Multipole Contribution from an Off–Axis Orbit in an IR Quadrupole and the Consequences on the Dynamic Aperture*

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Abstract

The low-energy beam of the proposed PEP–II B factory enters the first quadrupole (Q1) after the interaction point off axis in order to separate the low-energy beam from the high-energy beam. The off-axis beam orbit in Q1 gives rise to significant feed-down terms from higher multipoles that orginate from systematic effects and random fabrication errors. We study superconducting and permanent magnet designs of Q1, and look at the effect these different designs have on the dynamic aperture. Including a dipole field in a superconducting design allows us to offset the magnetic axis from the mechanical axis, thereby maintaining the separation of the beams while greatly reducing the feed-down effect. We illustrate relevant points of the discussion with tracking results for the PEP-II low-energy ring.

1 INTRODUCTION

The need to bring unequal-energy high-current beams into collision at a high luminosity asymmetric-energy B factory generally imposes the requirement that some of the magnetic elements near the Interaction Point (IP) be shared by both beams. These elements, where the β functions are large, must have high quality magnetic fields in order to minimize adverse effects to the beam dynamic aperture. This is especially true of shared magnetic elements where at least one, and sometimes both, of the beams have trajectories that are displaced from the axis of the element. The dynamic aperture also depends on the entire lattice, and upon the strengths of nonlinearities throughout the lattice. Other lattice features that are important are:

- the chosen working point in tune space,
- the dependence of the tune shift with energy and with amplitude, and
- the overall chromatic behavior of the lattice.

Changes in any of these variables can make the dynamic aperture of the latttice more or less sensitive to higher order multipoles in the high–β quads near the IP.

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2 PEP–II INTERACTION REGION

The proposed SLAC, LBL, LLNL PEP–II B factory has an Interaction Region (IR) design that employs a head-on collision. The unequal energy beams (9 GeV and 3.1 GeV) are initially separated by a bending magnet (B1). B1 separates the beams enough to avoid beam-beam effects from the first parasitic crossing at 0.63 m from the IP. The beams then enter Q1, a shared horizontally defocussing quadrupole, where the beams are further separated by placing the center of Q1 on the High-Energy Beam (HEB) trajectory. This maximizes the beam separation and allows Q2, a septum quadrupole for the Low-Energy Beam (LEB), to be placed 2.8 m from the IP.

B1 and Q1 will be inside a detector with a 1-1.5 T solenoidal field. This limits the technology choices for these magnets to Permanent Magnets (PM) or Superconducting Magnets (SC). The detector acceptance for physics events limits the available volume for B1, making permanent magnets the only practical solution for this bending magnet. In the case of Q1, both options are possible.

3 FEED-DOWN FROM THE OFF-AXIS ORBIT

By the time the LEB exits Q1, going away from the IP, the LEB orbit is substantially offset from the axis of Q1. To properly account for the effect higher order multipoles have on the off-axis LEB, the on-axis multipole coefficients are reexpanded about the LEB orbit position at 16 equally spaced locations throughout Q1. This results in a *feed-down* effect wherein higher order multipole coefficients produce lower order coefficients. In general, we have

$$B(z) \;=\; \mathrm{Re}\sum_{n=0}^{\infty} b_n \left(\frac{z}{r_0}\right)^n \qquad \text{where} \quad z = re^{i\theta}\ ,$$

and b_n is the field strength of the nth multipole at the reference radius r_0. The same field can be represented by an expansion about a new origin z' offset from the origin of the original expansion

$$B(z) = \mathrm{Re} \sum_{n=0}^{\infty} b'_n \left(\frac{z-z'}{r_0}\right)^n .$$

The new multipole coefficients b'_n for the same field in the transformed coordinate system can be derived by equating the two expansions of B and its derivatives at $z = z'$. The general form is

$$b'_n = \sum_{k=n}^{\infty} \frac{k!}{n!(k-n)!} b_k \left(\frac{z'}{r_0}\right)^{k-n} .$$

Since the LEB orbit is farthest from the axis of Q1 at the outside edge of Q1, the coefficients found for the sections near the end of the magnet dominate. Figure 1 shows plots of the systematic multipole coefficients and the multipole coefficients due to random fabrication errors for two PM cases and one SC case. The coefficients are all normalized to the strength of the quadrupole field.

The systematic multipoles of the PM Q1 result from the magnet being built from 16 or 32 blocks of uniform magnetization. These blocks are arranged azimuthally with magnetization vectors oriented to obtain a quadrupole field in the magnet aperture [1], with systematic multipoles at n = 18, 34, 50, ... for a 16-block magnet and n = 34, 66, ... for a 32-block magnet. The systematic multipoles in a SC magnet design result from the placement of the SC wires that make up the coils of the magnet. The SC magnet shown in the plot is a two-wedge design. The spacing of a wedge is an additional degree of freedom that can be used to further suppress higher order harmonics.

