

# On the Strong Intrinsic Focusing in Long Planar Wiggler\*

A.A Varfolomeev, A.V. Smirnov

Russian Research Center "Kurchatov Institute", Moscow 123182, Russia

## Abstract

The use of strong focusing in a short wavelength linac-driven FEL and some means for obtaining it are described. The principal purpose of such focusing is to maintain both high enough current density of the electron beam and phase synchronism between the electron beam and electromagnetic wave along the wiggler. Beam transport problem through the wiggler was treated with help of both K-V equations and special 3D code including phase detuning calculations. Possible way of improving the phase synchronism for sextupole periodic strong focusing is proposed.

## 1. INTRODUCTION

There is a great physical interest [1] in the development of free electron lasers (FELs) at wavelengths below 1000 Å. To achieve reasonable gain an FEL needs much higher density in the electron beam and longer wiggler length [2]. One of the most attractive and promising scheme is a single pass high-gain FEL operating in self-amplified-spontaneous-emission (SASE) mode. To achieve high enough gain at the shortest wavelength strong focusing of the electron beam is necessary [1-3]. In this paper we consider a somewhat alternative possibility for strong focusing realisation on the basis of unconventional undulator schemes having only intrinsic strong focusing, i.e. undulators having no external focusing elements (e.g. quadrupoles) but containing iron elements.

## 2. A BRIEF OVERVIEW OF MEANS OF STRONG FOCUSING

Various ways for improving the natural focusing have been proposed: edge focusing and separated function wigglers [4], alternating gradient focusing [3,4] and periodic sextupole alternating focusing [5,6]. Noticeable enhancement of natural focusing can be achieved in undulator having large amplitude of non-fundamental harmonic [7]. We have analysed the applicability of these means with respect to the Linac Coherent Light Source (LCLS) project [8]. The main requirements imposed in this project for  $\approx 10$  GeV electron beam optics are the following: beam confinement within radius about  $50 \mu\text{m}$  and phase synchronism between the e.m. wave and the e.b. along 40m undulator. Compact and feasible design could use undulator with intrinsic alternating focusing of quadrupole or periodic sextupole type which are considered below.

## 3. NUMERICAL EXAMPLES OF BEAM TRANSPORT

We have chosen FD lattice with period 6.4m for convenience and to minimise required field gradients. Generalised K-V equations taking into account undulator natural focusing were used to define the optimal beam matching conditions and focusing parameters presented in the Table 1.

Table 1. FD lattice parameters for intrinsic focusing.

Variant	Type	Gradient $ Q $ , T/m	Sextupole foc. rel. strength $ k_{xy}/k_w $
1	Quadrupole	1.74	0
2	Combined	1.12	5.92
3	Sextupole	0	13.6

Note, that  $k_x^2 + k_y^2 = k_w^2$  where  $k_y^2$ ,  $k_x^2$  have interchanging signs for the last two variants. All variants of the Table 1 corresponds to the same value of  $\beta$ -function, FD period (6.4m) and phase advance per cell ( $\approx 17.4^\circ$ ). Of course, 3D-simulations of FEL performance (see ref. [6]) can require to redefine the focusing parameters for smoothing of beam envelopes. However, our purpose is to consider beam transport and phase synchronism at focusing of different types. For solving these problems we used a special non-averaging multiparticle tracing code [9].

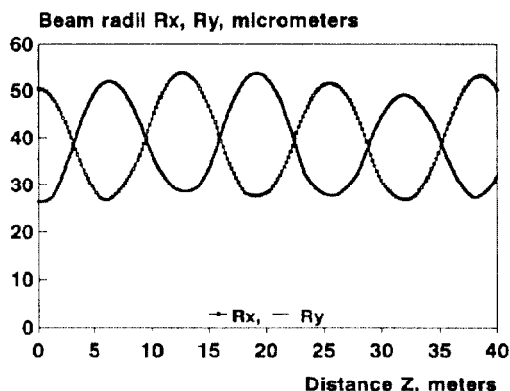


Figure 1. Electron beam radii plots in XOZ and YOZ planes.

