Self-field-limiting current density in field-photoemission of intense short relativistic electron beams

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Abstract

In various applications, such as high-power RF Free-Electron Lasers, both intense and very brilliant short relativistic electron beam pulses are required. For that, the RF photoinjector presently seems the best source. Electrons of an intense short pulse produced by laser irradiation are submitted, just after their photoemission, to a very strong RF field. The acceleration is so strong that relativistic acceleration and retardation effects have to be taken into account [1]. In this paper, the self-field-limiting current density will be theoretically discussed, as a function of beam parameters and RF field intensity. It will be compared to the relativistic Child-Langmuir current density, i.e. the corresponding steady state space-charge limit.

1. INTRODUCTION

1.1. Self-field limiting current. The case of the relativistic planar diode

They are many calculations of self-field limited current for charged particle beams which are injected in vacuum, in a drift space limited by conducting walls, often in presence of a longitudinal magnetic field.

For an accelerated beam, the same question has been extensively studied in the case of the planar electron diode where the cathode emission is unlimited, such as the modern high-power diodes. If d is the anode-cathode gap, and ϕ the gap voltage, the maximum current density is given by :

$$J_L = \frac{I_0}{8\pi d^2}\left[\int_1^{1+e\phi/mc^2} (x^2-1)^{-1/4}\, dx\right]^2,$$

where $I_0 = 4\pi\varepsilon_0 mc^3/e \approx 17$ kA. This relativistic Child-Langmuir formula reduces, in the non-relativistic case ($e\phi/mc^2 \ll 1$) to :

$$J_L = (\sqrt{2}I_0/9\pi d^2)(e\phi/mc^2)^{3/2} = (4\varepsilon_0/9)(2e/m)^{1/2}\phi^{3/2}/d^2.$$

To obtain this formula, one has to assume a 1D-steady state where thermal and edge effects as well as self-magnetic pinching may be neglected. (The 1D character may also be ensured by a very strong longitudinal magnetic field). On the cathode the electric field is zero.

A time-dependent analysis shows that the above formula gives a mean value for the current which, like the electric field on the cathode, undergoes relaxation oscillations.

An extension to the case where ϕ is slowly time-dependent (and non-relativistic : $e\phi/mc^2 \ll 1$) has been proposed.

In all these analyses, the potential is deduced from a Poisson equation.

1.2. The case of the RF photoinjector

As a source of intense, very short and very brilliant electron pulses, the RF photoinjector has appeared, over the past few years, as better than conventional sources, in particular for free electron lasers. Located in an RF cavity, and illuminated by a laser pulse, the cathode emits electrons which are rapidly accelerated to possibly relativistic energies.

In a FEL photoinjector, an exemple to which numerical applications will relate in this work, emitted electron pulses typically have some tens of ps in duration, some mm in diameter, some hundreds of A in intensity. As for the RF electric field intensity on the cathode, its scale is 10 to 100 MV/m.

The accelerating RF electric field seen by the emitted photoelectrons is the same for all of them as long as, on the one hand the beam radius is small compared to the cavity radius and, on the other hand, the pulse duration τ is small compared to the period $1/\nu$ of the RF wave. For ν=3 GHz, this corresponds to $\tau \ll 300$ ps ; for ν=150 MHz, to $\tau \ll 7$ ns. The latter conditions will hereafter be assumed fulfilled ; they are in the CEA "ELSA" free electron laser [2] (Bruyères-le-Châtel) for which ν=144 MHz.

2. THE (E,B) SELF-FIELD MAP IN THE EMITTED PHOTOELECTRON PULSE SUBMITTED TO AN EXTREME ACCELERATION

2.1. From the individual Lienard-Wiechert field to the beam pulse self-field

In a RF field of say 30 MV/m, a photoelectron emitted with an energy of 0.1 or 0.2 eV, i.e. with a velocity of 6 to $9\times10^{-4}c$, is accelerated to $c/4$ in 0.5 mm. For such an extreme acceleration, electrodynamic effects of acceleration (radiation) field and retardation have to be taken into account. The self-field effects are no longer space-charge effects to which a Poisson equation is applicable. One has to use either the full set of Maxwell equations with convenient boundary conditions, or an individual particle formalism founded on the Lienard-Wiechert equations, with images.