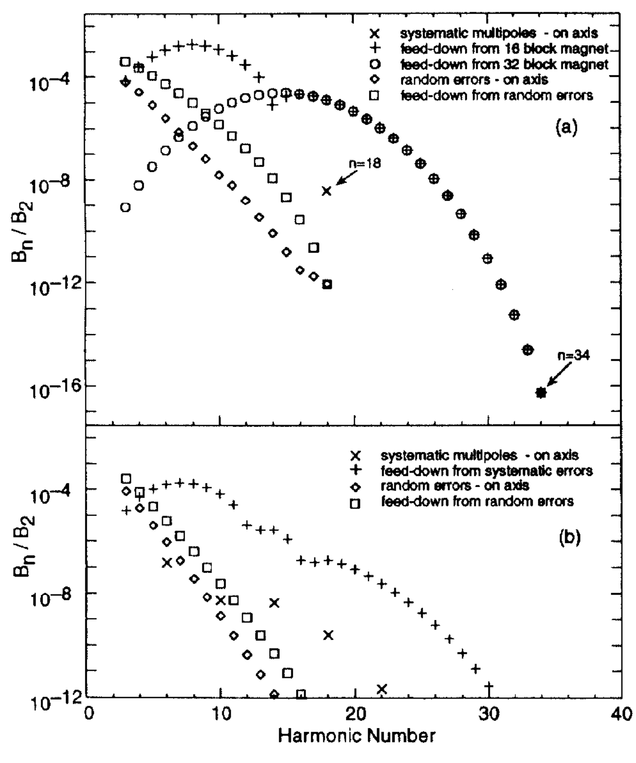

Figure 1. (a) Systematic and random harmonics for a 16-block and 32-block design. Systematic multipoles are at n=18 and 34. (b) Harmonics of a SC two-wedge design. In both plots, the feed-down harmonics result from the large offset of the LEB as it exits Q1.

4 TRACKING

PEP–II lattices are developed using two programs called MAD [2] and DIMAD [3]. Alignment and magnetic errors are introduced into a lattice with a program called TRACY II [4]. A lattice with errors is corrected in terms of closed-orbit, dispersion, beta-beating, and coupling. The corrected nonlinear lattice is then tracked and mapped in DESPOT [5]. Particles are tracked for 1024 turns through the lattice with synchrotron oscillation at 10 σ_E from the nominal energy. The required dynamic aperture is 10 σ of the beam size.

All of the aperture plots are for the LEB, and are shown at the injection point. Figure 2(a) shows the tracking results of the 16 block PM design, with the result for an ideal Q1 without higher harmonics. In Figure 2(b), we see that the dynamic aperture is sharply reduced when the multipoles due to random errors are doubled and the systematic multipoles in the 16-block magnet are multiplied by 2 and by 4.

Figure 3(a) shows plots of the dynamic aperture for the SC case shown in Figure 1(b), and Figure 3(b) shows the aperture of a 32-block design.

The 16-block design is clearly the weakest of the above three cases. Increasing the multipole strengths by as little as a factor of two has a marked result on the dynamic aperture. However, there is little or no effect on the dynamic aperture when we increase the 32-block multipoles by a factor of ten and the SC multipoles by a factor of five, even when the multipoles due to random errors are doubled.

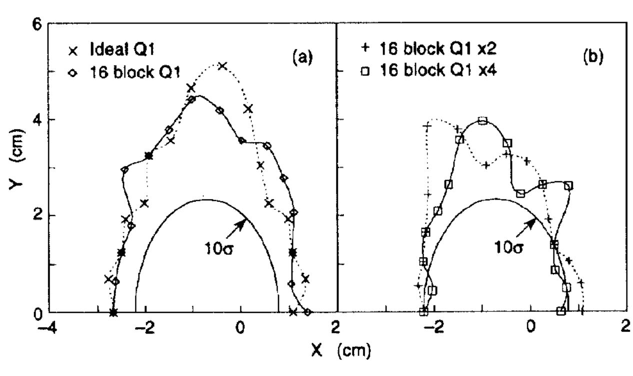

Figure 2. (a) Dynamic aperture plot of the LEB for an ideal Q1 magnet and a 16-block design. (b) Effect on the dynamic aperture when the 16-block systematic harmonics are multiplied by 2 and by 4.