Beam envelopes calculated for the matched beam propagation are shown in the Figure 1. For all variants corresponding plots are close to each other. For the last variant maximum beam radii slightly exceed that values for the first and second variants (the difference is  $4 \mu\text{m}$ ) because of  $\approx 22\%$  emittance

\* Supported in part by Grant N MQH000 from the International Science Foundation.

growth in the third variant. This growth shown in Figure 2 is due to non-linear effect resulted from higher order terms in undulator focusing strength expanded as a Taylor series [10].

In general case phase synchronism problem can be characterised by two main parameters: maximum phase detuning for individual particle and phase detuning averaged over particle ensemble. The last parameter describes mainly FEL resonance condition between the e.m. wave and particle ensemble in undulator having subsections, tapering or field errors whereas the first one gives phase shift due to betatron motion of a single particle and concerned with a fraction of particles trapped into ponderomotive well [11].

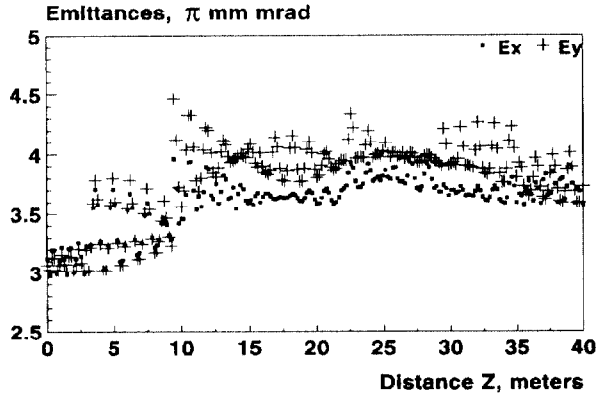


Figure 2. Normalised transverse emittances calculated along the undulator for the third variant of the Table 1.

Here we consider primarily the parameter of phase detuning averaged over particle ensemble which is calculated for the wavelength 4nm and energy spread 0.1%. In the Figures 3,4,6 we presented the simulations of the phase detuning averaged over particle ensemble and transverse momentum of the ensemble averaged over undulator period.

We can compare these plots obtained for the alternating gradient focusing (Figure 3) and periodic sextupole strong focusing (Figure 4). We see considerably lower phase deviations for the first variant (4°) than for the last variant (30°).

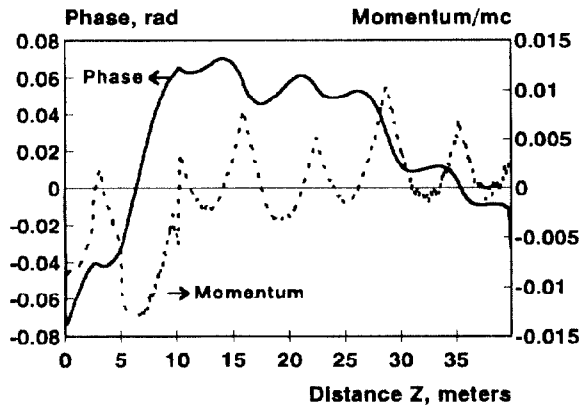


Figure 3. Average phase deviation from the synchronism and transverse momentum along the undulator for alternating gradient focusing (the first variant of Table 1).

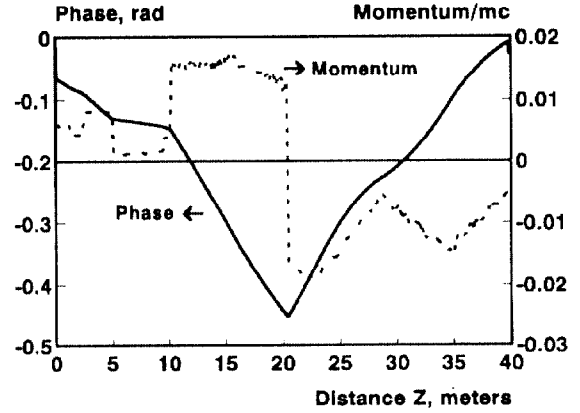


Figure 4. Average phase deviation from the synchronism and transverse momentum along the undulator for intrinsic sextupole focusing (the third variant of Table 1).

