Multi-Parameter Sorting of Dipoles for Large Superconducting Rings *

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Abstract

The presence of unavoidable multipole errors in superconducting magnets is known to be the main cause for limiting the dynamic aperture of large storage rings. Sorting of dipoles, in which dipoles are installed in the ring according to a certain sequence based on the measured multipole errors, is a way to reduce the adverse effects of random multipole errors without an increase in the cost. In this report, we present a multi-parameter sorting scheme which can systemically handle many multipoles simultaneously. Effectiveness of the sorting scheme has been studied for a test lattice which is similar to the high energy booster (HEB) of SSC. Advantage of the scheme for two-in-one magnets has been explained also with four uncorrelated normal and skew sextupole components in two rings.

1 MULTI-PARAMETER SORTING SCHEME

The magnetic field produced by magnet errors in a dipole can be represented by the multipole components (b_n, a_n) where (b_1, a_1) are normal and skew quadrupoles, (b_2, a_2) are normal and skew sextupoles, etc. In general, each a_n or b_n has two contributions: a systematic error which is the average of the error field and a random error which is the fluctuation of the error field from magnet to magnet. Since the systematic error is relatively easier to control, only the random contribution is considered in this work. Furthermore, random multipole components are assumed to be uncorrelated [1]. When the two-in-one design of the dipoles are considered, random multipole components in two rings are also assumed to be uncorrelated.

The idea of sorting is based on the fact that, if two magnets of approximately equal strength but opposite sign of the error are selected and placed one period of betatron oscillation apart (phase distance 2π), the adverse effects of field error can be substantially reduced [2]. If one particular multipole component is dominant, the selection can be done easily by using its value for sorting. When there are n equally important multipole components, however, the sorting of magnets must be done in such a way that effects of these n multipole components are taken into account simultaneously. For this purpose, a multi-dimensional vector is used to represent the n multipole components. The selection of magnets for sorting is then based on the Euclidean distance between the vectors. Since the effects of different order of multipoles can not be directly compared by their multipole coefficients, the nonlinear field arising from each multipole is used as the sorting parameter. The nonlinear field associated with the nth-order normal and skew multipoles at a phase-space location $x = y = x_0$ is characterized by $b_n x_0^n$ and $a_n x_0^n$ where x_0 will be used as a parameter to optimize the sorting. If the primary goal of sorting is to increase the dynamic aperture, the effectiveness of sorting should be peaked at x_0 near the dynamic aperture.

Let $\vec{S}^{(i)} = (S_1^{(i)}, ..., S_n^{(i)}, S_{n+1}^{(i)}, ..., S_{2n}^{(i)})$ denotes a $2n$-dimensional sorting vector where n is the highest order of multipoles considered and the superscript i specifies each magnet. Components of $\vec{S}^{(i)}$ are such that if b_k or a_k is included as a sorting parameter, $S_k^{(i)} = b_k x_0^k$ or $S_{n+k}^{(i)} = a_k x_0^k$, otherwise, $S_k^{(i)} = 0$ and $S_{n+k}^{(i)} = 0$. The strength of sorted error field of magnet i is defined by the Euclidean normal of $\vec{S}^{(i)}$,

$$|\vec{S}^{(i)}| = \sqrt{\sum_{k=1}^{2n} \left(S_k^{(i)}\right)^2}. \quad (1)$$

The strengths of normal multipole components S_b and skew multipole components S_a are defined, respectively, by

$$S_b = \sqrt{\sum_{k=1}^{n} \left(S_k^{(i)}\right)^2} \quad \text{and} \quad S_a = \sqrt{\sum_{k=1}^{n} \left(S_{n+k}^{(i)}\right)^2}. \quad (2)$$

Assume that there are N measured magnets available for installation at N consecutive locations with even N. These N magnets cover two periods of betatron oscillation. In our sorting scheme, the placement of magnets along the ring is done in the following manner.

(1) Cancellation of the error fields of each pair of magnets with (2π) phase separation.

We first group N magnets into $N/2$ pairs, $\left(\vec{S}^{(2i-1)}, \vec{S}^{(2i)}\right)$ for $i = 1, ..., N/2$, such that

$$|\vec{S}^{(2i-1)}| \geq |\vec{S}^{(2i+1)}| \quad (3)$$

and

$$\vec{S}^{(2i)} = \min_{l} \left(|\vec{S}^{(2i-1)} + \vec{S}^{(l)}|\right). \quad (4)$$

That is, magnets in each pair have almost the same strength but opposite sign of errors. They are therefore placed one period of betatron oscillation apart. Due to a finite number of magnets in a sorting group, $|\vec{S}^{(2i-1)} + \vec{S}^{(2i)}|$

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is in general not zero and only partial cancellation of the errors can be achieved. To characterize the degree of cancellation, we define

$$\left(\sum_{i=1}^{N/2} |\vec{S}^{(2i-1)} + \vec{S}^{(2i)}|\right) \Big/ \left(\sqrt{\sum_{k=1}^{N} \sigma_{S_k}^2}\right) \qquad (5)$$

as the residue of the cancellation with σ_{S_k} the rms value of the kth component of the sorting vector. As the number of magnets in each sorting group is increased, the residue decreases and the sorting becomes more effective. On the other hand, for a given number of magnets, the residue increases with the number of sorting parameters and the effectiveness of sorting will be diminished as more multipoles are included in the sorting.

