# Transverse Wakefields at High Dispersion

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# Abstract

In high energy linear colliders the transverse beam emittance has to be preserved in order to achieve small interaction spots. If the beam is off-center in an accelerating cavity, it excites transverse wakefields, mainly the dipole mode, which deflects the tail of the beam leading to an emittance growth. In a high dispersive region, even a well centered beam can excite wakefields of higher order modes since the energy spread of the beam causes a transverse beam size which is comparable with the beam pipe. During the bunch length compression in the ring-to-linac (RTL) section of the Stanford Linear Collider (SLC), the beam pipe of 75 mm diameter is practically filled in the horizontal plane. Especially, if beam losses start to be involved, the very outer parts of the beam excite wakefields at any pipe irregularities like bellows, diameter steps, etc. Measured evidences, theoretical estimates and comparisons to other effects, like higher order magnet optics, are presented.



# 1 Introduction

At the SLC, the 10 mm long bunch of the damping ring (DR) is compressed in the RTL section down to the necessary 1 mm length for the main linac. The compression is achieved by introducing an longitudinal energy correlation with a compressor cavity followed by a high dispersive region ( $\eta$ ), where particles of different energy travel along different trajectories. Particles at  $\pm 2\sigma_l$  (bunch length  $\sigma_l \approx 10$  mm) get an energy difference of  $\Delta E/E = \pm 2.4$ % and therefore are with  $\Delta x = \pm 39$  mm at  $\eta = 1.4$  m near the aperture of the pipe,

$$\Delta x = \eta \cdot \frac{\Delta E}{E}$$
 and  $\Delta E = E_{rf} \sin(2\pi\sigma_l/\lambda)$ , (1)

with an energy E = 1150 MeV, rf-amplitude of  $E_{rf} = 30$  MeV and  $\lambda = 105$  mm. Additionally to the intended path length difference, the off axis particles also experience any nonlinear kicks coming from wakefields or higher order magnetic fields. This causes an emittance blow up at the beginning of the linac, where the 39 mm size of the RTL should be reduced to the nominal beam size of about  $\sigma_x = 300 \,\mu$ m. Besides the direct emittance blow up, there is another even more disturbing mechanism: A small jitter of the DR phase and/or the bunch length  $\sigma_I$  will cause a different off axis beam population (or even losses at the

aperture) and therefore different amounts of particles away from the axis at the beginning of the linac. A few particles (say 5%) at an offset of 2mm (measured, see below) are worse than an unacceptable offset of the whole beam by 100  $\mu$ m (=0.05.2mm), if they are in the head of the bunch, since these particles are not fully compressed and therefore ahead of the main bunch distribution creating stronger transverse wakefields along the linac. The experimental observations are presented first. Then some theoretical investigations show the size and the effect of the transverse wakefields compared to magnetic errors and finally other effects especially of the longitudinal wakefield are estimated.

# 2 Experimental Observations

A limitation in going to higher beam currents (  $> 3 \cdot 10^{10}$ particles/bunch) resulted in unstable beam positions and emittance blow up at the end of the linac. At lower currents, this so called "linac instability" was tracked down to some occasions where particles were further away from the normal distribution ("fliers", see fig in complete paper [1]). The behavior was nearly invisible at the beginning of the linac, so the source was unclear, until a special BPM (beam position monitor, number 544) in the RTL showed something like a 90° correlation ("donut") with BPMs in the linac [1]. The BPM 544 was in at least two ways special: First, it was just downstream of some beam loss and might have been irritated by the spray of secondary particles. And second, the BPM is at a high dispersion and has therefore a bigger diameter than other BPMS, which enables that BPM to recognize particles near the aperture in x-direction. All BPMs are less sensitive for particles further off axis, since the BPM electrodes are arranged at  $45^{\circ}$  to the x and y plane [1]. So the attention was moved to the DR area and since BPMs in the RTL measure mainly the position of the beam, which corresponds to the DR phase and energy, these quantities were (unsuccessfully) investigated for any kind of jitter source.

Then the DR bunch length  $(\sigma_l)$ , was measured in the RTL with a screen showing some increases of the jitter from 0.5% to 3.5-5% of  $\sigma_l$  if the number of particles per bunch was above  $4 \cdot 10^{10}$ .

