Transverse Beam Parameter Measurements at the INR Proton Linac

I.N.Birukov, A.N.Mirzojan, P.N.Ostroumov, S.A.Petronevich
Institute for Nuclear Research of the Russian Academy of Sciences
Moscow, 117312, Russia

Abstract

During the commissioning of a two-part linac, it is important to determine the beam parameters in the first part in order to match the beam with the focusing system of the second part. A wire scanner is sufficient to determine the phase ellipse parameters at the focusing channel exit of a drift tube cavity. Five wire scanners placed at equivalent points in the periodic focusing channel are used to determine the phase ellipse parameters between the DTL and the DAWL of the INR linac.

1 INTRODUCTION

Two methods of emittance reconstruction are discussed in this paper. The first one uses results of the beam size measurements carried out by wire scanner under simultaneous gradient variation in all focusing elements. This method is used in the LAMPF linac in order to find the quality of the beam matching downstream of the first DTL tank [1]. The other method allows us to obtain beam parameters from profile measurements by using five wire scanners placed periodically along the focusing channel.

2 GRADIENT VARIATION METHOD

Due to a shortage of space between our DTL tanks, only wire scanner can be placed there. This is enough to determine the phase ellipse parameters of a beam in a periodic focusing channel. A small variation of the focusing gradients in a long channel causes a change of the phase advance of the transverse oscillations. For example, in the first DTL tank of the INR linac the phase advance is $\Delta\Phi = 12.84\pi$. Simultaneous deviation of focusing gradients in a range of $\pm 6\%$ in the vicinity of the nominal value provides a change of the phase advance of $\sim 2\pi$. The center and rms values of the beam sizes are measured as function of the focusing gradients in all the quadrupole lenses of the DTL tank.

Let us obtain equations for the phase ellipse parameters. The solution of the equation of motion on transverse plane can be written as:

$$x(\tau) = a\chi(\tau) + a^*\chi^*(\tau) = A\sigma(\tau) \cdot \cos(\Psi(\tau) + \Theta) \quad (1)$$

where χ, χ^* is the complex conjugate fundamental set of solutions, $\sigma(\tau)$, $\Psi(\tau)$ is the modulus and phase of fundamental solution, $d\tau = dz/S$, where S is length of the focusing period.

Let us consider the phase volume projection of the beam which is restricted by the ellipse:

$$\gamma(\tau)x^2 + 2\alpha(\tau)xx' + \beta(\tau)x'^2 = F_0 \quad (2)$$

At any point along the focusing channel, the parameters α, β, γ are determined by the equations:

$$\gamma(\tau) = \sigma'(\tau)^2 + \frac{1}{\sigma(\tau)^2}, \quad \alpha(\tau) = -\sigma(\tau)\sigma'(\tau),$$
$$\beta(\tau) = \sigma(\tau)^2, \qquad A = \sqrt{F_0} \quad (3)$$

For practical applications it is useful to transfer the ellipse equation to coordinates x, dx/dz with parameters $\beta = S \cdot \beta(\tau)$, $\gamma = \gamma(\tau)/S$, $\alpha = \alpha(\tau)$, $\epsilon = F_0/S$. The motion of particles in a periodical focusing channel can be matched. In this case the fundamental solution is expressed by modulus $\rho(\tau)$ and phase $\Phi(\tau)$ of the Floquet function. For an unmatched beam, the σ function is

$$\sigma(\tau)^2 = \rho^2(\tau) \cdot \{C_1^2 + C_2^2 + 2C_1C_2\cos(2\Phi(\tau) + \Theta)\} \quad (4)$$

$C_1^2 - C_2^2 = 1$ is the normalization condition. It is easy to obtain the relation between the beam size and the phase ellipse parameters

$$\frac{r_x^2}{F_{0x}\rho_{0x}^2} = u_1 + u_2\cos(2\Phi(\tau) + \Theta) \quad (5)$$

where $u_1 = C_1^2 + C_2^2$, $u_2 = 2C_1C_2$, ρ_{0x} is the modulus of the Floquet function at the exit of the focusing channel. From Eq. 5 we obtain:

$$\frac{r_{xM}^2 r_{xm}^2}{F_{0x}\rho_{0x}^4} = u_1^2 - u_2^2 = 1$$

where r_{xM}, r_{xm} are the maximum and minimum beam sizes measured as the gradients are varied (Fig. 1). It is easy to find:

$$F_{0x} = \frac{r_{xM}r_{xm}}{\rho_{0x}^2},$$
$$u_1^2 = \frac{r_{xM}^2 + r_{xm}^2}{2r_{xM}r_{xm}}, \qquad u_2^2 = \frac{r_{xM}^2 - r_{xm}^2}{2r_{xM}r_{xm}}$$