(2) Local cancellation of the error field.

In order to minimize the effect of error field of each magnet, magnet with larger strength of normal (skew) multipole components should be placed near a quadrupole which is defocussing in the x (y) direction. Since there will be a partial cancellation of the error field locally if the sign of the errors in magnets alternates, the adjacent magnet to the magnet of $\vec{S}^{(2i-1)}$ is chosen to be

$$\vec{S}^{(j)} = \min_{l \neq 2i} \left(|\vec{S}^{(2i-1)} + \vec{S}^{(l)}|\right). \qquad (6)$$

To install all magnets in a ring, we repeat this procedure in each segment covering two periods of betatron oscillation. When one arc does not contain an even number of betatron oscillation periods, there will be leftover cells with less than two periods of betatron oscillation. For these unbalanced cells, we may select a number of "good" magnets in each sorting group which contains a few extra magnets. It should be noted that if a single multipole component is included in the sorting vector $\vec{S}^{(i)}$, our sorting scheme will be reduced to the original Gluckstern-Ohnuma one-parameter sorting scheme [2].

2 TEST LATTICE

The test lattice used in this work is quite similar to the high energy booster of SSC. The total number of regular cells is 116 and each cell contains four dipoles. In addition, we have included two long-straight and four short-straight sections to make the model more realistic. The phase advance of a regular cell is 90^o in both transverse directions. Sorting is thus performed for each group of dipoles in eight cells with $N = 32$. The horizontal and vertical tunes are 39.42489 and 38.41437, respectively. All quadrupoles were assumed to be perfect. The magnetic field errors in each dipole are represented by a set of thin-lens multipoles, located in the middle and at both ends. Multipole components have been chosen randomly with Gaussian distributions centered at zero and truncated at $\pm 3\sigma$ where σ is the rms value of each multipole. In Table 1, we list values of σ used in this study [3].

Tracking of particle motions has been done without synchrotron oscillations, momentum deviations, or closed orbit distortions. No physical aperture limit was imposed in the ring. To improve the statistical significance of our simulations, we have used a number of different samples of random multiple components, usually fifty, generated with different seed numbers in a random number generator routine. In order to facilitate a study of many cases and increase efficiency, we have developed our own tracking codes. Two codes have been made independently in order to test the reliability of each. In order to reduce the sensitivity of dynamic aperture to the choice of initial launch point in phase space, we define a dynamic aperture as the shortest distance from the origin in the four-dimensional normalized phase space during the tracking. The dynamic aperture defined in this manner is relatively insensitive to the choice of launch point.

Table 1: The rms values of random multipole errors in units of 10^{-4}cm^{-n}.

n	1	2	3	4	5	6	7
σ_{b_n}	0.5	1.15	0.16	0.22	0.017	0.018	0.01
σ_{a_n}	1.25	0.35	0.32	0.05	0.05	0.008	0.01

3 RESULTS FROM TRACKING CALCULATIONS

3.1 *Sextupole components (b_2, a_2) only.*

It is already known that sorting sextupole components will improve the quality of rings [4]. With σ_{b_2} and σ_{a_2} given in Table 1, the dynamic aperture has been calculated for fifty random samples in 1000-turn tracking. Since the sorting is performed on the multipoles of the same order, the components of the sorting vector are simply taken to be the sextupole coefficients. For random arrangements of dipoles, the dynamic aperture averaged over fifty samples is $3.7 \times 10^{-3} m^{1/2}$. With b_2 alone is used for sorting, the average dynamic aperture increases to $9.3 \times 10^{-3} m^{1/2}$. With both b_2 and a_2 sorted, the average dynamic aperture is $9.8 \times 10^{-3} m^{1/2}$. For the two-in-one design of dipoles, b_2 and a_2 in both rings have been sorted simultaneously. The average dynamic aperture increases to $8.1 \times 10^{-3} m^{1/2}$ after this four-parameter sorting. It is understandable that there is no substantial improvement in the dynamic aperture between two-parameter sorting and one-parameter sorting since b_2 is dominant in this case. In order to examine the effectiveness of the two-parameter sorting scheme, we have also studied systems with different ratios of σ_{a_2} and σ_{b_2}. Improvements of the aperture due to the sorting are listed in Table 2. It is shown that the effectiveness of two-parameter sorting is independent of the ratio of σ_{a_2} and σ_{b_2} while the effectiveness of the one-parameter sorting diminishes considerably as $\sigma_{a_2}/\sigma_{b_2}$ approaches 1.

