ABOUT THE POSSIBILITY OF STORAGE AND ACCELERATION OF RIBBON BEAMS IN RING MAGNETS

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Abstract

A modification of usual spiral-sector magnets is described which gives strongly anisotropic focussing, $Q_x \ll Q_z$. It may enlarge the incoherent space charge limit by an order.

The radial growth of the average field in spiral-sector FFAG accelerators enables the holding of particles with different energies at different turns of the orbit, and the alternating gradient of the sector edge field gives nearly isotropic focussing, $Q_x \approx Q_z$. But the beam of particles occupies in these magnets relatively small part of the radial aperture.

We describe here a modification [1] of usual magnets which gives strongly anisotropic focussing, $Q_x \ll Q_z$. It enables one to fill with a "monoenergetic" beam all the radial aperture, which may exceed the pole gap e.g. by 20 times. This enlargement of the incoherent space charge limit may be useful in a storage ring or in a synchrotron with ribbon (radially wide) beam with the number of particles exceeding the usual value e.g. by 10 times.

Such machines may be useful e.g. for intense neutron pulses generation or for inertial ion nuclear fusion. The increased radial width of the beam may facilitate the slow extraction.

A simple limiting case of the anisotropic focussing is the "planar" focussing of the straight infinitely wide ribbon beam [2]; this focussing is the product of the large longitudinal velocity of the beam with the alternating gradient magnetic field of currents inclined to the beam; this field grows linearly with the distance from the median plane.

The maximal number of particles N at a circular orbit is determined usually by the incoherent shift of Q_x, Q_z. It is determined by the transverse field of a relatively long bunch with elliptical cross-section (semi-axes a, b in x,z directions),

$$E_x = \frac{\rho}{\varepsilon_0}\frac{bx}{a+b}, \quad E_z = \frac{\rho}{\varepsilon_0}\frac{az}{a+b},$$

which gives the defocussing forces

$$\delta f_x = eE_x/\gamma^2, \quad \delta f_z = eE_z/\gamma^2,$$

and the corresponding changes of betatron frequencies, e.g.

$$R.\delta(d^2x/ds^2) = x.\delta Q_x^2.$$

The last equation and analogous z-equation give the number of particles corresponding to $\delta Q_{x,z}$

$$N = 2\pi RS\rho/e = \frac{a+b}{b}\frac{S}{2r_0R}\beta^2\gamma^3\delta Q_x^2 = \frac{a+b}{a}\frac{S}{2r_0R}\beta^2\gamma^3\delta Q_z^2$$

where $S=\pi ab$ is the beam cross-section, $r_0=e^2/4\pi\varepsilon_0 mc^2$. If $a=b$ then we have the usual formula $N_0 = (S/r_0R)\beta^2\gamma^3\delta Q^2$.

In the general case, $a \neq b$, we have $\delta Q_x^2/\delta Q_z^2 = b/a$. For a ribbon beam, $a \gg b$, we have $\delta Q_z^2 \gg \delta Q_x^2$ and $N/N_0 \approx a/2b \gg 1$. For instance, if $a/b=25$, $Q_z \approx 3$, $\delta Q_z \approx 0.3$, then we have $\delta Q_z^2 \approx 2$, $\delta Q_x^2 \approx 0.08$

N/N_0=12.5 and the space charge lowers the Q_x from 0.4 to 0.3.

The radial aperture loss ΔR which is caused by the energy spread of a real "monoenergetic" beam is small; e.g. if $\Delta p/p$=0.15% and R=6m, then $\Delta R \gtrsim$ 7cm.

The realisation of the anisotropic focussing in spiral-sector magnets is possible by the reverse of the field index "k" sign: usually [3] k > 0 and the frequencies $Q_{x,z}$ are approximately equal. Our modification corresponds to $k \gtrsim -1$, that is to slow radial decrease of the field.

A more simple variant is a gradient magnet with a wide gap, slow radial decrease of the field ($-k = n>0$) and antisymmetrically inclined edges [1]. In the orbit plane the magnet poles look as "curvilinear parallelograms".

The corresponding magnet system consists of M periods OF_xMD_x for r-motion, OF_zMD_z for z-motion ($F_{x,z}$ - focussing and defocussing edges, M - gradient interval, O -field-free interval). Long straight sections of the type [2] may be included.

Multiplication of the correspondig matrices gives the the expressions for betatron phase shifts. For our case n>1 they are

$$\cos \mu_x = ch\left(\frac{m}{\rho}\sqrt{n-1}\right) + \frac{1}{2}\frac{l}{\rho}\sqrt{n-1}\, sh\left(\frac{m}{\rho}\sqrt{n-1}\right)\left(1-\frac{tg^2\theta}{n-1}\right).$$

$$\cos \mu_z = \cos\left(\frac{m}{\rho}\sqrt{n}\right) - \frac{1}{2}\frac{l}{\rho}\sqrt{n}\,\sin\left(\frac{m}{\rho}\sqrt{n}\right)\left(1+\frac{tg^2\theta}{n}\right)$$

Here θ is the angle between the orbit and the normal to the edge, m is the length of the gradient interval, l is the length of the field-free interval.

A computation of $\mu_{x,z}$ as functions of n and $tg^2\theta$ for a case M = 11, $0 < n < 10$, $0 < tg^2\theta < 20$, has given a rhomb-like stability region in the (n, $tg^2\theta$) plane. Its form is similar to the form of stability region for a usual spiral-sector accelerator ([4], fig. 79). If we take $n \gtrsim tg^2\theta \gtrsim 2$, then $\mu_z \gtrsim 1.5$, $\mu_x \gtrsim 0.15$, $Q_z \gtrsim 2.6$ and $Q_x \gtrsim 0.26$. This working point lies near the edge (μ_x=0) of the stability region. This fact is connected with the opposite actions of the constant gradient radial defocussing (n>1) and the edge alternating gradient radial focussing (which slightly prevails). But the corresponding tightening of tolerances for n and θ will not be excessive, because the needed values of n and $tg^2\theta$ are moderate.

The radial change of the field across the poles of the magnet may attain several tens of %% , e.g. ±30 % if R=6m , n=2 , a=1m ; the magnet gap is 10±5 cm. Such a magnet may be used e.g. in a storage ring with constant field (many-turn injection and one-turn ejection) or in a ribbon beam synchrotron.

As to the nonlinear betatron oscillations, one may suppose the existance of instability lines in the (n_x , n_z) plane, corresponding to the combinational resonances, $n_xQ_x + n_zQ_z = q$, $n_{x,z}$,q - integers. But the increments in our case ($Q_x << Q_z$) will be small, because the resonance conditions are fulfilled only for high orders of resonances when their strength is small.

So one can see that more detailed study of cirqular magnets with skewed edges and radial decrease of the field may give the possibility to increase the number of stored or accelerated particles by an order of magnitude.

References

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