The latter method is used in this work. So, the starting point is the electromagnetic field generated at ξ at the time t, in the laboratory frame, by one accelerated photoelectron located at $\mathbf{x}$ at time t, with the velocity βc :

$$\mathbf{E}(\xi,t\,|\mathbf{x}) = -(e/4\pi\varepsilon_0)\Big\{\left[(\mathbf{u}-\beta)\gamma^{-2}(1-\mathbf{u}.\beta)^{-3}\right]\|\xi-\mathbf{x}\|^{-2}$$

$$+\left[\mathbf{u}\times\{[\mathbf{u}-\beta]\times\dot{\beta}\}c^{-1}(1-\mathbf{u}.\beta)^{-3}\right]\|\xi-\mathbf{x}\|^{-1}\Big\}_{t=t'}$$

$$\mathbf{B}(\xi,t\,|\mathbf{x}) = \frac{1}{c}\,\mathbf{u}(t')\times\mathbf{E}(\xi,t\,|\mathbf{x})\ .$$

where : $\mathbf{u}(t)=[\xi-\mathbf{x}(t)]/\|\xi-\mathbf{x}(t)\|$, and where the retarded time t' is such that $\|\xi-\mathbf{x}(t')\| = c\,(t-t')$. The first term in $\mathbf{E}$ is the velocity field $\mathbf{E}_\beta$, which decreases in $1/r^2$, the second one the acceleration field $\mathbf{E}_{\dot{\beta}}$ -the radiation field far from the source- which decreases in $1/r$.

The point is now to formulate the field generated at (ξ,t) by all already emitted photoelectrons, in the presence of the equipotential conducting cathode.

Before embarking on the analysis, let us note some properties which allow important simplifying assumptions. At first, for electron motion, strong acceleration and self-field effects are dominant over thermal effects : the electron flow will be treated as laminar. Then it can be *a posteriori* verified that the radial force $F_r = -e(E_r - \beta_z c B_\theta)$ is very small as compared to the accelerating force $-e\mathrm{E}_0$: near from the cathode, where $F_r \approx -eE_r$ on account of the vanishing E_r imposed by the latter, but downstream also. Therefore, at a good approximation, the trajectories for $0\le t\le\tau$ are straight lines parallel to E_0.

Moreover, for given ξ and t, the mapping : $\mathbf{x}(t)\rightarrow\mathbf{x}(t')$ is a diffeomorphism $\mathcal{L}'_{\xi,t}$ (which can easily be analytically expressed in the case of what we call later the zero-order electron motion, i.e. the motion in E_0 alone ; then : $\mathcal{L}'_{\xi,t} = \mathcal{L}'_{\xi}$). So that, $n(\mathbf{x},t)$ being the electron number density, and the field $\mathbf{E}$ generated by images being taken into account :

$$\mathbf{E}(\xi,t) = \int_{\mathcal{D}'} n(\mathbf{x},t)\mathbf{E}(\xi,t|\mathbf{x})\,d^3\mathbf{x}$$

$$+\int_{\mathcal{D}''} n(\mathbf{x},t)\widetilde{\mathbf{E}}(\xi,t|\mathbf{x})\,d^3\mathbf{x}$$

where $\mathbf{E}(\xi,t\,|\mathbf{x})$ has the above-quoted Lienard-Wiechert (LW) expression in which $\mathbf{x}'=\mathbf{x}(t')$ is replaced by $\mathcal{L}'_{\xi,t}(\mathbf{x})$, and t' by :

$t-(1/c)\|\,\xi-\mathcal{L}'_{\xi,t}(\mathbf{x})\|$,

in : $\mathbf{u}'$, $\beta(\mathbf{x}',t')$ and $\dot{\beta}(\mathbf{x}',t')$.

$\widetilde{\mathbf{E}}$, the field generated by the image of the $(\mathbf{x},t)$ electron, conveniently retardated with respect to ξ , has again the general above LW expression with, instead of $\mathbf{x}'$ and t', $\mathbf{x}''$, and $t''=t-(1/c)\|\xi-\widetilde{\mathbf{x}}''\|$, and where the point $\widetilde{\mathbf{x}}''$ is symmetric of $\mathbf{x}(t'')$ with respect to the cathode plane z=0 ; moreover :

$$\beta(\widetilde{\mathbf{x}}'',t'') = -\beta(\mathbf{x}'',t''),\qquad \dot{\beta}(\widetilde{\mathbf{x}}'',t'') = -\dot{\beta}(\mathbf{x}'',t'').$$

