Theoretical Investigation of Traveling Wave RF Gun

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Abstract

In this paper a traveling wave type rf gun (TW gun) is investigated theoretically. Analytical formulae concerning energy gain, energy spread, and transverse emittance are derived. Some numerical results are calculated to demonstrate further the behaviours of the TW gun and to compare with those from analytical formulae.

1 INTRODUCTION

As a potential high brightness electron injector for FEL and future e^+, e^- linear colliders, rf gun have been extensively investigated theoretically and experimentally both for thermionic cathode [1][2] and photo-cathode [3][4][5]. Because of the historical reasons, almost all the experimental and theoretical works are confined within standing wave rf guns (SW guns). However, as pointed out by J. Le Duff [6], TW gun might be very interesting also to rf gun technology. In this paper TW gun will be theoretically investigated in a way different from that in ref. [7].

Before going on to the next section, it is assumed that the radius of the cathode is small, and the nonlinear force effects both from the space charge and the rf fields are neglected.

2 TW GUN THEORY

Quite different from a SW gun which is similar to a conventional DC gun except mainly that the accelerating electric field is oscillating with time, a TW gun is a little bit difficult to imagine at the first glance since there are almost no evident relations between a TW gun and a conventional DC gun. However, if one shifts his point of view from DC gun to a linear traveling wave accelerator, the confidence about a TW gun could be established without much difficulty. Now let's start with the "capture condition" of a linac [8] which says: if

$$E_z = E_0 sin(\phi) \quad (1)$$

then

$$cos(\phi_0) - cos(\phi_f) = \frac{2\pi}{\lambda_g}\frac{m_0c^2}{eE_0}\left(\frac{1-\beta_{z0}}{1+\beta_{z0}}\right)^{1/2} \quad (2)$$

where ϕ_0 is the phase when an electron is injected, ϕ_f is the asymptotic phase when the electron's velocity reaches almost light velocity c, λ_g is the wavelength in the accelerating structure, E_0 is the peak accelerating electric field strength, β_{z0} corresponds to the initial injectron electron's velocity, and m_0c^2 is the electron's rest energy. If we assume that a cathode is just put at the end plate of the coupler cavity of a linac with $\beta_{z0} = 0$, and require that $\phi_f = \pi/2$, one can determine the necessary electric field strength E_0 with respect to the injection phase ϕ_0

$$E_0 = \frac{2\pi}{\lambda_g}\frac{m_0c^2}{ecos(\phi_0)}\left(\frac{1-\beta_{z0}}{1+\beta_{z0}}\right)^{1/2} \quad (3)$$

For example, if $\lambda_g = 10cm$, $\beta_{z0} = 0$ and $\phi_0 = \pi/4$, from eq. 3 one gets $E_0 = 45$MV/m. It is obvious that $E_0 = 45$MV/m is a practically possibilbe value. It is now very easy to imagine that when a short laser pulse illuminates a photocathode at an initial rf phase $\phi_0 \leq \pi/4$ with $E_0 = 45$MV/m, the electron bunch will be accelerated continuously with its final asymptotic phase frozen at $\phi_f \leq \pi/2$. In the following section we will investigate in much more detail on the beam longitudinal and transverse motions.

2.1 *Longitudinal Motion Due to Rf Fields*

The longitudinal electric field inside a cylindrical symmetric TW structure is expressed as follows:

$$E_z(r,z,t) = E_z(r,z)sin(\omega t - \beta_g z + \phi_0) \quad (4)$$

where $\omega = 2\pi f$, β_g is the foundamental wave number of this slow wave TW structure and ϕ_0 is the emission phase of the center of the electron bunch. Since only linear term is kept, the electric field near the axis can be expressed as:

$$E_z(r,z,t) = E_z(0,z)sin(\omega t - \beta_g z + \phi_0) \quad (5)$$

In the following analytical treatment, $E_z(0,z)$ has been chosen as a constant. From eq. 5 we have

$$\frac{d\gamma}{dz} = \frac{qE_z(0,z)}{m_0c^2}sin(\phi) \quad (6)$$

where

$$\phi = \omega t - \beta_g z + \phi_0 = k\int_0^z \left(\frac{\gamma}{(\gamma^2-1)^{1/2}} - 1\right)dz + \phi_0 \quad (7)$$