3.2 *Effect of high-order multipoles.*

To understand the effect of high-order multipoles, we consider all the high-order multipoles up to 12-pole. The

Table 2: The average dynamic aperture for different values of $\sigma_{a_2}/\sigma_{b_2}$ with $\sigma_{a_2}+\sigma_{b_2} = 2.0m^{-2}$. The unit of σ is m^{-2}. D_0, D_1, and D_2 are the average dynamic aperture of the random arrangement, sorting b_2 only, and sorting both b_2 and a_2, respectively. The unit of the aperture is $10^{-3}m^{1/2}$.

σ_{a_2}	σ_{b_2}	D_0	D_1	D_2	$\frac{D_1-D_0}{D_0}$	$\frac{D_2-D_0}{D_0}$
0.5	1.5	2.75	6.86	7.67	145%	179%
0.7	1.3	2.91	6.04	7.82	108%	169%
1.0	1.0	2.93	4.59	7.69	57%	162%

aperture calculation shows that if the average over 50 sample is compared, the sorting becomes rather ineffectual when the high-order multipoles are included. The average dynamic aperture of the unsorted ring is $1.82 \times 10^{-3}m^{1/2}$. With sorting the sextupoles alone, it increases to $1.95 \times 10^{-3}m^{1/2}$, which is only 7% improvement. It is easy to understand that sorting sextupoles alone is no longer effective, since at the phase-space location near the dynamic aperture, the nonlinearity arising from various multipole components has the same order of strength. Because of this, 12-parameter sorting which includes all the multipoles has been tried with the sorting vector $\vec{S} = (b_2, b_3x_0, ..., b_7x_0^5, a_2, a_3x_0, ..., a_7x_0^5)$. The optimized sorting result is obtained when $x_0 = 2cm$, which is near the dynamic aperture with $\beta_x = 108m$ and $\beta_y = 19m$. Even though the 12-parameter sorting increases the aperture to $2.11 \times 10^{-3}m^{1/2}$. the improvement is much less than that of the case of sextupoles only. This reduction of the sorting effect can be understood as a result of an increase in the residue of the cancellation when more multipole components are included while the number of magnets in each sorting group is unchanged. The significance of multi-parameter sortings lies in the fact that it eliminates the worst unsorted case ("unlucky" case). For example, of fifty sorted cases, the smallest dynamic aperture, $2.0 \times 10^{-3}m^{1/2}$, is 34% larger than the worst unsorted case.

4 GENERAL DISCUSSIONS ON MAGNET SORTING

For the Tevatron at Fermilab, the goal of sorting dipoles was a rather limited one and the sorting scheme was straightforward [5]. A more ambitious sorting scheme, which was aimed at controlling many harmonic contents in a broad frequency range but still involving a manageably small number of magnets, was proposed by Gluckstern and Ohnuma [2] and this has become the basic sorting principle for the present scheme.

Sorting of course requires a reliable measurement (preferably "cold") of multipole components of *all* the magnets. If this is not possible for reasons of cost and schedule, we will be forced to come up with magnets satisfying the pre-determined tolerance requirements which, however, may create cost and schedule difficulties again because of their severity. The merit of sorting lies in the fact that it can coexist with any other correcting measures without introducing any harmful side effects and cost increase. It is, however, important to stress that magnet sorting should never be considered as "cure-all" in dealing with the nonlinear problems in superconducting rings. It is also important to realize that there is no unique way of sorting magnets. Different circumstances and different parameters certainly require different schemes optimum to the particular conditions.

With the strengths such as given in table 1, the dynamic aperture of the ring is determined by many multipole components and sortings based on sextupole components alone cannot increase the dynamic aperture to a substantial degree. Rather, high order multipoles create a magnetic aperture within which sorted sextupoles can enhance the linearity of the field. Although the resulting improvement in field linearity can be seen by evaluating the distortion function and from the reduced dependence of tunes on betatron oscillation amplitudes, it is not obvious from the dynamic aperture. This must be kept in mind in discussing the merit of sorting scheme based on the dynamic aperture alone.

Most of our tracking has been limited to one thousand to five thousand turns. This is justified since our goal is to compare the effectiveness of various sorting schemes and not to evaluate the aperture for long-term beam survival. For a few cases, the tracking has been extended to one million turns in order to see the reduction in the dynamic aperture. When all multipoles except sextupoles are taken to be zero in the tracking, there is a reduction of as much as $\sim 30\%$ in dynamic aperture. With all multipoles included, or if a realistic physical aperture (5cm, for example) is introduced in the tracking, there is very little change in the dynamic aperture even with a million turns.

Finally, we believe that the combined effect of errors in arc quadrupoles is much less than that of errors in arc dipoles. Problems associated with quadrupoles in the insertions are special when β-function is very large. Any compensation strategy there, including sorting of a few insertion quadruples, should be done independently from the sorting of dipoles in the arcs.

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