Parallel Reflector Beam Waveguide as a Microwave Undulator *

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Abstract

A parallel reflector beam waveguide system has been studied for application in a microwave undulator. This study is aiming for development of a tunable undulator for shorter wavelength synchrotron radiation. The wave propagation between two open parallel concave reflectors has been considered and the deflecting field strength is found for higher order transverse electric modes.

1 INTRODUCTION

Previously, microwave undulators using various types of waveguides have been studied. A ridge guide was used in the microwave undulator [1,2]. Rectangular waveguides were also studied for undulator properties [3]. An elliptical waveguide was used in a superconducting system [4]. In these works, standing waves were used. For higher field, short wavelength radiation with these regular hollow waveguides, the waveguide dimension becomes too small. This constrains the peak microwave power level and the beam aperture.

The undulator wavelength is given by

$$\lambda_u \simeq \frac{\lambda_g}{2} \qquad (1)$$

where λ_g is a guide wavelength. The radiation is related to the undulator period as

$$\lambda_r = \frac{\lambda_u}{2\gamma^2}(1 + \frac{K^2}{2} + \gamma^2\theta^2). \qquad (2)$$

The positron beam parameters in the APS storage ring are as follows:

$$\begin{aligned} \textit{Beam energy} &= 7\ GeV \\ \textit{Beam current} &= 100\ mA \\ \sigma_x &= 0.342\ mm \\ \sigma_y &= 0.091\ mm \\ \sigma'_x &= 24\ \mu rad \\ \sigma'_y &= 9\ \mu rad \end{aligned}$$

The on axis radiation ($\theta = 0$) first harmonic spectra for different field values as a function of photon energy is shown in Figure 1.

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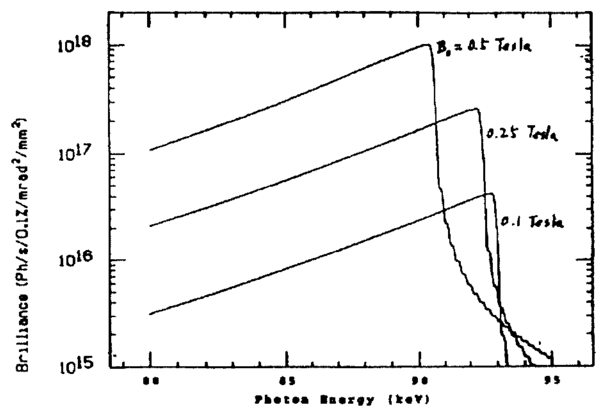

Figure 1: Brilliance vs. photon energy undulator radiation of the APS beam. λ_u=5mm, N=400 periods, L=2mm

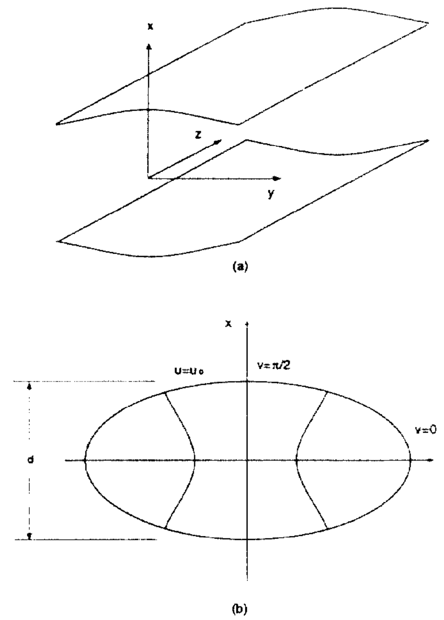

Figure 2: (a) Parallel reflector beam waveguide, (b) Elliptical coordinate system used for parallel reflector beam waveguide

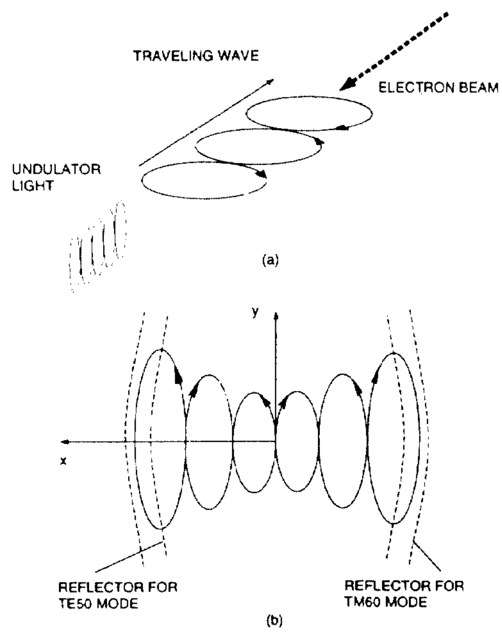

Figure 3: (a) Traveling wave undulator, (b) Transverse standing waves for TE_{50} and TM_{60} modes