$k = 2\pi/\lambda$ and λ is the electromagnetic wavelength in free space. β_g has been chosen equal to k (that is to say the phase velocity of this traveling wave equals to the speed of light). γ is the ratio between electron's relativistic energy and the rest energy m_0c^2. As the first order approximation of γ, eq. 6 can be integrated as follows:

$$\Gamma = 1 + \alpha sin(\phi_0 + \delta\phi)kz \quad (8)$$

$$\alpha = \frac{qE_z(0,0)}{m_0c^2k} \quad (9)$$

where $E_z(0,0)$ is the peak electric field on the cathode surface. Γ is an approximate expression of γ. In eq. 8 a very important parameter $\delta\phi$ is introduced. The physical reason of there existing this parameter is due to electron's low initial longitudinal velocity and the rapid increasing of ϕ in the cathode region. The introduction of $\delta\phi$ is to improve the accuracy of this first order approximation. It should be pointed out that for the theory of SW gun, this parameter should be introduced also. By using eq. 8, eq. 7 can be integrated as follows:

$$\phi = \frac{1}{\alpha sin(\phi_0 + \delta\phi)}((\Gamma^2 - 1)^{1/2} - (\Gamma - 1)) + \phi_0 \quad (10)$$

If $\Gamma \gg 1$ then ϕ will be frozen at its asymptotic value

$$\phi_f = \frac{1}{\alpha sin(\phi_0 + \delta\phi)} + \phi_0 \quad (11)$$

where $\delta\phi$ can be calculated from an empirical formula which is given as follows:

$$\delta\phi(degree) = 19E_z(0,0)^{-0.9} \quad (12)$$

where $E_z(0,0)$ is in MV/cm. It is obvious that $\delta\phi \approx 0$ corresponds to $E_z(0,0)$ approaching infinity. From eq. 8 and eq. 11 one can get the second order approximation of the final energy gain of a TW gun

$$\Gamma_f = 1 + \alpha sin(\phi_f)kL \quad (13)$$

where L is the length of the TW gun structure. For an electron bunch emitted from a cathode with the emission phase of the bunch center being ϕ_0, and the bunch length being $\Delta\phi_0$, the energy spread of this electron bunch can be calculated readily from eq. 13

$$\Delta\Gamma_f = \alpha kLcos(\phi_f)(1 - \frac{cos(\phi_0 + \delta\phi)}{\alpha sin(\phi_0 + \delta\phi)^2})\Delta\phi_0 \quad (14)$$

If $\phi_f = 90^o$

$$\Delta\Gamma_f = \frac{1}{2}\alpha kL(1 - \frac{cos(\phi_0^* + \delta\phi)}{\alpha sin(\phi_0^* + \delta\phi)^2})^2(\Delta\phi_0)^2 \quad (15)$$

where ϕ_0^* corresponds to $\phi_f = 90^o$. From eq. 11 the asymptotic bunch length can be calculated

$$\Delta\phi_f = \Delta\phi_0 - \frac{cos(\phi_0 + \delta\phi)}{\alpha sin(\phi_0 + \delta\phi)^2}\Delta\phi_0 \quad (16)$$

where $\Delta\phi_0, \Delta\phi_f$ are the initial and asymptotic bunch length respectively.