It is due to effect of smoothing of phase detuning at F-D transitions in the case of gradient focusing because of uncorrelated betatron oscillations of transverse momenta inside F or D section.

For a pure sextupole periodic strong focusing we have a step-wise function of averaged transverse momentum and a piece-wise linear function of the phase detuning (see Figure 5) because average transverse momenta are constant if the focusing strength is constant along the undulator [11]. We assume that this effect can be the main reason of shorter gain length of FEL having quadrupole focusing compared with FEL using periodic sextupole focusing according to simulations [6]. Emittance growth mentioned above can give an additional reducing of FEL performance.

One can try to improve the synchronism for the last case by imposing additional non-zero average transverse momentum to compensate the steps in the plot of the dimensionless momentum  $\langle P_{\perp} \rangle$  versus undulator length (see Figure 4). We have assumed that such a compensation can give a superimposed undulator field having the following amplitude  $B_a$ :

$$B_a(z) \sim k_a \frac{mc^2}{e} \sqrt{2(\langle P_{\perp} \rangle(z) - \langle P_{\perp} \rangle_{\min})},$$

where  $k_a$  is the wavelength of superimposed periodic field.

This small amplitude field can have either non-resonant period at any polarisation or the same period but other polarisation. As an example we found a suitable function  $B_a(z)$  presented in the Figure 5 to demonstrate such a possibility for the third harmonic. This function depends on focusing parameters and input beam matching.

Note, after additional iterations we could find more convenient a piece-wise constant function  $B_a(z)$ . In principle such correction can be made for quadrupole and combined focusing, however it will require rather complicated profile for  $B_a(z)$  (see Figure 3).

It is seen from Figure 6 that phase detuning in this case is considerably reduced (compared with the Figure 4) and became even lower than that for the case with quadrupole focusing (see the Figure 3).

All the results presented above are obtained for uniform ('K-V') particle distribution in transverse phase plane. However, the 'phase compensation' effect to be verified for others kinds of particle distribution.

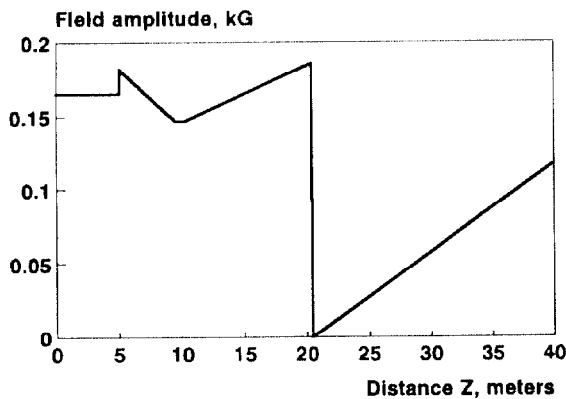


Figure 5. Amplitude profile of superimposed field (the third harmonic of fundamental undulator field) intended to reduce average phase detuning for sextupole focusing.

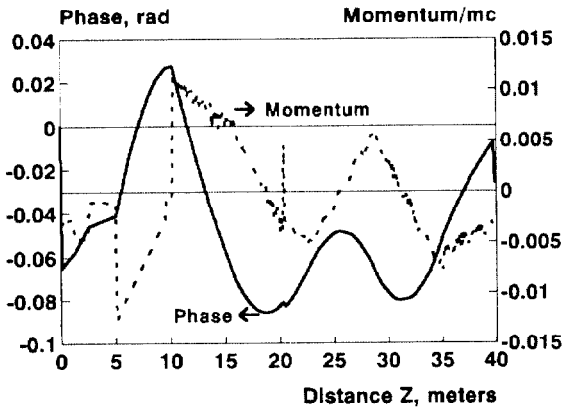


Figure 6. Average phase deviation from the synchronism and transverse momentum along the undulator with intrinsic sextupole focusing and superimposed periodic field (see Figure 5).