2 MODE OF OPERATION

A structure composed of two parallel cylindrical reflectors, as shown in Figure 2(a), is known as a beam waveguide which can support wave propagation. The fields of certain modes in open reflector waveguides resemble the fields in elliptic waveguides [5]. Thus the reflectors can be treated as the partial elliptical waveguide. For this structure the surface near the major axis of the ellipse can be removed. This structure can be used in a microwave undulator. The structure may have a few advantages. The beam will not touch the waveguide wall even at high frequency operation. The side openings can provide easier HOM damping and easier vacuum pumping.

Looking on the mode of operation, we can use either a standing wave (SW) or a traveling wave (TW). For SW undulation, one has to use a fixed cavity structure. The useful radiation occurs with the backward wave. Thus, a traveling wave opposite to the beam direction as shown in Figure 3(a) can achieve the same result. In a traveling wave, the electric and magnetic fields are in phase. The relativistic electron must move against the wave front so that both the electric and magnetic fields contribute to particle deflection.

The equivalent undulator magnetic field is given as

$$B_u = \frac{E_t}{c} + B_t \tag{3}$$

where E_t and B_t are the transverse fields on the beam axis.

3 ELLIPTICAL BEAM WAVEGUIDE

In this work the field expression is based on a beam mode description in the rectangular coordinates. An elliptical waveguide is shown in Figure 2(b) with the elliptical coordinates. The beam waveguide is a part of the elliptical waveguide . The fields of the structure are shown in Figure 3(b). The electric field for the odd order transverse electric (TE_{mo}) mode and the magnetic field for the even order transverse magnetic (TM_{mo}) mode are shown in Figure 3(b). These modes can be used for undulation of the beam. The positron beam is undulated in the $y-z$ plane and the $x-z$ plane for the TE and TM modes, respectively. The node width and the beam waist determine the undulator aperture size.

The wave function for these modes is [6]

$$\begin{aligned}\phi_{mo}(x,y,z) &= \frac{\frac{e^{-\beta^2_{mo}y^2}}{[1+\frac{(2\beta^2_{mo}x)^2}{k^2_{mo}}]}}{[1+\frac{(2\beta^2_{mo}x)^2}{k^2_{mo}}]^{1/4}} sin[(k_{mo}x \\ &+ \frac{2k_{mo}\beta^4_{mo}xy^2}{k^2_{mo}+4\beta^4_{mo}x^2} \\ &- (1/2)tan^{-1}(\frac{2\beta^2_{mo}x}{k_{mo}})]e^{\pm jh_{mo}z}\end{aligned} \tag{4}$$

where $k_o^2 = k_{mn}^2 + h_{mn}^2$, and β_{mn} is the wavenumber in the y-direction. For TE to z modes,

$$\begin{aligned}E_x(x,y,z) &= \frac{\partial\phi(x,y,z)}{\partial y} \\ E_y(x,y,z) &= \frac{\partial\phi(x,y,z)}{\partial x} \\ E_z(x,y,z) &= 0 \\ H_x(x,y,z) &= \frac{-h_{mo}}{\omega\mu}\frac{\partial\phi(x,y,z)}{\partial x} \\ H_y(x,y,z) &= \frac{-h_{mo}}{\omega\mu}\frac{\partial\phi(x,y,z)}{\partial y} \\ H_z(x,y,z) &= \frac{k^2-h^2_{mo}}{j\omega\mu}\phi(x,y,z).\end{aligned}$$

The transverse wavenumber $k_{mn} < k_o$ for a field evanesces in the y-direction. Then, the number of modes with $n=0$ as shown in Figure 3(b) is

$$p = \frac{1}{\pi}\{dk - tan^{-1}(\frac{d}{\sqrt{2bd-d^2}})\} \tag{5}$$

where b is the focal length of a reflector and d is the distance between the reflectors. For modes with $n=0$,

$$k_{mo}(m) = m\pi + tan^{-1}(\frac{d}{\sqrt{2bd-d^2}}) \tag{6}$$

and

$$\beta_{mo}(m) = \sqrt{\frac{k_{mo}(m)}{\sqrt{2bd-d^2}}}. \tag{7}$$

As in the rectangular waveguide, the modes in the beam waveguide may be treated as a sum of two waves propagating in the z-direction in the x-z plane. The decomposed wave propagation angle