#### 2.1 DR-Jitter

Just recently, in the ongoing running cycle, an even more complicated mechanism of bunch length blow up was recognized [2]: Instead of slowly damping down to a somewhat larger bunch length (at high current), the bunch length decreases further to a too small size. Then it blows up very quickly (0.1 ms) to a too big value afterwards it decreases again slowly by radiation damping (1.5 ms). So a "saw

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tooth" pattern is visible in a bunch length (or phase) versus time plot [1]. When the step occurs at ejection a flier might be seen in the linac. The donut is a little bit more complicated: Positive values at BPM 544 were also correlated with beam loss. So a larger bunch and therefore losses might indicate an extraction near the peak of the saw tooth. Damping down the bunch length leads to the nearly straight line on one side of the donut [1]. The opposite side is the difficult part: Why should a beam which is located at the same position in the RTL get an offset of up to 1.5mm at the end of the linac. During the blow up not only a quadrupole mode (bunch length) with nearly twice the synchrotron frequency  $(\nu_s)$  occurred, but also a frequency less than three times  $\nu_s$  showed some indication of a sextupole mode (bunch form). Different bunch forms and lengths might be responsible for the wider scatter of the other half of the donut. This model indicates that the saw tooth should get more symmetric at higher currents (not yet tested). The timing jitter of the saw tooth is currently stabilized by starting the instability externally by switching the rf off and on ("bunch munching" [3]) during the store time.

#### 2.2 RTL-Sensitivity

Any longitudinal jitter or distribution change of the DR bunch should be decreased to 10% by the RTL since the compression is not totally. A total compression should leave the RTL in first order unsensitive to changes. The nonlinearity of the rf-slope brings some particles in front and the back of the short linac bunch. An even more important aspect are the transverse kicks along different trajectories (see Fig. 1). The cause might be magnetic multipoles and/or transverse wakefields, the later one will be discussed in more detail.

### 3 Wakefields of Bellows

Besides the necessary compressor cavity in the north RTL, the smooth vacuum pipe is interrupted by about 40 bellows (30 in the south) and about ten other devices with bigger holes: profile monitors (screens) and T-connections to vacuum pumps. Since each bellow consists of 20 cells (some are different) let's concentrate on them.

#### 3.1 Wakefield Size

If the longitudinal wake is purely inductive, the transverse wakefield  $W_{\perp}$  is proportional to the current. This assumption is o.k. for bunches longer than the geometric variations of the bellow. An analytic formula [4] for the peak of the dipole kick

$$W_{\perp peak}[\text{Volt/cell}] = \frac{Z_o g \Delta}{\pi a^3} \cdot \frac{crQ}{\sigma_l}$$
(2)

gives about a peak  $W_{\perp} = 5.9 \,\mathrm{kV}$  for one bellow with 20 cells and a bunch length  $\sigma_l = r(\mathrm{offset}) = 6 \,\mathrm{mm}$ ,  $(Z_o = 377 \,\Omega)$ , gap width  $g = 2.5 \,\mathrm{mm}$ , bellow height  $\Delta = 6 \,\mathrm{mm}$ , bellow inner radius  $a = 24.5 \,\mathrm{mm}$  (see Fig. 2),  $c = \mathrm{velocity}$  of light and  $Q = 8 \,\mathrm{nC}$  (5  $\cdot 10^{10}$  particles).

Simulations with TBCI [5] showed that  $W_{\perp}$  is about 2.6 keV at  $\sigma = 6$  mm. Fig. 2 shows the shape of the input



Figure 1: Third order dispersion.