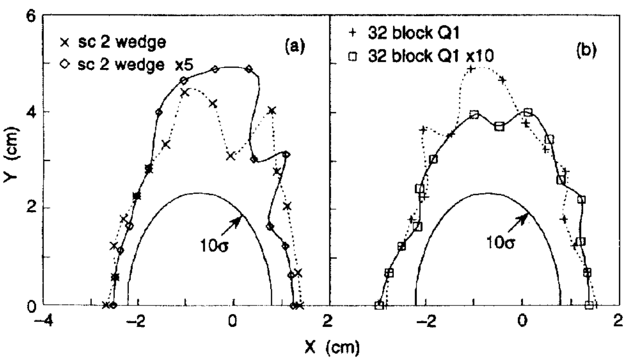

Figure 3. (a) Dynamic aperture plot for the SC case shown in Figure 1(b) and with the harmonics increased fivefold; (b) aperture plots from the 32-block design, standard and ten times over standard.

5 INTRODUCING A DIPOLE FIELD

Including a dipole field in the SC case allows us to displace the magnetic axis with respect to the mechanical axis of Q1. Consequently, the mechanical axis of Q1 is shifted closer to the LEB orbit without losing any beam separation. This greatly reduces the feed-down effect. Figure 4 shows the multipole components of the SC Q1 with the added dipole field harmonics at the new offset, and Figure 5 depicts the dynamic aperture for this case when the systematic harmonic values have been multiplied by 1000 and the random errors have been doubled. The aperture is little changed from the ideal case also shown.

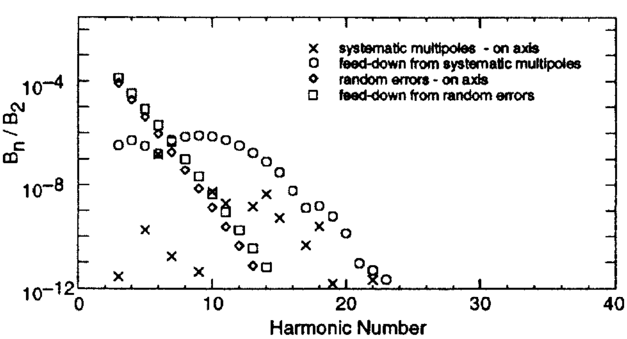

Figure 4. Multipole coefficients for the above SC two-wedge design, together with a dipole winding to reduce the offset of the LEB. The feed-down harmonics result from an offset that is about half the value used in the previous plots.

6 CONCLUSIONS

Multipole components that feed down from higher order harmonics must be carefully investigated for effects on the dynamic aperture of the lattice in any designs that include large off-axis orbit trajectories. We find that for the PEP-II B Factory design, the large off-axis LEB orbit in Q1 generates a dynamic aperture that has little safety margin for a 16-block PM design. However, a good dynamic aperture is found for a 32-block PM design and a SC design. By adding a dipole field to the SC design, the LEB orbit can be positioned closer to the mechanical axis, thus greatly reducing feed down from higher order harmonics. Increasing the systematic harmonics by as much as 1000 does not adversely affect the dynamic aperture. More work needs to be done. It is clearly important to construct an accurate map of the entire magnetic field for the permanent magnet and superconducting magnet designs, including the full effects of the Q1 fringe fields. This work is in progress.

7 ACKNOWLEDGEMENTS

We thank Gordon Bowden for his valuable assistance in helping us understand the harmonic content of permanent magnets.

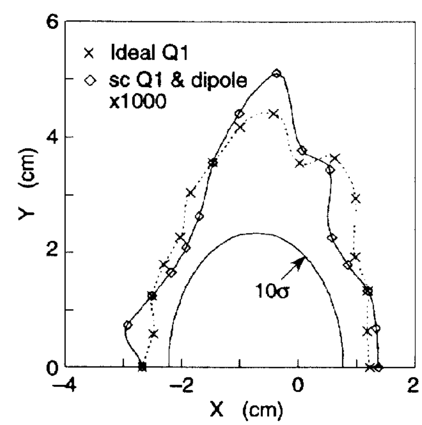

Figure 5. Dynamic aperture plots of the LEB for an ideal Q1 and for a SC Q1, with a dipole field in which the systematic harmonic values shown in Figure 4 have been increased by a factor of 1000.

8 REFERENCES

[1] K. Halbach, "Physical and Optical Properties of Rare Earth Cobalt Magnets," Nucl. Instånd Methods 187, pp. 109-117 (1981).

[2] F. C. Iselin, "The MAD Program (Methodical Accelerator Design) Version 8.4:Users Reference Manual," CERN-SL-90-13-AP-REV. 2, p. 120 (1991).

[3] R. Servranckx, K. Brown, L. Schachinger, and D. Douglas, "Users Guide to the Program DIMAD," SLAC Report 285/UC-28(A) (1985).

[4] J. Bengtsson, private communication, 1992.

[5] E. Forest, J. Bengtsson, and M. Reusch, "DESPOT Uses n-step Simplectic Integrators for Thick Elements," private communication.