Transverse Impedance of BESSY

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Introduction

The well established experimental technique adopted for the determination of the transverse impedance of an existing storage ring is according to Sacherer's theory the measurement of the head-tail growth rate and the measurement of the negative tune shift of the dipole mode as a function of beam current and chromaticity [1]. A measurement of the transverse impedance of BESSY with this technique is presented. After a brief introduction to the theoretical consequences a description of the experiment is given. The decay of the free coherent dipole oscillation is recorded and the damping rate is extracted. The tune shift is obtained after a Fourier transformation of the recorded decay. Measurements with long and short bunches are presented. The difficulties to find a unique broad band impedance model are discussed and necessary modifications of the theory will be pointed out.

Theory

The interaction of an oscillating bunch with the vacuum chamber shifts the frequency and leads to damping or growth of the bunch motion. Both effects depend on the beam current and are expressed as a complex frequency shift given at relativistic velocity by [2]:

$$\Delta\omega = j \frac{e\, I\, c\, R}{2\, Q_\beta\, E\, L} \frac{\Sigma_p Z_\perp(\omega_p)\, h_0(\omega_p - \omega_\xi)}{\Sigma_p h_0(\omega_p - \omega_\xi)}$$

with	$-\infty<p<+\infty$	c:	speed of light
$j=$	$\sqrt{-1}$	R:	average machine radius
e:	elementary charge	Q_β:	betatron tune
I:	beam current	E:	energy

The bunch length, L, varies for Gaussian bunches from author to author: $L = 4\,\sigma$ [3] ... $3.5\,\sigma$ [4] ... $\pi\,\sigma$ [5] ... $2\cdot\sqrt{2}\,\sigma$ [6]. The power spectrum of the dipole oscillation, $h_0(\omega_p-\omega_\xi)$, consists of lines at $\omega_p = (p + Q_\beta)\cdot\omega_0$ with a Gaussian envelop shifted by the chromatic frequency $\omega_\xi = \xi \frac{Q_\beta}{\alpha} \omega_0$ [3]:

$$h_0(\omega_p-\omega_\xi) = \frac{1}{\sqrt{\pi}} e^{-(\omega_p-\omega_\xi)^2\, \sigma^2/c^2}$$

with the revolution frequency, $\omega_0 = c/R$, the chromaticity, $\xi = dQ_\beta/Q_\beta\, /\, dp/p$, and the momentum compaction factor, α. The complex transverse impedance, $Z_\perp(\omega)$, is approximated by a broad band resonator [5]:

$$Z_\perp(\omega) = \frac{Q|Z_\perp(0)|}{\frac{\omega}{\omega_{res}} + j\, Q\left((\frac{\omega}{\omega_{res}})^2 - 1\right)}$$

with the resonance frequency ω_{res} and the quality factor Q (close to one).

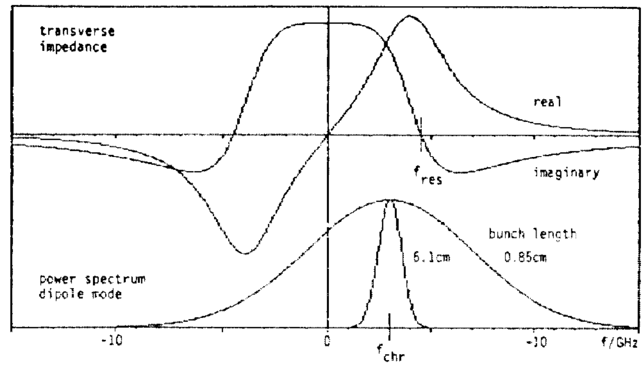

Fig. 1: Transverse broad band impedance and envelop of the dipole power spectrum for $\sigma = 0.85$ cm and $\sigma = 6.1$ cm.