2.2 Transverse Motion Due to Rf fields

From $\nabla \cdot \mathbf{E} = 0$ and $E_\varphi = 0$

$$\frac{1}{r}\frac{\partial(rE_r)}{\partial r} + \frac{\partial E_z}{\partial z} = 0 \quad (17)$$

$$E_r(r,z,t) = -\frac{r}{2}(\frac{dE_z(0,z)}{dz}sin(\omega t - \beta_g z + \phi_0) - \beta_g E_z(0,z)cos(\omega t - \beta_g z + \phi_0)) \quad (18)$$

From $\nabla \times \mathbf{H} = \epsilon_0\frac{\partial\mathbf{E}}{\partial t}$, only H_φ exist, therefore

$$\frac{1}{r}\frac{\partial(rH_\varphi)}{\partial r} = \epsilon_0\frac{\partial E_z}{\partial t} \quad (19)$$

From eq. 19 we get

$$H_\varphi = \frac{\epsilon_0\omega}{2}rE_z(0,z)cos(\omega t - \beta_g z + \phi_0) \quad (20)$$

From

$$F_r = qE_r - q\mu_0 v_z H_\varphi \quad (21)$$

We know from eq. 18, eq. 20, and eq. 21 that

$$\begin{aligned} F_r &= -\frac{qr}{2}(\frac{dE_z(0,z)}{dz}sin(\omega t - \beta_g z + \phi_0) \\ &+(\mu_0\epsilon_0\omega v_z - \beta_g)E_z(0,z)cos(\omega t - \beta_g z + \phi_0)) \\ &= -\frac{qr}{2}(\frac{dE_z(0,z)}{dz}sin(\omega t - \beta_g z + \phi_0) \\ &+ (k\beta_z - \beta_g)E_z(0,z)cos(\omega t - \beta_g z + \phi_0)) \end{aligned} \quad (22)$$

where $\beta_z = v_z/c$. From eq. 22 one can get the transverse momentum gain due to rf field

$$P_{rf} - P_{r0} = \int_{t_0}^{t_f} F_r dt \quad (23)$$

$$p_r = \frac{P_r}{m_0 c} = \beta_z\gamma r' \quad (24)$$

where $r' = \frac{dr}{dz}$, p_r is normalized transverse momentum. If we assume that at the cathode surface $P_{r0} = 0$, during the acceleration r keeps constant value, and at the exit of the TW gun $\gamma \gg 1$, $\beta_z \approx 1$ and $E_z(0,z)$ from constant value $E_z(0,0)$ in any manner drops to $E(0,L) = 0$, by using eqs. 13 and 22 we get

$$p_{rf} = \frac{\alpha rk}{2}(sin(\phi_f) + \frac{1}{\alpha}cot(\phi_f)) \quad (25)$$

Since electrons emitted from cathode around emission phase ϕ_0 have an initial phase spread $\Delta\phi_0$, at the exit of the TW gun there is also a transverse momentume spread Δp_{rf} which can be derived from eq. 25

$$\Delta p_{rf} = \frac{\partial p_{rf}}{\partial\phi_0}\Delta\phi_0 + \frac{1}{2}\frac{\partial^2 p_{rf}}{\partial\phi_0^2}(\Delta\phi_0)^2 \quad (26)$$

Δp_{rf} is the source of the electron bunch's transverse emittance produced by the linear rf field. From eq. 25 one can derive the analytical formula for the transverse r.m.s. emittance

$$\epsilon_r(\pi m \cdot rad) = 4(< p_r^2 >< r^2 > - < p_r r >^2)^{1/2} \quad (27)$$

where r denotes x or y components. Since this analytical formula is complicated, it is omitted here. On the contrary we use the definition that the transverse emittance ϵ_r is the area occupied by the electrons in the phase space divided by π, we have

$$if \quad \frac{\partial p_{rf}}{\partial \phi_0} \neq 0 \qquad (28)$$

$$\epsilon_r^{rf} = \frac{\alpha k r_c^2}{2\pi}|(cos(\phi_f) - \frac{1}{\alpha sin^2(\phi_f)})(1 - \frac{cos(\phi_0+\delta\phi)}{\alpha sin^2(\phi_0+\delta\phi)})|(\Delta\phi_0) \qquad (29)$$

$$if \quad \frac{\partial p_{rf}}{\partial \phi_0} = 0 \qquad (30)$$

$$\epsilon_r^{rf} = \frac{\alpha k r_c^2}{4\pi}|sin(\phi_f) + \frac{2cos(\phi_f)}{\alpha sin^3(\phi_f)}|(1 - \frac{cos(\phi_0+\delta\phi)}{\alpha sin^2(\phi_0+\delta\phi)})^2(\Delta\phi_0)^2 \qquad (31)$$

where r_c is cathode radius.