Thus, the ellipse parameters can be written as

$$\beta = \frac{S\rho_0^2}{2r_{xM}r_{xm}}\{r_{xM}^2 + r_{xm}^2 + (r_{xM}^2 - r_{xm}^2)\cos\Theta_{0x}\}$$
$$\alpha = -\frac{1}{2r_{xM}r_{xm}}\{\rho_{0x}\rho'_{0x}(r_{xM}^2 + r_{xm}^2) +$$
$$+\sqrt{1 + \rho_{0x}^2\rho_{0x}'^2} \cdot (r_{xM}^2 - r_{xm}^2)\cos(\Theta_{0x} + \Psi_{0x})\} \quad (6)$$
$$\gamma = \frac{1 + \alpha^2}{\beta}, \qquad \epsilon = \frac{r_{xM}r_{xm}}{S\rho_{0x}^2}$$

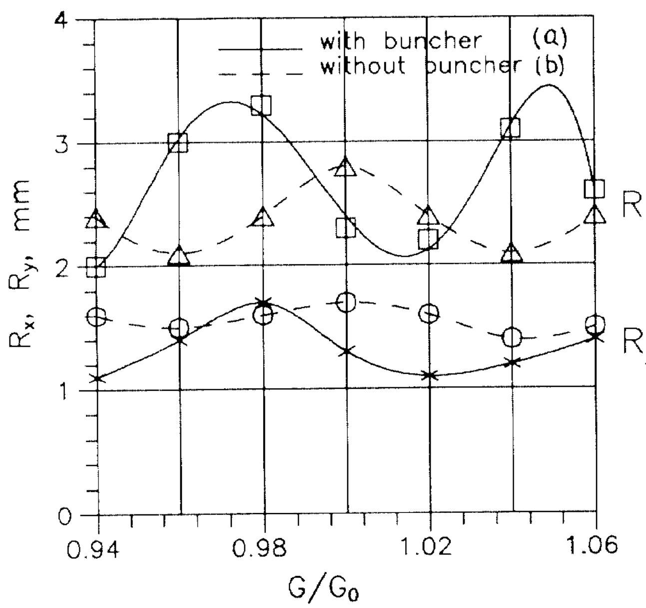

Figure 1: Measured beam sizes

where

$$\Psi_{0x} = \arctan \frac{1}{\rho_{0x}\rho'_{0x}}, \qquad \rho'_{0x} = \left(\frac{d\rho}{d\tau}\right),$$

$$\cos\Theta_{0x} = \frac{2r_{x0}^2 - (r_{xM}^2 + r_{xm}^2)}{(r_{xM}^2 - r_{xm}^2)}$$

The sign of Θ_{0x} coincides with a sing of derivative r' at $G/G_0 = 1$. It is well known that ρ and ρ' can be expressed through the elements of focusing period matrix m_{ij} :

$$\rho_{0x} = \frac{1}{\sqrt{\nu\cos\epsilon_a}} \qquad \rho'_{0x} = -\frac{\tan\epsilon_a}{\rho_{0x}}$$

where

$$\nu = \sqrt{-\frac{m_{21}}{m_{12}}} \qquad \sin\epsilon_a = \frac{m_{11} - m_{22}}{2\sqrt{-m_{12}m_{21}}}$$

It is clear that expressions like Eq. 6 can be written for the y-plane also. In that way it is enough to know the rms sizes r_{xM}, r_{xm} , r_{x0} to determine the rms-emittance and its orientation in phase space. The experimental curves obtained at the exit of the first DTL tank for 20-MeV beam are plotted in Fig. 1. Using these data it is possible to calculate the degree of mis-match. Particularly, from Fig. 1a it is obvious that the bunchers have considerable influence on the matching conditions. This comes from an increase in the space charge forces. Fig. 1b shows the curves which have been obtained after careful matching of the beam.

3 PERIODIC MEASUREMENTS METHOD

In the transition region between the DTL and the DAWL of the INR linac the focusing structure changes from the FODO to FDO. Therefore it is important to determine the phase ellipse parameters for beam matching between the section. The beam rms-sizes are used to reconstruct the phase ellipse. The profile measurements are carried out with the wire scanners (WS) placed periodically, through one focusing period along the first module of the DAWL. The beam matching is done with two quadrupole doublets placed between the DTL and the DAWL.

