Sources of amplitude-dependent tune shift in the PEP-II design and their compensation with octupoles*

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Abstract

Higher order contributions from sextupoles and dipole fringe fields, quadrupole fringe fields, kinematic terms in the Hamiltonian and systematic or random multipole errors may degrade the dynamic aperture by shifting the particles in the tune plane in an undesired direction. We compare the various sources quantitatively and discuss their impact on the dynamic aperture of the PEP-II low energy ting (LER) lattice [1]. A tunable compensation scheme for the LER lattice, which permits an independent control of the tune shift with amplitude in the two transverse planes is presented.

INTRODUCTION

Tune spread in a ring resulting from a variation of tune with amplitude can cause a crossing of "dangerous" resonances and a subsequent dynamic aperture reduction. However having too little tune shift can be undesirable because as the tune shift decreases the widths of the resonances become broad causing particles to be moved to large amplitudes. There is no simple rule which tells us how much (or how little) amplitude-dependent tune shift can be tolerated in our storage rings. In addition, beam-beam tails are effected by the amount of tune shift with amplitude. It is therefore a good idea to have a way to adjust the tune shift in the ring.

In this paper we elaborate on the various sources of amplitude-dependent tune shift and list the contributions from each source in the LER. We also discuss the effect of the tune shift on the particle motion and present compensation strategies. Throughout this discussion we will be using the following notation: To first order in invariant amplitudes, the change in the tune of a particle is

$$\begin{pmatrix} \Delta\nu_x \\ \Delta\nu_y \end{pmatrix} = \begin{pmatrix} \frac{\partial\nu_x}{\partial J_x} & \frac{\partial\nu_x}{\partial J_y} \\ \frac{\partial\nu_y}{\partial J_x} & \frac{\partial\nu_y}{\partial J_y} \end{pmatrix} \begin{pmatrix} J_x \\ J_y \end{pmatrix} \tag{1}$$

where J_x and J_y are the horizontal and vertical action of the particle respectively. (Note: The two off-diagonal cross terms in equation 1 are equal.)

*This work was supported by the Director, Office of Energy Research, Office of Basic Energy Sciences, Materials Sciences Division, of the U.S. Department of Energy under Contracts No. DE-AC03-76SF00515, DE-AC03-76SF00098, and DE-AC03-76ER70285.

SOURCES OF AMPLITUDE-DEPENDENT TUNE SHIFT

Sextupoles

The first source of amplitude-dependent tune shift is sextupoles which in the case of the LER are mainly located in FODO cells in the ring's arcs (except for four doublets located in the IR region). The arc sextupoles are placed in an interleaved pattern (one focussing, one defocussing, one focussing, ...). Each arc in the LER is made up of strings of 4 cells with 90° phase advance. The sextupoles in the IR region are placed π-apart. As a result the ring is made up out of first order "achromats".

In the case of sextupoles, the cross term is a factor of four larger than the two direct terms (see table 1). We expect the beam to populate the tune plane area to the lower left of the working point which is $\nu_x = 0.57$, $\nu_y = 0.64$ if the only source of amplitude-dependent tune shift is sextupoles.

As an illustration of this we tracked about 1500 particles each with a different transverse starting position, (x_0, y_0), for 1024 turns and if the particle survived 1024 turns (only 770 survived), we performed a FFT on the horizontal and vertical turn by turn position data and calculated the particle's fractional horizontal and vertical tunes. Fig.1 shows these 770 points, each corresponding to one particle launched on a grid inside the dynamic aperture of the LER. We see that the horizontal (vertical) tune is dominantly dependent on the vertical (horizontal) amplitude showing the dominance of the cross term.

Other sources of amplitude-dependent tuneshift

Fringe fields

The change of the longitudinal vector potential A_s at the edge of a dipole or a quadrupole is accompanied by higher-order magnetic fields. As a first step we model the fringe field as a hard edge. The map over the quadrupole edge can be written in terms of a Lie operator [2]

$$\begin{aligned} \mathcal{E}_\pm &= \exp\{: \pm\frac{k}{12(1+\delta)} \times \\ &\quad (3x^2yp_y - 3y^2xp_x + y^3p_y - x^3p_x) :\} \end{aligned} \tag{2}$$

where the positive sign refers to the entrance and the negative sign to the exit of the quadrupole and k is the strength of the quadrupole. This map reproduces to first order the original expression derived by Lee-Whiting [3].

