Beam Interaction with Pumping Holes in Vacuum-Chamber Walls

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Abstract

The longitudinal and transverse coupling impedances produced by a small pumping hole in the walls of an accelerator vacuum chamber are analytically evaluated at frequencies below cut-off. The method developed is based on the Bethe theory of diffraction by small holes. The estimates of the contribution from such elements to the coupling impedance of the UNK and LHC vacuum chambers are obtained.

1 INTRODUCTION

There is a general tendency to minimise beam-chamber coupling impedances to avoid beam instabilities and reduce heating. In doing so one tends to shield enlargements of the vacuum chamber, i.e. vacuum boxes, bellows, etc. On the other hand, the presence of vacuum pumping holes in the shields is required to provide high vacuum inside the beam pipe. The number of such elements can be very large in big machines. For example, so called liners in the UNK chamber have nearly 10^5 slots. The LHC design includes a thermal screen with 10^7 small holes for pumping. So, the evaluation of the coupling impedances for these chamber elements is of great importance. Due to the absence of axial symmetry, a numerical solution to the problem is to be essentially three-dimensional. It implies time-consuming computations even for simplified models. This paper presents the analytical calculation of the coupling impedances for a small pumping hole in the chamber with an arbitrary cross section.

2 EVALUATION METHOD

To evaluate the coupling impedance we have to calculate the fields induced in the chamber by a given current perturbation. The fields produced by a relativistic point charge in the chamber without hole are evaluated easily and then can be considered as incident electromagnetic waves on the hole. According to the Bethe theory of diffraction by small holes [1], the diffracted fields can be obtained as those radiated by effective surface "magnetic" currents or, in the case of a small hole, simply by effective electric and magnetic dipoles. Thus, when this approach is applicable, one can replace the excited hole by effective dipoles, evaluate the fields radiated by them inside the chamber and obtain the coupling impedance.

We consider an infinite cylindrical pipe with an arbitrary cross section S and perfectly conducting walls. The z axis is directed along the pipe axis, a hole is located at the point $(\vec{b}, z=0)$, and a typical hole size h satisfies $h \ll b$. The point charge q moves with velocity $v=c$ along the chamber axis with a transverse offset $\vec{s}$. Then the e.m. fields harmonics $\vec{E}^{(0)}, \vec{H}^{(0)}$, which would be produced by this charge on the chamber wall without hole, can be expressed as a series

$$E_\nu^{(0)}(\vec{b}, z; \omega) = Z_0 H_\tau^{(0)}(\vec{b}, z; \omega) = -Z_0 q e^{ikz} \sum_{n,m} \lambda_{nm}^{-1} \psi_{nm}(\vec{s}) \nabla_\nu \psi_{nm}(\vec{b}) , \quad (1)$$

where λ_{nm}, $\psi_{nm}(\vec{r})$ are eigenvalues and orthonormalized eigenfunctions (EF) of the 2D boundary problem in S:

$$\left(\nabla^2 + \lambda_{nm}\right)\psi_{nm} = 0 \; ; \; \psi_{nm}|_{\partial S} = 0 \; . \quad (2)$$

All other components vanish on the wall. In the case of a circular cross section with radius b the field harmonics on the hole, i.e. in $(r=b, \varphi=\varphi_h)$, are

$$E_r^{(0)}(b, \varphi_h, 0; \omega) = Z_0 H_\varphi^{(0)}(b, \varphi_h, 0; \omega) = \frac{Z_0 q}{2\pi b}\left[1 + 2\sum_{n=1}^{\infty}\left(\frac{s}{b}\right)^n \cos n(\varphi_h - \varphi_s)\right] , \quad (3)$$

where φ_s is the azimuth angle of the beam.

Under conditions $h \ll b$, $\omega h/c \ll 1$ and $\Delta \ll h$, where Δ is the wall thickness, one can consider the hole excitation by the fields (1) in the spirit of Bethe's approach. To satisfy the boundary conditions on the hole, the effective surface "magnetic" charge density ρ_{mag} and current $\vec{J}_{mag}$ have to be introduced. To calculate the fields produced by this current at distances R from the hole, $R \gg h$, one can replace the excited hole by effective dipoles, which are expressed simply in terms of the incident fields (1):

$$\vec{M} = \alpha_m \vec{H}_\tau^{(0)} \; ; \; \vec{P} = \epsilon_0 \alpha_e \vec{E}_\nu^{(0)} , \quad (4)$$

where the magnetic α_m and electric α_e polarizabilities can be analytically evaluated in the case of an elliptic hole [2].