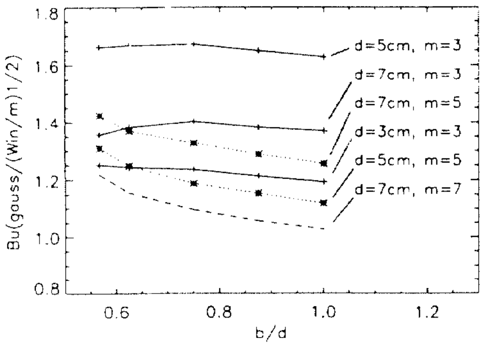

Figure 4: Undulator field strength of TE_{mo} modes vs. b/d. f=30GHz

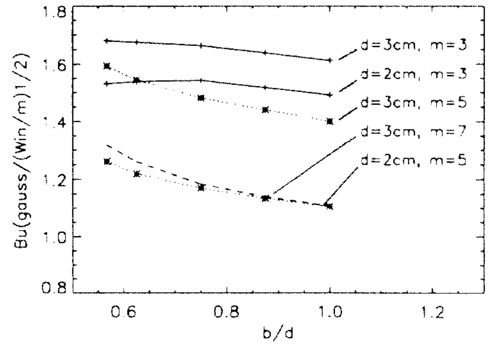

Figure 5: Undulator field strength of TE_{mo} modes vs. b/d. f=60GHz

$$\theta_{mo}(m) = sin^{-1}\frac{k_{mo}(m)}{\sqrt{2bd-d^2}}. \qquad (8)$$

This angle is an important parameter for launching a specific mode in the parallel reflector beam waveguide. The beam waists at $x = 0$ and on the mirror are

$$\omega_o(m) = \sqrt{\frac{\sqrt{2bd-d^2}}{k_{mo}(m)}} \qquad (9)$$

$$\omega_s(m) = \sqrt{\frac{2d}{\sqrt{\frac{d}{b(2-d/b)}}k_{mo}(m)}}. \qquad (10)$$

Dissipated power in the structure is given by

$$P_d = R_s \int_0^{2\pi}\int_0^{\ell} |H_z(x(u_o,v),y(u_o,v),z)|^2 f(u_o,v)dvdz \qquad (11)$$

where R_s is the surface resistance of the reflector, ℓ is the length of the beam waveguide, and $f(u,v) = f_o\sqrt{sinh^2(u) - sin^2(v)}$ is the scale factor in an elliptical coordinate system. The time average power flow is given by

$$P = \int_0^{u_o}\int_0^{2\pi} (E_xH_y - E_yH_x)\ f^2(u,v)\ dudv. \qquad (12)$$

The dissipated power in the waveguide wall is normalized to the power flowing through the waveguide so that the field strength for a unit input power can be found. For a unit dissipated power, the undulator field is

$$B_u = (\frac{E_y}{c} + B_x)/\sqrt{P_d}. \qquad (13)$$

Figures 4 and 5 show B_u vs. b/d for a few different cases at $30GHz$ and at $60GHz$, respectively. The field strength becomes higher as b/d approaches 0.5 where the resonance is unstable. For TM modes, steps similar to the above can be used to predict the equivalent undulator field. The undulator fields for TM modes are lower than the fields of TE modes.

4 CONCLUSION

A beam waveguide type microwave undulator has been considered. The effective undulator field B_u of the reflector beam waveguide shows that the field can exceed the field of the rectangular waveguide operated at a frequency around $15GHz$ for a useful beam aperture. The width and the waist of the node at the transverse field center determine the effective beam aperture. Using a traveling wave requires an output coupler to either reuse the power by feedback or dissipate the unused power.

5 REFERENCES

[1] T. Shintake, K.Huke, and J. Tanaka, "Microwave Undulator", Japanese Journal of Applied Physics, Vol.21, No.10, October 1982

[2] T. Shintake, K.Huke, J. Tanaka, I. Sato, and I. Kumabe, "Development of Microwave Undulator", Japanese Journal of Applied Physics, Vol.22, No.5, May 1983

[3] K. Batchelor, "Microwave Undulator", Linear Accelerator Conference Proceedings, Stanford, June 1986

[4] R. Boni, A. Savia, B. Spataro, F. Tazzioli, P. Fabbricatore, R. Parodi, and P. Fernandes, "Superconducting Microwave Undulator", Rev. Sci. Instrum., July 1989

[5] F.T.Tischer and J.R.Potukuchi, "Waves Guided between Open Parallel Concave Mirrors", IEEE Trans. on Microwave Theory and Techniques, July 1974

[6] T. Nakahara and N. Kurauchi, "Guided Beam Waves between Parallel Concave Mirrors", IEEE Trans. on Microwave Theory and Techniques, Vol. MTT-15, No. 2, February 1967