The dependence of the linac position (at maximal excursion) versus the energy in the RTL show a strong third order term (fit to measurements solid). The measured 12-pole field of 2% at the pole tip of the quadrupole at the highest dispersion gives a 5th order contribution (dashed) of about the right magnitude. The calculated effect of the wakefield contribution at  $5 \cdot 10^{10}$  is about half that effect (dashed-dotted, see below).

bellow with 4 cells and the longitudinal  $(W_z)$  and transverse  $(W_{\phi})$  wakefields for the transverse mode m = 3,  $\sigma = 2 \,\mathrm{mm}$  and r = a. The peak longitudinal wake for  $m \geq 1$  is about twice that for m = 0 and the transverse wakefield scales with m like expected [4]:

$$W_{zm} = \left(\frac{r'}{a}\right)^m \left(\frac{r}{a}\right)^m \cos m\theta \sum_n^\infty 2k_{mn} \cos \frac{\omega_{mn}s}{c} \quad s > 0$$
(3)

$$W_{\perp m} = m \left(\frac{r'}{a}\right)^m \left(\frac{r}{a}\right)^{m-1} \left(\hat{r}\cos m\theta - \hat{\theta}\sin m\theta\right)$$
$$\cdot \sum_{n=1}^{\infty} \frac{2k_{mn}}{(\omega_{mn}a/c)}\sin \frac{\omega_{mn}s}{c}$$
(4)

following the Panofsky-Wenzel Theorem

$$\frac{\partial}{\partial s}W_{\perp m} = \nabla_{\perp}W_{zm}.$$
(5)

A test particle has a radius r and a distance s behind the exciting particle (r'),  $\hat{r}$ ,  $\hat{\theta}$  are unit vectors and  $\omega_{mn}$  is the frequency of the mode with the loss parameter  $k_{mn}$ . Since the scaling with m, particles near the aperture have an enormous effect and even orders of  $m_{max} \approx \pi a/\sigma \approx 30$  have to be taken into account. At 70% of the aperture the contribution of the higher order transverse modes decreases slowly with  $m/2^{m-1}$ , m = 6 has still about 20% of the dipole value.

#### 3.2 Effects of Wakefield Kicks

A bunch with  $5 \cdot 10^{10}$  particles and an offset of 17.4 mm (corresponds to  $\Delta E = +20$  MeV at  $\eta = 1$  m) will get a

dipole wakefield kick of about  $6.6 \,\mu$ rad from one bellow. This has to be compared with the angular divergence  $\sigma'_x = \sqrt{\varepsilon/\beta}$  of the beam. With a range in the beta function  $3 < \beta < 50$  m and a normalized emittance of  $\gamma \cdot \varepsilon = 1.6 \cdot 10^{-5}$  m rad ( $\gamma$  = relativistic energy factor) the range of  $\sigma'_x$  is from  $12 - 50 \,\mu$ rad. Since the effects of the 30 bellows cancel partly each other due to different phase advances, about  $\sqrt{30} \cdot 6.6 \,\mu$ rad =  $36 \,\mu$ rad is expected which is of the



Figure 2: Wakefield calculation.

The input shape for TBCI and the calculated longitudinal and transverse wake potentials for m=3 and  $\sigma = 2$  mm is shown.

same order of  $\sigma'_x$ . The detailed effects with m up to 6 for each bellow at its specific RTL lattice place was simulated and their summed up maximal influence at the beginning of the linac is shown in Fig. 1.

The beam is normally more in the center of the pipe, therefore only the higher modes contribute which are excited by a broad centered beam. Since both, this effect and the magnet errors can be cancelled up to third order by octupoles, two of 8-pole magnets were installed last down time and they improved the emittance by 5-10%.

#### 3.3 Longitudinal Wakefield

The longitudinal wakefield scales with  $1/2^m$   $(m \ge 1)$  for 70% of the aperture and is about 1 MeV for m = 0 or 1. With 3 MeV which is about  $3\sigma_E$  (DR energy spread) the bunch compression might be disturbed. Since the bunch shortens in the RTL, there are less wakefields at the beginning causing a factor of about 1/2 and also the higher orders cancel or decrease with a centered beam, so the maximal real effect is more like  $\sigma_E/2$ . The effect of a longer bunch can be adjusted by a stronger compression.

Besides effects on the 10-to-1 longitudinal compression, the induced energy spread in the RTL might indirectly effect also the 100-to-1 transverse beam size reduction!

# 4 Conclusion

The transverse wakefields of a beam at high dispersion lead to dispersive abberations of higher order, which can be mainly compensated by magnetic elements, like for higher order magnetic errors. The longitudinal wakefield has a small effect on the compression process, but seem to have an indirect effect on the transverse beam size reduction. This chromatic like effect should be avoided be reducing the amount of wakefields, e.g. with sleeves in the bellows.

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