$\mathcal{D}'$ it is that part of the beam at time t : $\mathcal{F}(t)$, consisting in electrons which have antecedents with respect to ξ :

$$\mathcal{D}' = \{\mathbf{x};\ \mathcal{L}'_{\xi,t}(\mathbf{x})\in\Omega\},$$

where Ω is the z>0 half-space. $\mathcal{D}''$ has the same definition, $\mathcal{L}'_{\xi,t}$ being replaced by $\mathcal{L}''_{\xi,t}$: $\mathbf{x}(t)\rightarrow\mathbf{x}(t'')$.

Another, and in fact more suitable, expression of $\mathrm{E}(\xi,t)$ corresponds to the use of $\mathbf{x}'$ or $\mathbf{x}''$ instead of $\mathbf{x}$ as integration variable :

$$\mathbf{E}(\xi,t) = \int_{\Delta'} n(\mathbf{x}',t')\widetilde{\mathbf{E}}(\xi,t|\mathbf{x}')\det(D\mathcal{L}'_{\xi,t})\,d^3\mathbf{x}'$$

$$+\int_{\Delta''} n(\mathbf{x}'',t'')\widetilde{\mathbf{E}}(\xi,t|\mathbf{x}'')\det(D\mathcal{L}''_{\xi,t})\,d^3\mathbf{x}''$$

where $\Delta = \mathcal{L}_{\xi,t}^{-1}(\mathcal{D})$.

2.2. Electron motion : iteration procedure

As it has been *a posteriori* verified, the just emitted short pulse is paraxial at a good approximation for the FEL RF injector conditions described in 1. Axial and radial electron motion can thus be decoupled, and all the electrons of a given slice z have the same γ.

Taking advantage of the decoupling, and considering an axisymmetric beam, where (r,z) are the cylindrical coordinates, the following iteration procedure has been used to calculate the electromagnetic field map inside the pulse, at different times $t\in(0,\tau)$, where τ is the pulse duration.

At order 0, the $(\mathbf{E},\mathbf{B})^{(0)}$ map is determined from the slice trajectories $z^{(0)}(t\,|t_0)$ in the axial RF field alone, where t_0 is the emission time for the considered slice. Then, order 1 slice trajectories $z^{(1)}(t\,|t_0)$ are calculated for electron slices submitted to both the RF field and the (0)-axial proper electric field on the beam axis $E_z^{(0)}(r=0,z,t\,|t_0)$. From $z^{(1)}(t\,|t_0)$ a new field map $(\mathbf{E},\mathbf{B})^{(1)} = [\mathbf{E}^{(1)}(r,z,t),\ \mathbf{B}^{(1)}(r,z,t)]$ is deduced, etc. Three iterations have shown to be sufficient.

3. SELF-FIELD-LIMITING CURRENT. COMPARISON WITH THE RELATIVISTIC CHILD-LANGMUIR LIMIT

Figures 1 to 3 show the maximum or self-field limiting current density J_{max} as a function of the pulse duration τ for various RF-field intensities on the cathode : E_0=15, 30, 45 MV/m. For comparaison, the relativistic Child-Langmuir (RCL) limit is also plotted.

For smallest τ, J_{max}(RCL) is slightly greater than J_{max}(LW) ; for greatest τ, it is smaller. These discrepancies have quite different origins.

For smallest τ, it may be explained by the limited transverse extension of the studied photoemited beam (for τ=20 ps, and E_0=30 MV/m, the pulse length is $L(\tau)$=1 mm, while the beam radius, for S=1 cm^2, is 5.6 mm), which contrasts with the theoretically unlimited extension of the planar diode. $J_{max}(LW)/J_{max}(RCL) \rightarrow 1$ when $\tau \rightarrow 0$ (and $L(\tau)/R \rightarrow 0$).

For the greatest τ, the discrepancy is due to the retardation effects : the last emitted photoelectrons no longer experience the electromagnetic influence of the electrons located in front of the beam pulse. Beyond some pulse duration τ, the maximum current density no longer decreases at all.

At the end, Figure 4 shows the maximum field-photoemitted charge Q_{max}, for various E_0, compared to J_{max}(RCL).$S.\tau$.