For the particular case of a circular hole with radius h,

$$\alpha_m = 4h^3/3 \; ; \quad \alpha_e = -2h^3/3 \; . \quad (5)$$

For a narrow longitudinal slot with width $w \ll l$

$$\alpha_m = \pi l w^2/24 \; ; \quad \alpha_e = -\pi l w^2/24 \quad (6)$$

and the condition for applying Bethe's theory is $\omega l/c \ll 1$.

The longitudinal impedance can be defined as

$$Z(\omega; \vec{s}, \vec{t}) = -\frac{1}{q}\int_{-\infty}^{\infty} dz e^{-ikz} E_z(\vec{t}, z; \omega) , \quad (7)$$

where $k = \omega/c$, $\vec{t}$ is the transverse offset of a test charge, and the component E_z of the e.m. field, radiated by the effective currents inside the chamber, depends on $\vec{s}$. The usual definition corresponds to $\vec{s} = \vec{t} = 0$. We expand the radiated fields in a series in waveguide eigenmodes [2]

$$\vec{F} = \sum_{nm} \left(a_{nm} \vec{F}^{+}_{nm} \theta(z) + b_{nm} \vec{F}^{-}_{nm} \theta(-z) \right) , \quad (8)$$

where $\vec{F}$ means either $\vec{E}$ or $\vec{H}$ and superscripts '±' denote fields with propagation factors $\exp(\mp\Gamma_{nm} z)$, $\Gamma_{nm} = (\lambda^2_{nm} - k^2)^{1/2}$, radiated respectively in the positive (+, $z > 0$) or negative (−, $z < 0$) direction. The unknown coefficients a_{nm} and b_{nm} can be found from the Lorentz reciprocity theorem (e.g., [2]) as

$$2i \frac{k}{Z_0} \lambda_{nm} \Gamma_{nm} \begin{Bmatrix} a_{nm} \\ b_{nm} \end{Bmatrix} = \iint_{hole} dS \vec{J}_{mag} \vec{H}^{\mp}_{nm} \quad (9)$$
$$= -i\omega (\mu_0 \vec{H}^{\mp}_{nm} \vec{M} - \vec{E}^{\mp}_{nm} \vec{P} + \ldots) ,$$

in which $\vec{M}$ and $\vec{P}$ are given by Eqs. 4, and the expansion of the integral is justified for a small hole.

3 LONGITUDINAL IMPEDANCE

Substituting Eqs. 8-9 into Eq. 7 one can easily integrate over z when $\omega < \omega_{cut}$ (all $\Gamma_{nm} > 0$). Taking into account Eq. 4, we get the longitudinal impedance of a hole:

$$Z(\omega; \vec{s}, \vec{t}) = -iZ_0 \frac{\omega}{c} (\alpha_e + \alpha_m) e_\nu(\vec{s}) e_\nu(\vec{t}) , \quad (10)$$

where $$e_\nu(\vec{r}) \equiv \frac{E^{(0)}_\nu(\vec{r})}{Z_0 q} = -\sum_{n,m} \lambda^{-1}_{nm} \psi_{nm}(\vec{r}) \nabla_\nu \psi_{nm}(\vec{b})$$

is just a normalised beam electric field on the hole, i.e. the solution of a standard 2D electrostatic problem in region S: to find an electric field on a conductive boundary produced by a charge placed in point $\vec{r}$. For simple cases e_ν can be obtained easily from the Gauss law, e.g., for a circular chamber cross section $e_\nu(0) = 1/(2\pi b)$, cf. Eq. 3, and

$$Z(\omega) = -iZ_0 \frac{\omega}{c} \frac{(\alpha_e + \alpha_m)}{4\pi^2 b^2} . \quad (11)$$

This result was obtained earlier by direct summation [3]. In the case of a circular hole of radius h the polarizabilities are given by Eq. 5 and

$$Z(\omega) = -i \frac{Z_0}{6\pi^2} \frac{\omega h^3}{c b^2} , \quad (12)$$

which shows an inductive contribution of the hole (we use $\exp(-i\omega t)$ time-dependence). Numerical result [4] obtained by using the 3D code T3 coincides well with this analytical result (factor ≈ 0.017 instead of the exact one $1/(6\pi^2) = 1.69 \cdot 10^{-2}$). It should be noted, that for a circular hole the dependence on the hole radius h and chamber one b can be simply derived in a qualitative way.[1] $Re\, Z$ is much smaller than $|Im\, Z|$, see [3] and [5].