Table 1: Phase ellipse parameters

Plane		β cm/mrad	α	ϵ_n $\pi\cdot$cm$\cdot$mrad
X	Design	0.121	0.047	
	Measured	0.14	0.19	0.11
Y	Design	0.301	1.386	
	Measured	0.26	0.91	0.075

The problems with reconstructing α, β, γ can be seen from two models. In the first case the beam envelope can be expressed through Floquet function using Eqs. 1– 4. A practical focusing system is not strictly periodic because of small changes of a focusing period's length. Therefore it is better to use the transfer matrices of each focusing period. Let us consider α_0, β_0, γ_0 the parameters of the phase ellipse at location of the first WS. In the position of the i-th WS the parameter β is

$$\beta_i = C_i^2\beta_0 - 2C_iS_i\alpha_0 + S_i^2\gamma_0$$

where

$$\begin{pmatrix} C_i & S_i \\ C_i' & S_i' \end{pmatrix}$$

is the transfer matrix between the first and i-th scanners. Parameter β_i can be expressed as a function of the beam envelope

$$\beta_i = \frac{r_i^2}{\epsilon} = \frac{r_i^2\kappa_i}{\epsilon_n}$$

where ϵ_n is the normalized emittance, r_i is the beam size obtained from measurements, $\kappa_i = (\beta\gamma)_i$ is the relativistic factor. Then

$$\kappa_i r_i^2 = C_i^2\epsilon_n\beta_0 - 2C_iS_i\alpha_0\epsilon_n + S_i^2\frac{1+\alpha_0^2}{\beta_0}\epsilon_n, \; i = 1 \div 5$$

The solution of this equations is found using the least square technique. The values of rms-emittances are shown in Table 1. After calculating the ellipse parameters for each plane, the beam envelopes have been reconstructed (solid line in Fig. 2). Results of the measurement are shown by points and the dashed line corresponds to the matched beam envelope. The reconstructed phase ellipses at the first WS position as well as the tangents obtained by the transformation of the measured beam sizes are shown in Fig. 3. The error of the rms-size determination is 0.2 mm and from Fig. 3 we can see sufficiently high precision of the emittance reconstruction. The data shown in Fig. 2 correspond to the design gradient set in all focusing lenses

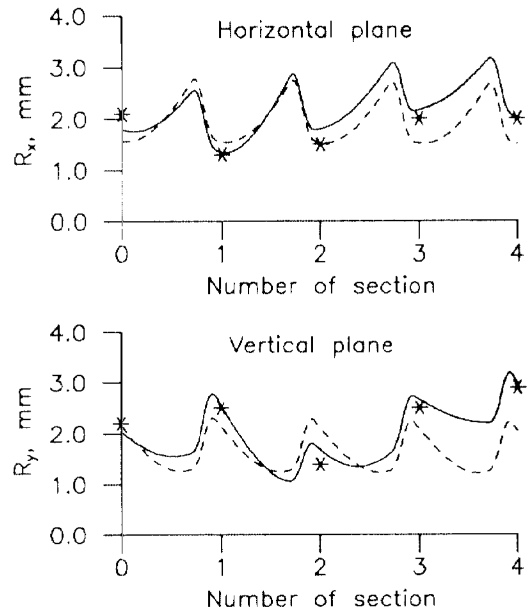

Figure 2: Beam envelopes along sixth tank

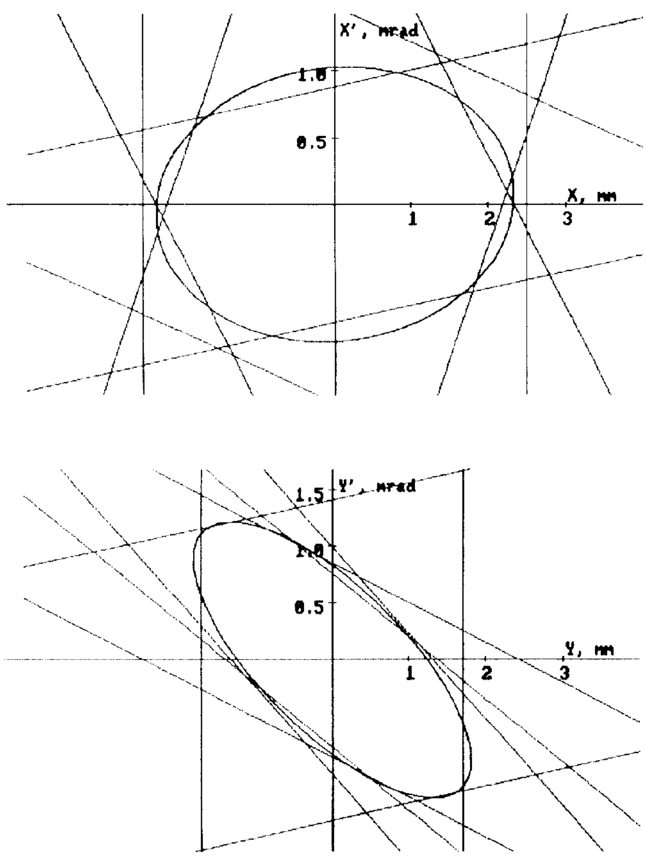

Figure 3: Reconstructed phase ellipse and transformed tangents

along DTL. The comparison of experimental and calculated beam envelopes shows the real focusing channel is close to the design one.

4 REFERENCES

[1] A.Browman and J.Hurd. Los Alamos Scientific Laboratory, LAMPF, Private communication