# Arrangement in Sectors of CLIC Linac Focusing 

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#### Abstract

For the main Linac of the CERN Linear Collider (CLIC), the emittance growth due to wake fields and misalignments has been investigated with a new scaling of the betatron function. Since a continuous scaling is not feasible in practice, a more realistic implementation, based on a division of the Linac into a small number of sectors in which focusing is kept constant, is presented. The MTRACK code has been modified in order to deal with thick quadrupole transfer matrices, to take into account the geometric dimensions of enginecred elements and the modularity of the supports, and to allow for betatron coupling due to tilt misalignments and for radiation effects. Tracking results show that such a division in sectors is possible, provided that the matching between sectors is adjusted: this was done using the MAD code.


## 1 INTRODUCTION

The lattice of the main Linac should be scaled with energy in order to balance the effects of wake fields (low energy) and chromatic dispersion (high energy). It was found that an independent scaling of the cell length $L_{\mathrm{c}}$ and of the focal distance $f$ allows a better control of the emittance growth [1], [2]. Moreover, one RFQ should be installed at each quadrupole to prevent instabilities, and the relative amount $\alpha_{\mathrm{rf}}$ of microwave gradient with respect to magnetic gradient should be scaled with energy. The selected laws are:

$$
\begin{equation*}
\frac{L_{\mathrm{c}}}{L_{\mathrm{c}_{0}}}=\left(\frac{\gamma}{\gamma_{0}}\right)^{0.3} \frac{f}{f_{0}}=\left(\frac{\gamma}{\gamma_{0}}\right)^{0.4} \quad \frac{\alpha_{\mathrm{rf}}}{\alpha_{\mathrm{ff}}}=\left(\frac{\gamma}{\gamma_{0}}\right)^{-0.1} . \tag{1}
\end{equation*}
$$

The result is an increasing amplitude of the betatron functions and a decrease of the phase advance per cell.

The vertical emittance growth was evaluated running the tracking code MTRACK7 that uses thin-lens treatment, a continuous succession of accelerating cavities, one RFQ and a pick-up at each quadrupole, a one-to-one correction scheme and $50-5-5 \mu \mathrm{~m}$ r.m.s. misalignments (respectively for quadrupoles, cavities and pick-ups) [2]. Different values for the RFQ phase and/or for $\alpha_{\mathrm{rf}}$ were chosen in four or five different parts of the Linac in order to get the right compromise between minimum energy spread and minimum blow-up [3], [4]. This was done for each particular machine with different misalignments, looking at the emittance 'all along' the Linac and minimizing beating. The final $\varepsilon_{\nu}$, after one further iteration of the one-to-one correction and optimization of $\alpha_{\mathrm{rf}}$, is on average slightly above the target value of $2 \times 10^{-7} \mathrm{rad} \mathrm{m}$. Nevertheless,
this target was reached by adding pick-ups and running MTRACK8 with the one-to-few correction scheme [4].
This approach has some limitations. First, MTRACK7 underestimates the Linac length, as quadrupole sizes and drift spaces between elements are not considered. Second, it does not take into account some fixed geometric features such as the accelerating section length (an integer number of them must be fitted between two quadrupoles), their position on each supporting girder in groups of four, and the girder dimensions. Third, some modularity should be introduced in order to avoid hundreds of different quadrupoles: we must think about sectors with constant focusing.

## 2 LAY-OUT REQUIREMENTS

In order to preserve the periodicity given by the girders, it was decided that the physical length of a quadrupole, constant within each sector, should be a multiple of that of the accelerating section so that one or some sections might be substituted by quadrupoles without modifying the overall structure. In the first sector we can use a 34 cm quadrupole with magnetic length $L_{\mathrm{q}}=26 \mathrm{~cm}$ (Fig. 1).


Figure 1. Schematic lay-out for a girder in the first sector.
The distance $L_{\mathrm{c}}$ between two quadrupoles, constant within each sector, should be a multiple of the physical length of a girder (including the space between them), so that a quadrupole is always the first element on a girder. Choosing $L_{\mathrm{c}_{\mathrm{o}}}=4.09 \mathrm{~m}$ in Eq. (1), we may have in the first sector a cell length of three girders, close to the one we had in the continuous scaling. There will be six sectors (Fig. 2), whose energy ranges are given in Table 1, together with the number $n_{g}$ of girders between two quadrupoles and the cell length $L_{\mathrm{c}}$.
Table 1: Main parameters of each sector

| Sector | $n_{\mathrm{g}}$ | $L_{\mathrm{c}}(\mathrm{m})$ | $E_{\mathrm{i}}(\mathrm{GeV})$ | $E_{\mathrm{f}}(\mathrm{GeV})$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 3 | 4.23 | 9.00 | 16.85 |
| 2 | 4 | 5.64 | 16.85 | 38.93 |
| 3 | 5 | 7.05 | 38.93 | 76.00 |
| 4 | 6 | 8.46 | 76.00 | 132.63 |
| 5 | 7 | 9.87 | 132.63 | 213.69 |
| 6 | 8 | 11.28 | 213.69 | 250.00 |



Figure 2. Proposed division in sectors with $L_{\mathrm{c}_{0}}=4.09 \mathrm{~m}$.