[1] K. Bane, private communication and [4]

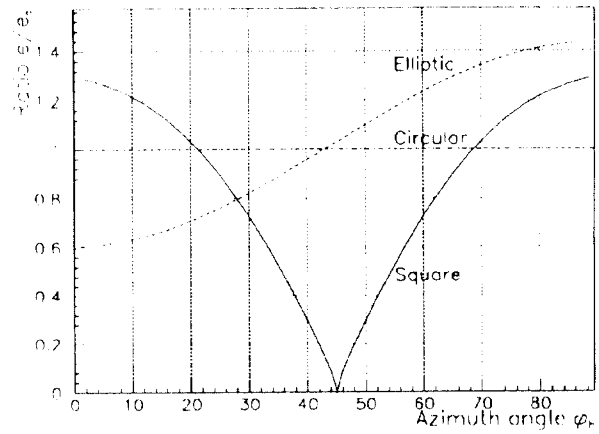

Figure 1: Ratio e_ν/e_c versus azimuth angle

For a very narrow longitudinal slot with account of Eq. 6 we would get $Z(\omega) = 0$, i.e. the impedance vanishes to the first-order approximation of our approach. It seems natural since such a slot does not interrupt essentially induced wall currents. Taking into account next-to-leading terms (when $w \ll l < b$) leads to

$$Z(\omega) = -i \frac{Z_0}{96\pi} \frac{\omega}{c} \frac{w^4}{b^2 l} \left(\ln \frac{4l}{w} - 1 \right) . \quad (13)$$

For a rectangular chamber with width a and height b, in which a hole in the side wall ($x_h = \pm a/2$) is displaced from the plane $y = 0$ by y_h, $|y_h| \le b/2$, Eq. 10 yields

$$Z(\omega) = -iZ_0 \frac{\omega}{c} \frac{(\alpha_e + \alpha_m)}{b^2} \Sigma^2 , \quad (14)$$

where $$\Sigma \equiv \sum_{m=0}^{\infty} \frac{\cos(2m+1)\pi y_h/b}{\cosh(2m+1)\pi x_h/b}$$

is the fast-converging series.

For an arbitrary cross section the best way to calculate the longitudinal impedance produced by a hole is to solve numerically the 2D problem for e_ν and obtain a result simply from a comparison with that for a circular cross section:

$$Z = Z_c (e_\nu/e_c)^2 , \quad (15)$$

where Z_c is given by Eq. 11. Fig. 1 shows ratios e_ν/e_c for the square chamber 70×70 mm^2 and elliptic one with semiaxes 40 and 30 mm. The code MGD2 [6] has been used and e_c was taken at radius b =35 mm.

4 TRANSVERSE IMPEDANCE

The dipole transverse impedance is defined by

$$\vec{Z}_\perp(\omega; \vec{s}, \vec{t}) = -\frac{i}{qs} \int_{-\infty}^{\infty} dz e^{-ikz} \left[\vec{E} + Z_0 \vec{\beta} \times \vec{H} \right]_\perp , \quad (16)$$

where the diffracted fields in the RHS are taken at $(\vec{t}, z; \omega)$, $\vec{\beta} \to (0,0,1)$ and the limit of $s \to 0$, $t \to 0$ is usually assumed. It is clear that both the E- and H-eigenmodes contribute to the integral dislike Eq. 7. After calculations similar to those for the longitudinal case we get

$$\vec{Z}_\perp(\omega;\vec{s},\vec{t}) = -\frac{i}{qs}\sum_{n,m}\Big[\lambda_{nm}^{-1}(\vec{\nabla}\psi_{nm})_t(\vec{\nabla}\psi_{nm}\cdot\vec{P}/\varepsilon_0)_h + \kappa_{nm}^{-1}(\vec{\beta}\times\vec{\nabla}\chi_{nm})_t(\vec{\nabla}\chi_{nm}\cdot Z_0\vec{M})_h\Big] , \quad (17)$$

where κ_{nm}, $\chi_{nm}(\vec{r})$ are eigenvalues and EF of Eq. 2 subjected to another boundary condition, $\chi_{nm}|_{\partial S} = 0$. The effective moments in the RHS are assumed to be produced by a dipole beam-field component, i.e. $P, M \propto s$, cf. $n = 1$ term in Eq. 3.