Figure 1 shows real and imaginary part of the impedance together with the shifted envelop of the spectrum for short and long bunches. In the small BESSY ring ($\omega_0/2\pi = 4.8$ MHz) with the low emittance lattice ($\alpha = 0.014$) chromatic frequencies up to $2\pi\cdot 3$ GHz can be reached. As long as $\omega_{res} >> |\omega_\xi| + c/\sigma$ the real frequency shift, $\Delta\omega$, and the growth rate, $1/\tau$, are approximated by:

$$\Delta\omega = -\frac{e\, I\, c\, R}{2\, Q_\beta\, E\, L} |Z_\perp(0)|$$

$$\frac{1}{\tau} = \frac{e\, I\, c\, R}{2\, Q_\beta\, E\, L} |Z_\perp(0)|\, Q \frac{\omega_\xi}{\omega_{res}}$$

Experiment and Results

The experimental setup is similar to other beam response measurements in time domain [6, 7, 8]. A single bunch of electrons is excited every 100 ms with injection kicker magnets. The initial vertical amplitude amounts to a few mm. A stripline is used for the measurement of the center of mass motion. The 500 MHz component of the stripline signal is fed into a constant output amplifier and is demodulated with a diode. The demodulated signal goes through a low pass filter with a cut-off frequency of 2.5 MHz and is recorded with an 8 bit-ADC every 200 ns for more than 1.6 ms. The revolution time is 208 ns and the vertical single particle damping time amounts to 16 ms at 800 MeV. The bunch length, σ, can be varied from 0.85 cm to 6.1 cm with the existing two rf-systems [9, 10].

Usually the bunch length is fixed during one experiment and measurements are performed as a function of beam current and for eight different sextupole settings. The decay curves for bunches of 6.1 cm are shown in Fig. 2a left. In order to extract the head-tail growth rates the 8K data points are divided into 15 overlapping sets of 512 data points. Each set is Fourier transformed and the peak values close to the vertical tune of the 15 spectra are

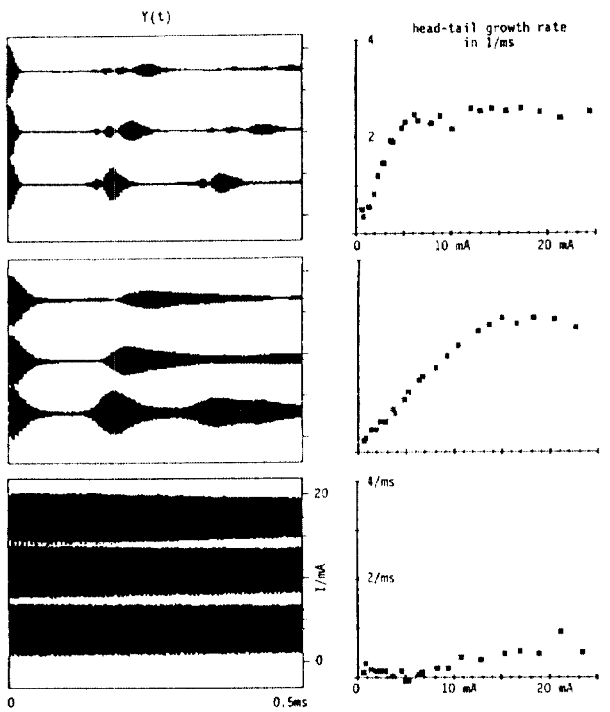

Fig. 2a: Decay of the center of mass motion with long bunches and the extracted head-tail growth rate as a function of beam current. The chromaticity increases from bottom to top.

