filter and amplifiers is proportional to the bunch deviation:

$$V_k[n] = S_P\sum_{m=0}^{n}\mathbf{K}_a[m]x_k[n-m,s_P]. \quad (6)$$

For this k-th bunch at n-th turn the beam angle is changed after a kicker by

$$\Delta x'_k[n] = \frac{S_K}{\sqrt{\beta_K \beta_P}} R_K I_k[n], \tag{7}$$

where the amplitude of the current pulse in the kicker is

$$I_k[n+1] = \frac{f_0}{R_K} V_k[n+1] + \frac{f_1}{R_K} \left[\sum_{m=0}^{n+1} \sum_{j=1}^{k-1} V_j[m] \times \right.$$
$$\times \exp\left(-\frac{k-j}{M}\alpha_K - (n+1-m)\alpha_K \right) -$$
$$- \sum_{m=0}^{n} \sum_{j=k}^{M} V_j[m] \times$$
$$\left. \times \exp\left(-\frac{M+k-j}{M}\alpha_K - (n-m)\alpha_K \right) \right]. \tag{8}$$

Here

$$f_0 = 1 - \exp\left(-\frac{\delta\tau}{\tau_K}\right); \qquad \tau_K = \frac{L_K}{R_K};$$
$$f_1 = \exp\left(\frac{\tau - \delta\tau}{\tau_K}\right) - \exp\left(-\frac{\delta\tau}{\tau_K}\right); \qquad \alpha_K = \frac{T_0}{\tau_K}; \tag{9}$$

$\tau \le T_0/M$ is the current pulse duration in the kicker, and $\delta\tau \le \tau$ is a delay between a voltage jump and a kick.

2.3 General Solution

The Z-transform approach is used to obtain a general solution. This transformation is

$$\tilde{f}(z,s) = \sum_{n=0}^{\infty} f[n,s] z^{-n}.$$

It gives for (3) and (4):

$$\Delta\tilde{Q}_{kj}(z, \Delta s_{kj}) =$$
$$-2\nu_b \sum_{m=1}^{\infty} \frac{z\, h_m}{z - \exp(-\alpha_m)} \exp\left(-\frac{\Delta s_{kj}}{C_0}\alpha_m\right); \tag{10}$$
$$\hat{\mathbf{X}}_k(z, s + C_0) = z\hat{\mathbf{X}}_k(z,s) - z\hat{X}_k[0,s]. \tag{11}$$

Here $\hat{X}_k[0,s]$ is a column matrix determining the k-th bunch initial state after injection. A column matrix $\hat{\mathbf{X}}_k(z,s)$ consists of the elements $\tilde{x}_k(z,s)$ and $\tilde{x}'_k(z,s)$. Let us introduce a column matrix $\hat{Y}(z,s)$ with M elements $\tilde{x}_k$. After Z-transformation in (2) and in (6,7,8) the Eq.(1) can be written as

$$\left[\left(\frac{d^2}{ds^2} + K(s) + \frac{2Q_0 \Delta Q_0}{R^2} \right) \hat{I} - \right.$$
$$\left. - \frac{4Q_0\nu_b}{R^2} \sum_{m=1}^{\infty} \frac{\hat{q}(\alpha_m)\, h_m}{z - \exp(-\alpha_m)} \right] \hat{\mathbf{Y}}(z,s) =$$
$$= \frac{S_K S_P \tilde{\mathbf{K}}_a(z)}{\sqrt{\beta_K \beta_P}} \left[f_0 \hat{I} + \right.$$
$$\left. + \frac{\hat{q}(\alpha_K)\, f_1}{z - \exp(-\alpha_K)} \right] \hat{\mathbf{Y}}(z, s_P) \delta(s - s_K). \tag{12}$$

Here $\hat{q}(\alpha)$ is a matrix $M \times M$ with elements

$$(\hat{q}(\alpha))_{kj} = z \exp\left(-\frac{k-j}{M}\alpha\right), \quad j < k;$$
$$(\hat{q}(\alpha))_{kj} = \exp\left(-\frac{M+k-j}{M}\alpha\right), \quad j \ge k. \tag{13}$$