Figure 3. Proposed lay-out for the six sectors of the Linac.

Though we always under-focus in the last sector and over-focus in the first one, this helps to control better the wake fields at low energy and the dispersion at high energy. In this arrangement (Fig. 3) the quadrupoles become longer as the energy increases, but still only three different sizes are needed.

As for the focal distance $f$, there are two possibilities: keeping the focusing strength $K$ or the gradient $G$ constant within a sector. In the first case, as $K=e C / p$, the gradient $G$ increases with the particle momentum $p$ : although quadrupoles are identical, an independent power supply for each one is needed. In the second case $G$ is frozen and only one power supply per sector is needed, which indeed seems simpler and cheaper. Nevertheless, this solution relies strongly on the nominal energy of the beam and does not work at a different one: even if a single cavity fails it is a problem. Furthermore, the scaling of $f$ in a scetor would be proportional to $\gamma$, i.e. far from $\gamma^{0.3}$, causing a larger emittance growth. Thus the first possibility ( $K=$ constant) has been retained, as it allows tuning; then $G / G_{0}=p / p_{0}$, with a different $G_{0}$ in each sector, switching to a longer quadrupole when reaching the limit of $100 \mathrm{~T} / \mathrm{m}$.

An initial value for $f_{0}$ is now imposed in Eq. (1); using

$$
\begin{equation*}
f=\frac{L_{\mathrm{c}}}{2} \sin \frac{\mu}{2} \quad \beta_{\min , \max }=\frac{2 L_{\mathrm{c}}}{\sin \mu}\left(1 \pm \sin \frac{\mu}{2}\right) \tag{2}
\end{equation*}
$$

with $\mu=90^{\circ}$ (close to the one we had in the continuous scaling and very useful in practice), we get $f_{0}$ and $\beta_{\text {min,max }}$ in the first sector; then $f, \mu$ and $\beta_{\text {min,max }}$ can be found in each sector (Table 2).

Table 2: Twiss parameters in each sector

| Sector | $f(\mathrm{~m})$ | $\mu(\mathrm{deg})$ | $\beta_{\min }(\mathrm{m})$ | $\beta_{\max }(\mathrm{m})$ |
| :---: | ---: | ---: | ---: | ---: |
| 1 | 2.991 | 90.000 | 2.478 | 14.442 |
| 2 | 4.389 | 79.949 | 4.096 | 18.816 |
| 3 | 5.910 | 73.225 | 5.944 | 23.510 |
| 4 | 7.537 | 68.282 | 7.991 | 28.434 |
| 5 | 9.257 | 64.433 | 10.217 | 33.549 |
| 6 | 11.061 | 61.316 | 12.603 | 38.829 |

## 3 SECTOR PARAMETERS

The actual length of each sector must still be evaluated by the tracking code, modified in order to cope with the above constraints on $L_{\mathrm{q}}, L_{\mathrm{c}}$ and $G$, to fit an integer number of accelerating sections between two quadrupoles, to consider drift spaces (at the edges of each quadrupole, cavity and girder), and to use a thick- instead of a thin-lens treatment.

Running this new code version, named MTRACK9, it is possible to set $L_{\mathrm{q}}$ to find the length $L_{\mathrm{s}}$ of each sector, the number $n_{c}$ of cells (and quadrupoles), the number $N_{g}$ of girders, and the minimum and maximum magnetic gradient (Table 3).

Table 3: MTRACK9 computed parameters

| Sector | $L_{\mathrm{q}}$ <br> $(\mathrm{m})$ | $L_{\mathbf{s}}$ <br> $(\mathrm{m})$ | $n_{\mathrm{c}}$ | $N_{\mathrm{g}}$ | $G_{\min }$ <br> $(\mathrm{T} / \mathrm{m})$ | $G_{\max }$ <br> $(\mathrm{T} / \mathrm{m})$ |
| :---: | :---: | ---: | ---: | ---: | ---: | :---: |
| 1 | 0.26 | 135.36 | 32 | 96 | 39.42 | 71.36 |
| 2 | 0.60 | 417.36 | 74 | 296 | 22.11 | 50.44 |
| 3 | 0.60 | 676.80 | 96 | 480 | 37.84 | 72.89 |
| 4 | 0.60 | 1015.20 | 120 | 720 | 57.52 | 99.48 |
| 5 | 0.94 | 1480.50 | 150 | 1050 | 52.54 | 83.89 |
| 6 | 0.94 | 676.80 | 60 | 480 | 70.29 | 82.17 |

The total length of the Linac is 4402 m (it was 3245 m with continuous scaling); there are 32 short, 290 medium and 210 long quadrupoles: the total number is therefore 532 quadrupoles (it was 430).