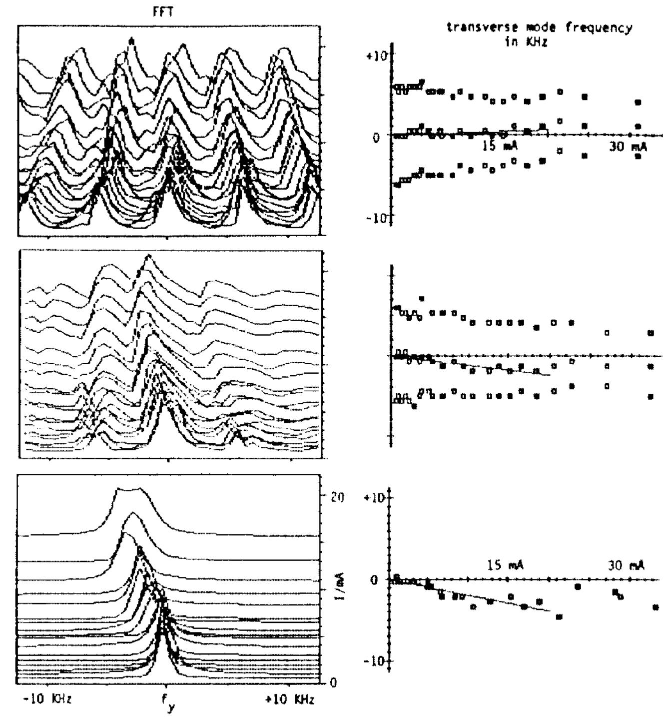

Fig. 2b: Fourier spectra of the motion from fig. 2a and the frequencies of the lowest modes as a function of beam current. The chromaticity increases from bottom to top.

used to fit the exponential decay. Obviously, the decay is not exponential, especially for larger chromaticities. The vertical center of mass motion is modulated with the synchrotron frequency [11]. The finite chromaticity and the energy spread lead to a periodical dephasing and phasing of the initial coherent vertical motion of the particles. The effect is most pronounced for small synchrotron frequencies. Never-theless, the extracted head-tail growth rates increase, as expected, linearly with beam current but level-off at around 3/ms, due to the small dynamic range of the measurement and the way the data are analysed. For comparison with theory the slope of the growth rate is extracted and displayed in figure 3 as full squares. In addition the recorded motion was Fourier transformed in order to find the shift of the dipole frequency (fig. 2b). The expected negative tune shift of the dipole mode, f_y, is clearly visible at least at low beam currents. The frequency of the dipole mode and the $|m|=1$ modes [3] have been extracted from the spectra and are shown in fig. 2b right. The slope of the dipole tune is deduced and displayed as full squares in fig. 3. Similar measurements were performed for shorter bunches.

Discussion in terms of the broad band impedance model

All experimental results are collected in fig. 3 together with the theoretical results obtained with the broad band impedance model. There is fair agreement in the case of the growth rates, but the tune shifts are not reproduced. Table 1 gives the results of individually fitted measurements as well as of a simultanous fit to all measurements. The impedance $|Z_\perp(0)|$ extracted from the tune shifts is generally found to be larger than the one used to fit the growth rates.

Table 1: resulting broad band impedance (Q=1)

bunch length σ [cm]	resonance frequency ν_{res} [GHz]	$\lvert Z_\perp(0)\rvert$ [MΩ/m] head-tail growth rate	dipole mode tune shift	Symbol in fig. 3
0.85	3.0	0.27	1.2	●
1.6	4.5	0.18	0.31	○
3.3	4.5	0.22	0.19	□
6.1	3.0	0.13	0.34	■
all σ's	4.5	0.18	0.49	

The behaviour of the growth rates at large chromatic frequencies suggests a resonance frequency of at least 2.5 GHz and the ratio of the tune shifts for different bunch lengths at small chromatic frequencies demand even higher resonance frequencies: only if the imaginary part of the impedance is effectively constant over the mode spectrum, the tune shift ratio will be proportional to $1/\sigma$ as found in the experiment. On the other hand a resonance frequency of 2 GHz seems more appropriate to fit the experimental tune shift for longer bunches. Finally resonance frequencies of at least 3 GHz were chosen since there are more experimental results available for the growth rates than for the tune shifts. The reason has been the insufficient stability of one of the quadrupole power supplies during parts of the measurements.