3 DISCUSSIONS

The main advantage of TW gun is that it is suitable for providing high electron beam current. Limited by the length of this paper the space charge effects and the comparisons between TW gun and SW gun have been omitted [9]. Figs. (1-4) from theoretical formulae and numerical calculation are used to show the behaviour of a TW gun.

As for the practical TW structure design it has been suggested that a backward wave TW gun structure as shown in Fig. 5 could be used in order to avoid the problem of the unsymmetric field perturbation caused by the waveguide-TW structure coupling aperture in the case of a forward traveling wave structure [10].

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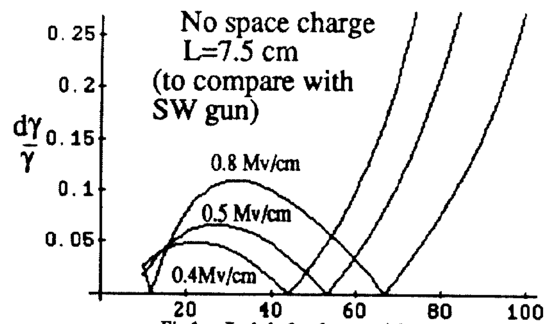

Fig.1 Initial phase (degree)

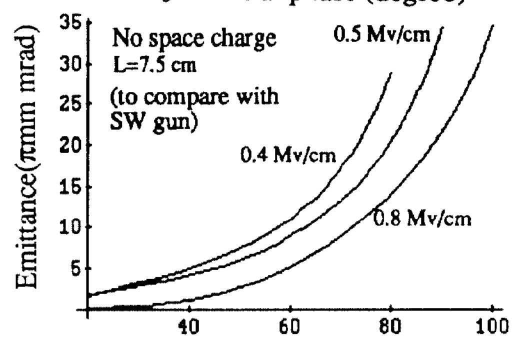

Fig. 2 Initial phase (degree)

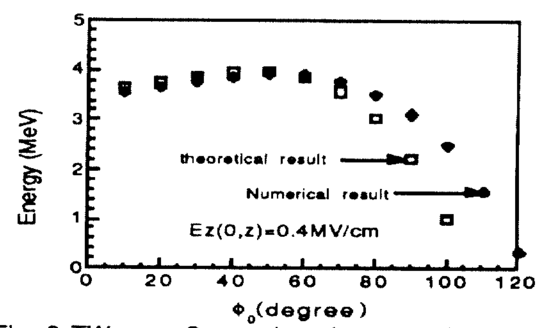

Fig. 3 TW gun: Comparison between the numerical results and theoretical results

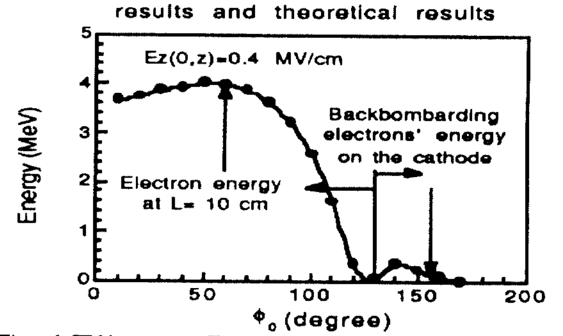

Fig. 4 TW gun: Energy vs initial injection phase

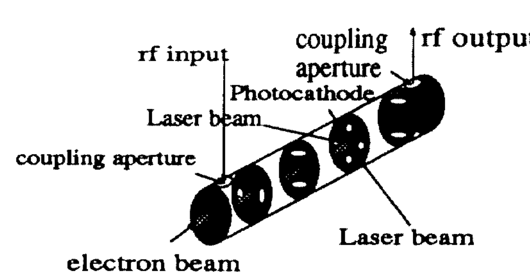

Fig. 5 Backward wave TW gun