Table 1 shows that the quadrupole fringe field have a dominant effect on the tune shift. With a listing of the

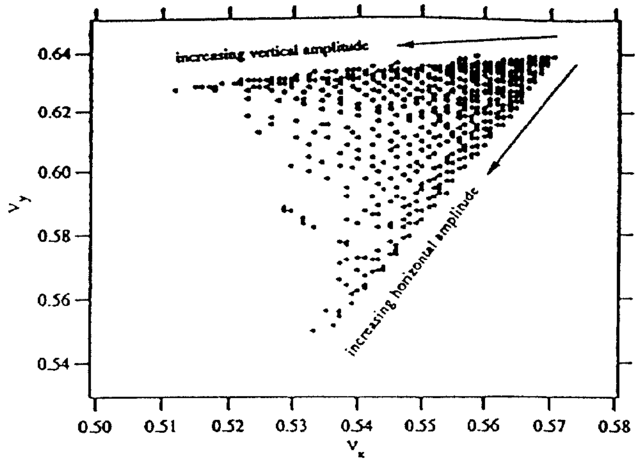

Figure 1: Fractional tune portrait of all particles which survived 1024 turns.

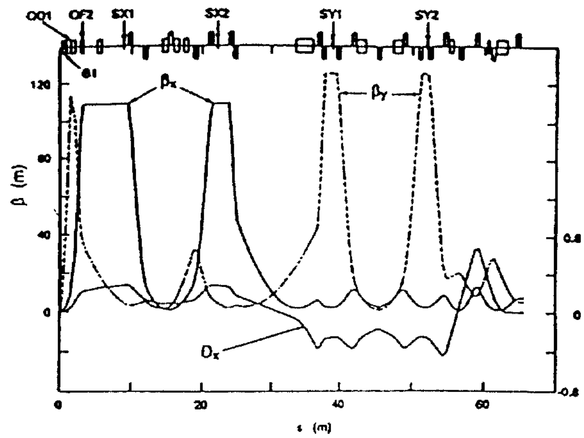

Figure 2: Twiss parameters and magnet layout for the right half of the interaction region straight section.

Twiss parameters around the ring and the Floquet transformation $x \to (\sqrt{\beta_x})\,x$, $p_x \to (-\alpha_x/\sqrt{\beta_x})\,x + (1/\sqrt{\beta_x})\,p_x$, $y \to (\sqrt{\beta_y})\,y$, $p_y \to (-\alpha_y/\sqrt{\beta_y})\,y + (1/\sqrt{\beta_y})\,p_y$ we find that the major contributors are located in the IR region. The first quadrupole but also the quadrupoles, which are located around the strong chromaticity sextupoles (see fig.2), have the strongest effect.

The effect of a dipole fringe field is to introduce an additional sextupole component into the lattice. We observed an effect on the dynamic aperture when dipole fringe fields were included in the tracking (see table 2).

Kinematic effects

In a drift space, the expansion of the square root in the Hamiltonian, leads to a fourth-order term in p_x, p_y

$$H = -\delta + \frac{p_x^2 + p_y^2}{2(1+\delta)} + \frac{(p_x^2 + p_y^2)^2}{8(1+\delta)^3} + O(p^6) \ , \qquad (3)$$

To locate the leading contributors around the ring we transform the momentum with into Floquet variables[1], $p_x \to \sqrt{\gamma_x}p_x$, $p_y \to \sqrt{\gamma_y}p_y$, from which follows that the magnitudes of γ_x^2, $\gamma_x\gamma_y$, and γ_y^2 will control, respectively, the magnitudes of $\partial\nu_x/\partial J_x$, $\partial\nu_x/\partial J_y$, and $\partial\nu_y/\partial J_y$.

The largest values of γ_x^2 occur at the first quadrupole close to the IP, and at quadrupoles close to the chromaticity sextupoles SX2. The largest values of γ_y^2 occur at the drift space and the first weak bend after the IP up to the first quadrupole. Another large value of γ_y^2 occurs near the quadrupole just before the chromaticity sextupole SY1. The effect of the cross term $\gamma_x\gamma_y$ is somewhat smaller. The magnitude of the tune shift with amplitude which are obtained from kinematic terms are listed in table 1.

[1] This is a sort of reverse Courant-Snyder transformation which is convenient when the perturbation depends on the momentum only.

	$\partial\nu_x/\partial J_x$	$\partial\nu_x/\partial J_y$	$\partial\nu_y/\partial J_y$
Sextupoles	-800	-2810	-634
Quad. fringes	1628	1368	3010
Bend. fringes	0	10	44
kin. terms	188	80	800

Table 1: Tune shift in the LER from various sources

In the case of the KEK B-Factory design it was mentioned, that the major limits to the dynamic aperture arise from kinematic terms of the drift space around the IP and the fringe field of the first quadrupole on both sides of the IP [4]. For the PEP-II design we found one source of amplitude-dependent tune shift located around the IP and a second source, almost equally strong, which is located around the two chromaticity sextupole pairs.

Effect of other sources on the dynamic aperture

Table 2 displays the dynamic aperture for on- and off-momentum motion at the injection point $\beta_x = 34$m , $\beta_y = 169$m . Besides the horizontal and the vertical dynamic aperture also the number of surviving particles is listed, which is roughly proportional to the area inside the dynamic aperture. Table 2 shows that the dynamic aperture is reduced, both in the horizontal and in the vertical plane, if all fringe fields and kinematic effects are included in the tracking.