Let us transform $\tilde{x}_k$ to new variables $\tilde{v}_n$ being elements of a column matrix $\widehat{\mathbf{W}}(z,s)$:

$$\widehat{\mathbf{W}}(z,s) = \hat{F}\hat{\mathbf{Y}}(z,s); \quad F_{nk} = \exp\left(-i\frac{2\pi k}{M}\left(n - i\frac{\ln z}{2\pi}\right)\right).$$

After this transformation in Eq.(12) the system of independent M equations for $\tilde{v}_n$ is obtained:

$$\left[\frac{d^2}{ds^2} + K(s) + \frac{2Q_0}{R^2}\left(\Delta Q_0 - \right.\right.$$
$$\left.\left. -2\nu_b \sum_{m=1}^{\infty} \frac{h_m}{\exp(i\frac{2\pi n}{M} + \frac{\alpha_m + \ln z}{M}) - 1} \right)\right] \tilde{v}_n =$$
$$= \frac{\tilde{\mathbf{K}}(z)}{\sqrt{\beta_K \beta_P}} \tilde{v}_n(z, s_P)\delta(s - s_K), \tag{14}$$

where

$$\tilde{\mathbf{K}}(z) = S_P S_K \tilde{\mathbf{K}}_a(z) \left(f_0 + \right.$$
$$\left. + \frac{f_1}{\exp(i\frac{2\pi n}{M} + \frac{\alpha_K + \ln z}{M}) - 1} \right). \tag{15}$$

The gain [6] for a digital filter of the first order and a wide-band amplifier with a gain $|\mathbf{K}|$ is

$$S_P S_K \tilde{\mathbf{K}}_a(z) = \frac{z + a_1}{z - b_1} |\mathbf{K}|. \tag{16}$$

Eqs. (14) are the same as in [4] for a coasting beam. The solution is fully determined by eigenvalues $z_{(n)k}$ that can be found from equation

$$z_{(n)k}^2 - z_{(n)k} \mathrm{Tr}\widehat{\mathbf{M}}_n(z_{(n)k}) + \det \widehat{\mathbf{M}}_n(z_{(n)k}) = 0, \tag{17}$$

where

$$\widehat{\mathbf{M}}_n(z) = \widehat{\mathbf{M}}_{0n}(z) + \frac{\tilde{\mathbf{K}}(z)}{\sqrt{\beta_K \beta_P}} \widehat{\mathbf{M}}_n(z; s_P + C_0, s_K)\hat{T}.$$

Here $\widehat{\mathbf{M}}_{0n}(z)$ is the unperturbed revolution matrix from point s_P; $\widehat{\mathbf{M}}_n(z; s_P + C_0, s_K)$ is the transfer matrix from s_K to $s_P + C_0$; $\hat{T}$ is 2×2 matrix in which $T_{21} = 1$ and the other elements are zero. For all these matrices the phase advances must be calculated for the tune Q_n that in accordance with (14) is

$$Q_n^2 = Q_0^2 + 2Q_0\Delta Q_0 -$$
$$-4Q_0\nu_b \sum_{m=1}^{\infty} \frac{h_m}{\exp(i\frac{2\pi n}{M} + \frac{\alpha_m + \ln z}{M}) - 1}. \tag{18}$$

The motion of the bunches is stable if

$$|z_{(n)k}| \le 1.$$

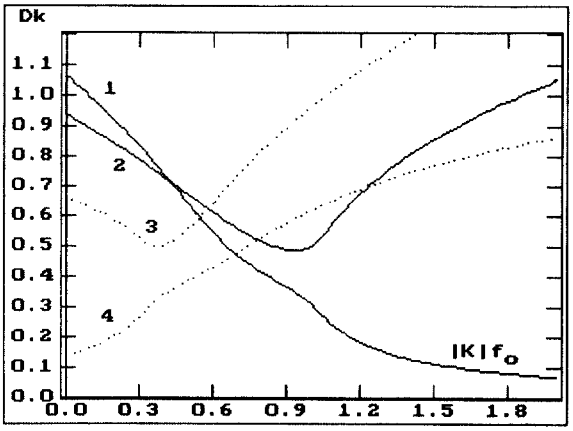

Figure 1: D_k dependencies on $|\mathbf{K}|f_0$.
$\mathrm{Re}Q_0 = 70.31$; $|\mathrm{Im}Q| = 0.01$; $\mathrm{Re}\psi_{PK} = 140.5\pi$.