In the case of a circular cross section

$$\vec{Z}_\perp(\omega) = -iZ_0\frac{\alpha_m+\alpha_e}{\pi^2 b^4}\vec{a}_h\cos(\varphi_h-\varphi_b) , \quad (18)$$

where $\vec{a}_h$ is the unit vector directed to the hole, φ_h, $\varphi_b = \varphi_s = \varphi_t$ are asimuth angles of the hole and beam. This means that the deflecting force is directed to (or opposite to) the hole and its value depends on the azimuth angle between the beam-offset vector and the direction to the hole. For two particular cases we can obtain from Eq. 18: the transverse impedance of a circular hole is

$$\vec{Z}_\perp(\omega) = -iZ_0\frac{2h^3}{3\pi^2 b^4}\vec{a}_h\cos(\varphi_h-\varphi_b) \quad (19)$$

and that of a narrow slot is

$$\vec{Z}_\perp(\omega) = -iZ_0\frac{w^4}{24\pi b^4 l}\left(\ln\frac{4l}{w}-1\right)\vec{a}_h\cos(\varphi_h-\varphi_b) . \quad (20)$$

It can be concluded from Eqs. 12,13,19,20 that both the longitudinal and transverse coupling impedances of a narrow slot are essentially lower than those of a circular hole with the same pumping area.

If we consider M ($M \geq 3$) holes uniformly spaced in one cross section, the resulting impedance is

$$\vec{Z}_\perp(\omega) = -iZ_0\frac{\alpha_m+\alpha_e}{\pi^2 b^4}\frac{M}{2}\vec{a}_b , \quad (21)$$

where $\vec{a}_b$ is the unit vector in the direction of the beam transverse offset. It is seen that the deflecting force is now directed along the beam displacement, i.e. some restoration of the axial symmetry occurs, and the maximum value of $Z_\perp$ is only $M/2$ times larger than that for $M = 1$.

In the case of a rectangular cross section

$$\vec{Z}_\perp(\omega) = -iZ_0\pi^2(\alpha_m+\alpha_e)/b^4 \cdot[\vec{e}_x\Sigma_1+\vec{e}_y\Sigma_2](\Sigma_1\cos\varphi_b+\Sigma_2\sin\varphi_b) , \quad (22)$$

$$\text{where } \Sigma_1 \equiv \sum_{m=0}^{\infty}\frac{(2m+1)\cos(2m+1)\pi y_h/b}{\sinh(2m+1)\pi x_h/b} ;$$

$$\Sigma_2 \equiv \sum_{m=0}^{\infty}\frac{2m\sin 2m\pi y_h/b}{\cosh 2m\pi x_h/b} .$$

It follows from Eqs. 14 and 22 that the impedances of a hole in the rectangular chamber vanish when $|y_h| \to b/2$, i.e. a hole is near one of the corners.

5 ESTIMATES AND CONCLUSIONS

At frequencies below cut-off, small discontinuities contribute additively to the coupling impedance. We assume this additivity for estimates, but for higher frequencies the problem still remains to be examined, see [5].

Let us estimate the impedances produced by the pumping slots in the UNK liners. Approximately $N = 3260$ vacuum boxes with bellows are to be shielded by these liners and every liner has $M = 26$ pumping slots with width $w = 0.6$ cm and length $l = 6$ cm. We take the chamber radius $b = 3.5$ cm and the machine one $R = 3306$ m. Since in this case $l > b$ the figures obtained are rough. Approximating slots by long elliptic holes of the same area, with the length l and effective width $w_{eff} = 4w/\pi$, we get the coupling impedances:

Table 1: Impedance Estimates for UNK Slots

	$\lvert Z/n\rvert$ / Ohm	$Z_\perp$ / (Ohm/m)
One slot	$4.3\cdot10^{-8}$	0.46
One liner	$1.1\cdot10^{-6}$	6.0
Total	$3.6\cdot10^{-3}$	$2\cdot10^{4}$

In the LHC design it is supposed to shield the cold chamber walls by an internal thermal screen, which has $N = 10^7$ pumping holes with radius $h = 2$ mm. With longitudinal spacing $d = 1$ cm and machine radius $R = 4243$ m, there will be nearly $M = 4$ holes in a chamber cross section. We take the mean radius of the thermal screen $b = 1.5$ cm for estimates. The figures are shown below.

Table 2: Impedances of the LHC Holes

	$\lvert Z/n\rvert$/ Ohm	$Z_\perp$/ (Ohm/m)
One hole	$5.3\cdot10^{-8}$	4.0
One cross section	$2.1\cdot10^{-7}$	8.0
Total	0.53	$2\cdot10^{7}$

The values of the longitudinal and especially transverse total impedance are very large. So, we conclude that some modifications of this thermal-screen construction (say, replacing holes by slots) are necessary.

6 REFERENCES

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