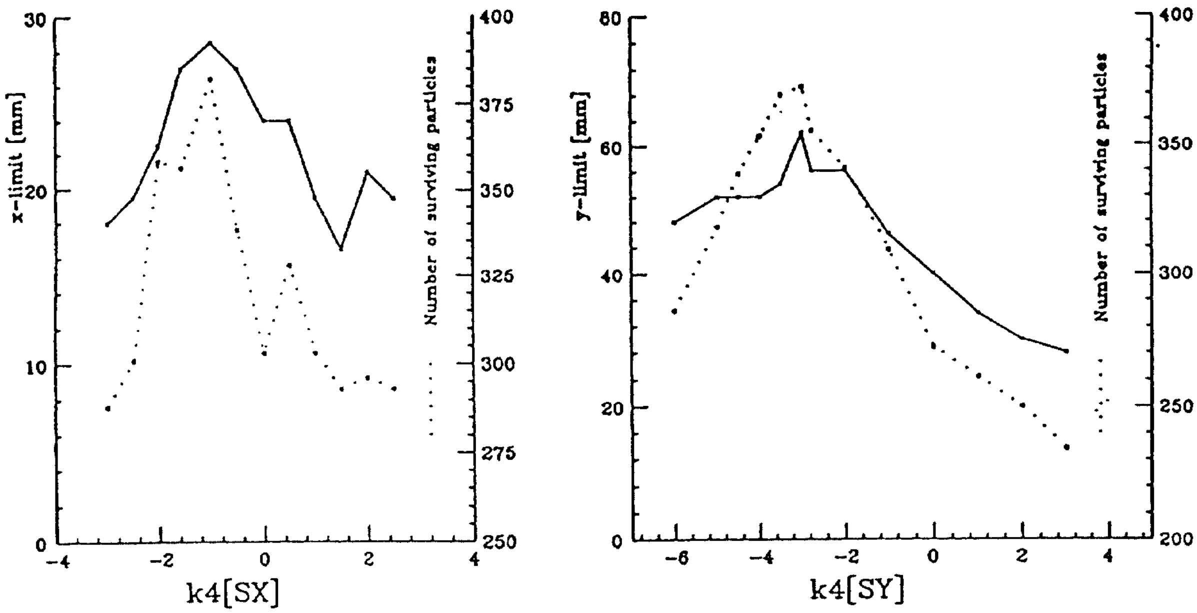

Figure 3: Horizontal and vertical dynamic aperture verses octupole strength.

	x_{apr} [mm]	y_{apr} [mm]	area
Sxt.	46.5 (24)	86 (56)	770 (335)
Sxt. + Qd. frng.	34.5 (22.5)	86 (48)	668 (331)
Sxt. + Dip. frng.	46.5 (24)	84 (52)	715 (319)
Sxt. + Kin.	45 (24)	88 (50)	792 (327)
All terms	36 (21.5)	74 (40)	632 (256)

Table 2: Dynamic aperture at the injection point for different nonlinear sources the values in parenthesis account for particles launched at $\delta = 0.008$.

1 Control of amplitude dependent tune shift

Octupole compensation

One needs three knobs, e.g., three octupoles with separate power supplies, in order to vary $\partial\nu_x/\partial J_x$, $\partial\nu_x/\partial J_y$ and $\partial\nu_y/\partial J_y$ independently. We will focus here only on the terms which are mostly effected by fringe fields and kinematic terms namely $\partial\nu_x/\partial J_x$ and $\partial\nu_y/\partial J_y$ in order to see if we can restore the dynamic aperture to its original size.

The phase-advanced Hamiltonian for an octupole is $h_{K_4} = (K_4/4)(x_i^4 - 6x_i^2y_i^2 + y_i^4)$ where K_4 is the strength of the octupole. We chose to insert two pairs of octupoles on each side of the IP in the high β locations close to the chromaticity sextupoles pairs SX and SY. By placing the octupoles in this region we are able to compensate the tune shift coming from quadrupole fringes and kinematic terms. Also by having a -I transformation between the octupole pairs we do not introduce a first order chromatic sextupole. Octupoles with only a weak strength are necessary to cover a wide range of tune shift.

Effect on the dynamic aperture

Fig.3 shows the horizontal (vertical) dynamic aperture of the LER with fringe fields and kinematic terms and sextupoles for different SX (SY) octupole excitations. The runs were done for an initial relative energy deviation of $\delta = 0.008$. The number of surviving particles is shown as a dotted line. We find a peak of the dynamic aperture at $K_4(SX) = -1.0\mathrm{m}^{-4}$ and $K_4(SY) = -3.0\mathrm{m}^{-4}$. The tune shift with amplitude at that octupole strength is not far from its original values. The horizontal and the vertical dynamic aperture should be compared to the values shown in parenthesis in Table. 1.

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