The number of independent main power supplies can be limited by grouping under the same one a set of quadrupoles whose gradient range is below $20 \%$, say, and then using independent weaker 'trimmers' for the single quadrupoles. For example, all the 60 quadrupoles of the last sector may be grouped, as $G_{\max }$ is about $17 \%$ higher than $G_{\min }$; in the first sector we may have 4 groups of 8 quadrupoles. In conclusion, the total number of main power supplies needed decreases from 532 to 24 , although we now need 532 'trimmers'.

Owing to the thick-lens treatment, the simple relations (2) do not apply any more: so MTRACK9 cannot obtain the values of Table 2 and a $\beta$-beating appears. Using the MAD code [5], a single cell of each sector must be matched for the given $\mu$, and a new algorithm is needed to compute the initial values of $\beta_{\text {min,max }}$ the focal distance must be redefined as well. The resulting parameters are, however, not so different from those found previously.

When passing from one sector to the next, a strong $\beta$ beating appears, which grows up to a maximum amplitude that is twice the wanted one. On a set of five machines,
even after one iteration of the one-to-one correction, $\varepsilon_{y}$ grows up to $7 \div 18 \times 10^{-7} \mathrm{rad} \mathrm{m}$, well above the target value.

## 4 SECTOR MATCHING

In order to reduce the emittance growth, we matched the sectors; at each transition point, four conditions must be imposed (on $\alpha_{x}, \alpha_{y}, \beta_{x}, \beta_{y}$ ); another two conditions (less restrictive) were added to limit the oscillation amplitude.

To avoid moving quadrupoles or changing their length, we act on their strength: MAD found sufficient to change five strengths for each matching region, and MTRACK9 got for $\beta(z)$ the expected flat shape in each sector (Fig. 4). The phase of the cavities was changed from $7^{\circ}$ to $9^{\circ}$ to minimize both energy spread and blow-up. The final emittance is now about a factor two better than without matching, but still above the target value after one iteration.


Figure 4. Betatron function along the Linac after matching.
Up to this point, the $\alpha_{\mathrm{ff}}$ parameter was scaled with energy according to Eq. (1) with $\alpha_{\mathrm{rf}}{ }^{=} 0.59$ coming from previous calculations; but it should be optimized by sectors for each particular machine, as the best values depend on the selected set of random misalignments. A new iterative technique was tested: it consists of finding out for each sector the $\alpha_{\mathrm{rf}}$ that gives the smallest emittance growth at its exit. This gave a much lower emittance on the set of five machines previously run with scaled $\alpha_{\mathrm{rf}}$.

Eventually, adding an extra pick-up at cach second girder and applying the one-to-few correction algorithm gave the required emittance for the same set of five machines as before (Fig. 5).


Figure 5. Emittance growth after matching, after one iteration of the one-to-few correction, with optimized $\alpha_{\mathrm{tf}}$.

## 5 BETATRON COUPLING

The analysis of coupling between transverse planes was introduced into MTRACK9 passing from a $2 \times 2$ to a $4 \times 4$ matrix formalism, both for transfer and sigma matrices. It reads or generates a random tilt angle for each quadrupole, according to a Gaussian distribution with given r.m.s. $\vartheta$ (the same approach used for misalignments); it evaluates the transfer matrix of a tilted quadrupole; it computes the emittance in both planes from a $4 \times 4$ sigma matrix as the determinant of the proper $2 \times 2$ sub-matrices.

Setting $\vartheta=0$, results do not change. A small $\vartheta$ yields a strong coupling between transverse planes; this is due to a resonance, because the lattice is identical in both planes (in particular $\mu_{x}=\mu_{y}$ ). This effect grows with $\vartheta$ and is enhanced by the large emittance ratio ( $\varepsilon_{x} / \varepsilon_{y}=29$ ).

In order to break the symmetry, a different phase advance in the two planes was chosen, using two families of quadrupoles (the strengths $K_{\mathrm{f}}$ and $K_{\mathrm{d}}$ of which are obtained from MAD for each kind of single cell), and then re-matching the sectors; MTRACK9 was again modified in order to deal with $K_{\mathrm{f}} \neq K_{\mathrm{d}}$.

It was found that a r.m.s. tilt of $0.5 \mu \mathrm{rad}$ can be tolerated with $\Delta \mu=5^{\circ}$, and of $1 \mu \mathrm{rad}$ with $\Delta \mu=10^{\circ}$ : in both cases the final emittances $\varepsilon_{x}, \varepsilon_{y}$ were kept around the target values, though $\beta$-beating (Fig. 6) indicates coupling.


Figure 6. Betatron function in presence of coupling.

## 6 CONCLUSIONS

The emittance dilution is not worse than in the case of continuous scaling, after adding thick-lens treatment and taking into account the geometric dimensions of the engineered elements as well as the modularity of the supports. Nevertheless, the real Linac will be almost 1.2 km longer.

Coupling effects can be tolerated if the r.m.s. tilt is below $1 \mu \mathrm{rad}$ with $\Delta \mu=10^{\circ}$, but coupling corrections might relax this tolerance. Radiation effects are negligible [6] and therefore have not been taken into account.

## 7 REFERENCES

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