An alternative analysis of the experimental results was made for the measurements with long bunches. For chromatic frequencies smaller than π·GHz the measured growth rates are significantly higher than the calculation in fig. 3 and are better approximated by: $1/(\tau\, I\, \omega_\xi) = 360/(\text{s mA } 2\pi \text{ GHz})$ and the tune shift is nearly

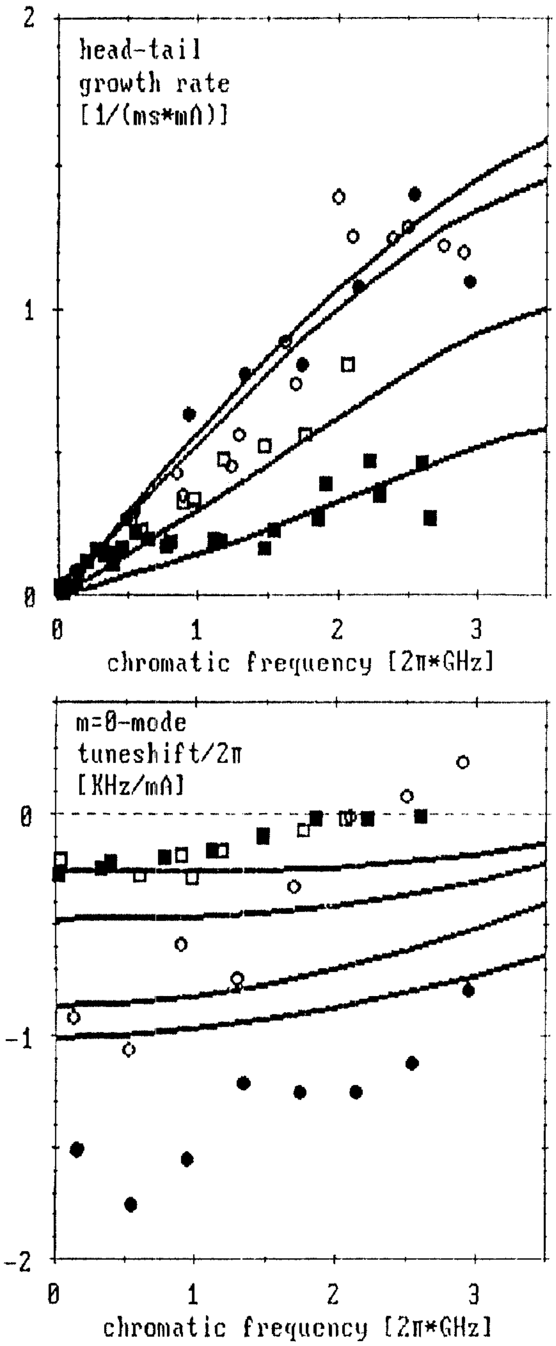

Fig. 3: Growth rates and tune shifts of all measurements as a function of chromatic frequency. The meaning of the symbols is found in table 1. The solid lines are the theoretical result for a broad band impedance with the parameters given in the last line of table 1.

constant with a value of $\Delta\omega/I \approx -2\pi\, 250$ Hz/mA. The ratio of the two slopes leads to a resonance frequency of $\omega_{res}/Q = 2\pi{\cdot}4.4$ GHz and the impedance is found to be: $|Z_\perp(0)| = 0.47$ MΩ/m. This broad band impedance is identical to the approximation chosen for all measured tune shifts but over estimates the growth rates. For small chromatic frequencies the experimental growth rates do not show the expected $1/\sigma$ behaviour.

I have no explanation for the discrepancies but there are reasons to suspect the theory in case of small synchrotron and large chromatic frequencies because the dominating dephasing and phasing effect is not included in Sacherer's theoretical approach. Under these circumstances the center of mass motion is zero most of the time and finite values are reached again only after one synchrotron periode as seen in fig. 2 a. The same must be true for the wake fields that are responsible for the damping of the motion. When there is damping always the same particles are in the head and the tail of the distribution. Maybe a naive approach could be to readjust the time scale in order to cut out the times when the bunch center does not move. This would lead to large corrections for the damping rates and should also alter the frequency shifts. With short bunches the correction is small because the dephasing is less pronounced. Even if this was a valid approach it would not explain that for small chromatic frequencies the experimental tune shifts do and the growth rates do not scale as $1/\sigma$.

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