3 RESULTS

It was found in [6] that for a classical feedback the Eq.(17) is

$$z^2 - \left(2\cos(2\pi Q_n) + \widetilde{\mathbf{K}}(z)\sin(2\pi Q_n - \psi_{PK})\right) z + \\ +1 - \widetilde{\mathbf{K}}(z)\sin\psi_{PK} = 0, \quad (19)$$

where ψ_{PK} is a phase advance from PU to DK. Multi-bunch instabilities due to Q_n dependencies (18) are analyzed in [5]. It was found that the well-known modes

$$\omega_n = (n \pm Q_n)\omega_0$$

correspond to such values of Q_n in (18) that

$$i2\pi n + \ln z = i\omega_n T_0. \quad (20)$$

The problem of a beam stability and the peculiarities of Eq.(19) solutions are discussed in [4, 6] for TFS with a digital filter and $L_K = 0$. But if $L_K > 0$, then additional criteria for the beam stability will have place. The beam stability will be analysed further for a kicker with $L_K > 0$.

3.1 Single Bunch Instability Damping

For a single bunch ($M = 1$) one obtains in (15):

$$\widetilde{\mathbf{K}}(z) = \frac{(z+a_1)[z + (f_1/f_0 - 1)\exp(-\alpha_K)]}{(z-b_1)[z - \exp(-\alpha_K)]}|\mathbf{K}|f_0.$$

So we get in (19) the equation of the fourth power for z_k. As a rule, $f_0 \simeq f_1$. Hence for the kicker with $L_K > 0$ the new solution z_4 of (19) corresponds to the solutions with recursive filter in feedback circuit. Fig.1 shows $D_k = |z_k|$ dependencies on $|\mathbf{K}|f_0$ when the phase advance from PU to DK is adjusted closely to an odd number of $\pi/2$ radians. The solid curves 1 and 2 correspond to the oscillations with the tune in the neighbourhood of Q_0. The dotted curve 3 corresponds to the new root determined by the IIR–filter parameters. The new oscillation mode (curve 4) is conditioned with $L_K > 0$. To provide the independence on $|\mathbf{K}|$ of the feedback action on the closed orbit displacement and for a better suppression of noise, it is necessary to set $a_1 = -1$ [4]. The parameter b_1 is chosen to optimize the damping factor ($b_1 = 0.66$ in Fig.1). Because D_k depends on $|\mathbf{K}|f_0$, the kicker parameters L_K and R_K are chosen to minimize the gain $|\mathbf{K}|$ and to increase f_0 up to 1. The influence of the fourth solution on the stability region is excluded by chosing $\exp(-\alpha_K)$ value (it equals 0.14 in Fig.1).

3.2 Multi-Bunch Instability Damping

Taking into account (20), for a large number of bunches we get:

$$\widetilde{\mathbf{K}}(z) = \frac{z+a_1}{z-b_1}\left(1 + \frac{M\tau_K}{(1+i\omega_k\tau_K)T_0}\right)|\mathbf{K}|f_0,$$

where $f_0 \simeq f_1$ is assumed. Hence the influence of L_K on a beam stability will be small if $M\tau_K < T_0$. In this case the stability criteria for a small $|\mathbf{K}|$ are

$$\frac{1}{2}|\mathbf{K}|f_0 \sin\left(\mathrm{Re}\psi_{PK} \pm \frac{\omega_k M\tau_K^2/T_0}{1 + M\tau_K/T_0}\right) > 2\pi|\mathrm{Im}Q|.$$

So a phase shift due to L_K must be taken into account.

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