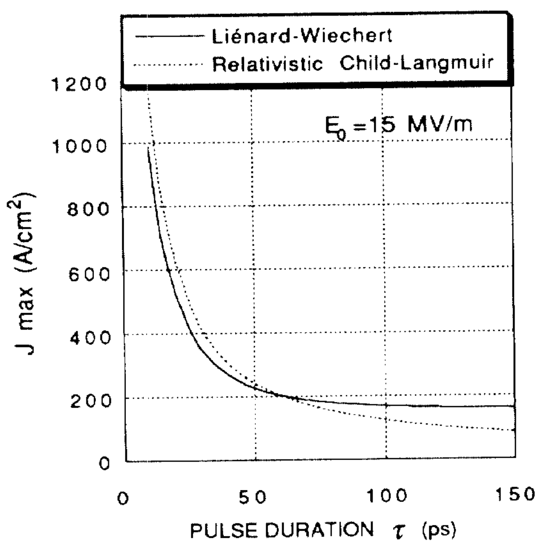

Figure 1

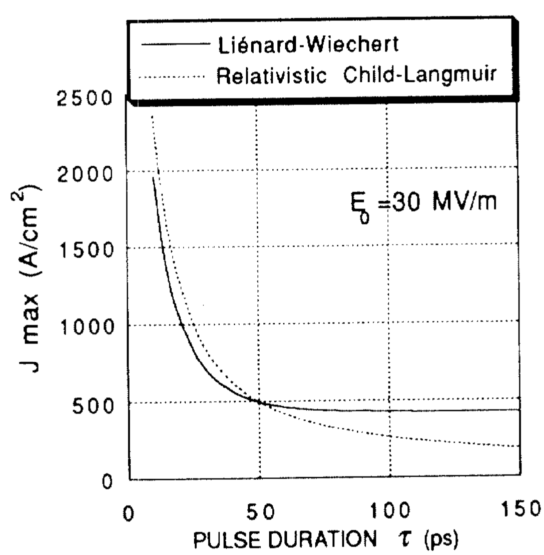

Figure 2

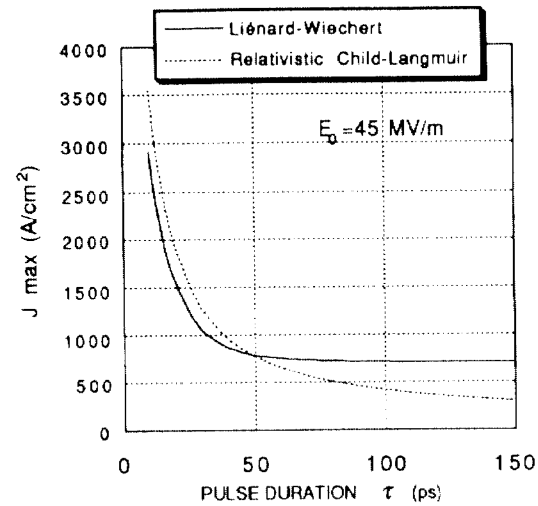

Figure 3

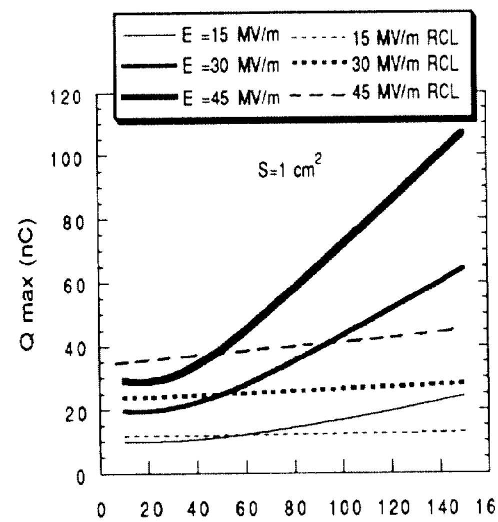

Figure 4

5. REFERENCES

[1] J.-M. Dolique and J.C. Coacolo, "Relativistic Acceleration and Retardation Effects on Photoemission of Intense Electron Short Pulses", in 1991 IEEE Particle Accelerator Conference Proceedings, San Francisco, USA, May 1991, pp.233-235

[2] R. Dei-Cas, S. Joly and the FEL group, "Overview of the FEL activities at Bruyères-le-Châtel", in 1990 Int. Conf. on Free Electron Lasers, Paris, France, September 1990