# Slow Particle Loss in Hadron Colliders 

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#### Abstract

The dynamic behaviour of particles in large superconducting accelerators is strongly affected by the unavoidable non-linearities of the bending and focusing fields. Due to the very high order of the non-linear components, particle losses can take place after a very large number of turns. As tracking simulations cannot be taken to an infinite numher of turns, other methods have to be used to evaluate the long-term behaviour of such machines. For the LHC we have used survival plots, together with the detection of the border between regular and chaotic motion. In this paper the features of the survival plots are discussed, with the help of the analysis of the motion in the transverse tune space. It is shown how the local nature of the instability influences the particle lifetime, and how difference resonances play an important role in the loss process.


## 1 INTRODUCTION

The prediction and understanding of the long-term behaviour of conservative systems has always been a major concern in the field of non-linear dynamics. In particle accelerator physics this becomes more and more important in view of the large hadron colliders, which require a long lifetime of the beam in the presence of strong non-linear fields.

In general the results from tracking simulations are summarized in survival plots[1, 2], which depict the particleloss turn number as a function of the amplitude. We give here a summary of a more extended study[3] that, while giving a phenomenological view of the chaotic motion in a strongly non-linear accelerator, allow us to explain the typical features of the survival plots.

## 2 SURVIVAL PLOTS

A survival plot is used to evaluate below which amplitude a safe operation of the accelerator can be expected; to this end a coarse scan of amplitudes is sufficient when combined with the amplitude at which chaos sets in. Using a finer scan of the amplitudes to produce the survival plots, one typically finds a wide spread of the loss turn number. It is very important to understand the cause of this spread because its lower bound will be directly observed in the actual accelerator.

From the theory of the Lyapunov exponents we expect the chaotic behaviour to be a local phenomenon; to test

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Figure 1: Survival plot of companion particles
these local properties of the instability, we started 15 companion particles located on a four-dimensional sphere, at a distance less than $10^{-4} \mathrm{~mm}$ for 14 different amplitudes spaced roughly by $5 \%$ of the expected dynamic aperture, and tracked them through the LHC structure including all non-linearities. Fig. 1 shows those companion particles that are lost before $10^{5}$ turns.

The spread in the loss turn numbers when scanning the phase space in such a microscopic way compares well with the one found when the particles are started with a macroscopic separation. The local strength of the instability, rather than macroscopic differences in phase space, is therefore the cause of this spread. Moreover, as seen in Fig. 1, this local instability tends to become smaller when the dynamic aperture is approached, resulting in an increase of the spread in loss turn numbers for decreasing amplitudes. It has to be noted that some of the particles close to the dynamic aperture are not yet lost, so that the spread is even wider there, as indicated by the open dashed lines.

The dense survival plots present another very interesting feature: we can identify different amplitude ranges for which the average loss turn number and its spread, as a


Figure 2: Tune diagram for chaotic particles with different starting amplitudes (the $\star$ indicates the nominal tune)
possibility of an exchange between transverse and longitudinal motion. Such a coupling can be invoked to explain the variation of the four-dimensional amplitude seen in Fig. 3c. At the end of the life of the particle, at some 60,000 turns (point $F$ in Fig. 3), another exchange of energy takes place between the horizontal and the vertical planes, until both amplitudes increase rapidly, indicating the effect of a sum resonance.

The variations of the amplitude can be nicely described by means of the evolution in the tune diagram of Fig. 4, where we show the phase advance averaged over 760 turns, which correspond to 5 synchrotron oscillations. Our chaotic particle of Fig. 3 moves wildly in the tune diagram in a region spanned by resonances of at least $12^{\text {th }}$ order: soon after it has been started (point A in Figs. 3 and 4), it is attracted by a resonance of order 23 and spends some time there (point B); then it is quickly displaced towards a difference resonance of order between 12 and 29 (point C ) which leads to the energy exchange mentioned above; after this very short event (less than 4000 turns), the particle is captured again on two main nests of resonances (points $D$ and E), and bounces back and forth between them, until it is brought back to the same region of difference resonances (point $F$ ), and is finally driven out by a sum resonance. The difference resonance at point $C / F$, as well as the lethal sum resonance at the end, cannot be determined precisely, owing to the short time for which they are active.
function of the amplitude, stay approximately the same. This can be related to different high-order resonances, or even to different nests of resonances. In the tune diagrams of Fig. 2 we see that at the beginning particles exhibit tune-shifts that vary smoothly with amplitude (Fig. 2a). However, shortly thereafter, chaotic particles belonging to different ranges of amplitudes are attracted towards different nests of resonances (Fig. 2b), each of them determining a different average lifetime.

## 3 TUNE WANDERING

To get a phenomenological understanding of the slow particle losses in large superconducting accelerators, we will now choose a chaotic particle at large amplitude and will follow in detail its tune evolution and that of one of its companions during their whole lifetimes.

In Figs. 3a and 3b we first show the averaged horizontal and vertical amplitudes for one of these particles. This particle has a rather strong stochastic behaviour, which results in wild and irregular amplitude variations. After about 12,000 turns (point C in Fig. 3) a coupling resonance becomes active, leading to an increase of the horizontal amplitude and a decrease of the vertical amplitude. The effect of this coupling resonance can be interpreted as an exchange of energy between the two transverse planes, in which case the four-dimensional amplitude remains unchanged at this specific point ( C in Fig. 3c). However, the motion being six-dimensional, there is also in general the


Figure 3: Evolution of amplitudes for a chaotic particle


Figure 4: Evolution of the transverse tunes for a chaotic particle (the $\star$ indicates the nominal tune)

The role of difference resonances turns out to be very important: though not being capable of causing a particle loss themselves, they seem to have a catalytic effect, moving the particle to places where it finally reaches a sum resonance strong enough to extract it.

It is very instructive to follow the tunes of another par ticle (Fig. 5) started at a microscopic distance from the particle described above; after following the same path as the first particle in the tune diagram for a while (until point B in Figs. 4 and 5), suddenly the particle 'decides' to follow a different track, which leads to its loss due to a completely different resonance and after a different period of time.

We can explain now why at a lower amplitude the spread of the particle-loss turn number becomes wider: as the motion becomes less chaotic, particles move more slowly in the tune diagram; they thereby have more time to 'choose' a path different from the paths 'chosen' by the other companions; consequently they will have a much wider spread of lifetime.

## 4 CONCLUSIONS

We demonstrated that the spread of the particle-loss turn number in the survival plots is due to the local nature of the instability. We also found that this spread is widened when the dynamic aperture is approached and that the step-wise variation of the average lifetime as a function


Figure 5: Evolution of the transverse tunes for a companion particle (the *indicates the nominal tune)
of amplitude can be related to the influence of different nests of resonances. Moreover, analysing the evolution of the tunes of a chaotic particle, we have seen that very high-order resonances, and a large number of them, may determine the loss mechanism, and that difference resonances can play an important rolc in it. Finally it became clear that the spread of the particle lifetime is due to the different paths in the tune diagram.

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## 6 REFERENCES

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