We found that this effect (see Figure 6) has almost the same result for the 'water bag' and Gaussian distributions. Calculations of the maximum phase detuning done for a single reference particle (it is not shown in the Figures) indicated approximately the same value 0.5rad for all variants from the Table 1 (with exception of Gaussian distribution).

It is not a difficult problem to provide the intrinsic gradient values required (see Table 1) or even rather more high gradients [4], however strong sextupole intrinsic focusing requires advanced schemes [12] or novel configurations. Such possible configurations are under consideration and the problem of effective aperture decreasing to be investigated.

#### 4. CONCLUSION

1. Simulations indicated that intrinsic sextupole-type strong periodic focusing has two disadvantages: large phase

deviation from the resonance and emittance growth which are due to effects of a step-wise behaviour of transverse momentum and focusing strength nonlinearities respectively. Alternating gradient focusing and combined-function nonlinear wigglers, in which quadrupole and sextupole fields are superposed, have relatively small phase detuning averaged over particle ensemble.

2. Phase synchronism can become better in undulator with strong periodic sextupole focusing compared with alternating gradient focusing if an appropriate additional periodic field with small amplitude modulated along the undulator is introduced. This phase detuning compensation takes place independently on the kind of particle distribution in transverse phase space.

#### 5. ACKNOWLEDGEMENTS

The authors would like to thank Drs. A.S. Khlebnikov and A.H. Hairtadinov for fruitful discussions of concrete magnetic systems providing strong intrinsic focusing.

#### 6. REFERENCES

- [1] Proc. Workshop on Fourth Generation Light Sources, SLAC, February 24-27, 1992, SSRL 92/02.
- [2] B.E. Newman, J.L. Goldstein, J.S. Fraser, R.C. Cooper. A linac-driven XUV Free Electron Laser. AIP Conf. Proc. No 118 (1983), 190.
- [3] L.H. Yu, C.M. Hung, D.Li, S. Krinsky. Scaling Function For Free-Electron-Laser Gain Including Alternating Gradient Focusing. Preprint BNL 49122 (1993).
- [4] J. Claus. Strong Betatron Focussing in Linearly Polarized Wigglers. Preprint BNL (1992)
- [5] G. Travish, J. Rosenzweig. Strong Focusing for Planar Undulators. AIP Conf. Proc. No 279. (1992), 277.
- [6] G. Travish, J. Rosenzweig. Numerical Studies of Strong Focusing in Planar Undulators. Proc. of the IEEE Particle Accelerator Conf., (1993), 2081
- [7] A.A. Varfolomeev, A.H. Hairtadinov. Focusing Properties of an Undulator with Anharmonic Magnetic Field. Presented at the 16-th Int. FEL Conf. and to be published in NIM A(1995)
- [8] C. Pellegrini, J. Rosenzweig, H.-D. Nuhn, P. Pianetta, R. Tatchyn, H. Winick, K. Bane, P. Morton, T. Raubenheimer, J. Seeman, K. Halbach, K.-J. Kim, J. Kirz. A 2-4 nm High Power FEL on the SLAC Linac. Nucl. Instrum. and Meth., NIM A331 (1993), 223.
- [9] A.V. Smirnov. 3D-code UNDYNA for electron beam transport simulations in planar undulators. Preprint IAE-5655/14, Moscow, Russia (1993)
- [10] J.J. Barnard. Anharmonic betatron motion in free electron lasers. Nucl. Instr. and Meth. A296(1990) 508.
- [11] E.T. Scharlemann. Wiggle plane focusing in linear wigglers. J. Appl. Phys. 58 (6), (1985), 2154
- [12] A.A. Varfolomeev, A.H. Hairtadinov. Advanced Hybrid Undulator Schemes Providing Enhanced Transverse E-Beam Focusing. Nucl. Instrum. and Meth., NIM A331 (1994), 462.