Observation and Analysis of Nonlinear Parametric Coupling of Longitudinal Modes in Synchrotrons

Patrick L. Colestock, Gerald P. Jackson and Linda Klamp
Fermi National Accelerator Laboratory*
P.O. Box 500, Batavia, Il 60510

Abstract

Observations of coupling between longitudinal modes on a coasting beam have been made in the Fermilab Tevatron. A model for the intermode coupling is developed based on a three-wave scattering formalism. This analysis leads to a frequency selection rule and an intensity threshold, which determines the onset of the coupling phenomena. We present the results of these experiments and analysis.

1 INTRODUCTION

In a stored, coasting beam, a well-known class of longitudinal oscillations that occur at harmonics of the revolution frequency can be induced by external excitation or destabilized by a ring impedance of sufficient magnitude and appropriate phase.[1] A commonly-used method of empirically determining the ring impedance is by applying external excitation to the beam and measuring the transfer function[2] to a calibrated pickup.

During measurements of the beam transfer function in a Tevatron 150 GeV coasting beam, we have observed, in addition to the expected linear response, the excitation of a number of neighboring revolution harmonics. The response is suggestive of parametric[3,4], or three-wave coupling whereby the external excitation couples to other frequencies according to a specific set of selection rules.

In this paper, we describe details of these measurements and develop a theory based on an extension of the linearized Vlasov equation to include the effects of weak nonlinear coupling between longitudinal modes. In Section 2, we present the experimental results. In Section 3, we give a brief derivation of the nonlinear coupling theory and in Section 4 make a comparison of the theory and experiment. In Section 5, we discuss the implications of the phenomenon and applicability of the model.

2 MEASUREMENTS

A proton beam of intensity 5×10^{12} and momentum spread $\sigma_p/p \sim 2\times10^{-3}$ was coasting at 150 GeV. The beam was excited with a wideband longitudinal kicker and the response at the pump frequency was measured with a network analyzer using a wideband pickup. The neighboring frequency spectra were simultaneously monitored with a spectrum analyzer. Measurements were made by slowly scanning the pump frequency across a revolution line ($\dot{\omega}_{rf} \sim 160\ \mathrm{sec}^{-2}$).

The measured response was observed to contain harmonic lines *below* the pump frequency, as shown in Fig. 1(a), as well as a corresponding series of harmonic lines near zero frequency, as shown in Fig. 1(b). Further investigation indicated that the coupling to neighboring harmonics only occurred when the beam intensity was above about 2×10^{12}. The single-sideband response was observed for both positive and negative slewing directions, but disappeared altogether when the excitation frequency was stationary.

3 VLASOV MODEL

Longitudinal modes in a coasting beam are described by the Vlasov equation in the following form

$$\frac{\partial g}{\partial t} + i\,n\omega g + \dot{\varepsilon}\frac{\partial g_o}{\partial \varepsilon} = 0$$

where g is the perturbed distribution function for the n^{th} Fourier harmonic, g_o is the unperturbed distribution function, and ω is the instantaneous revolution frequency. This equation is linear and, apart from a small frequency spread, will not give rise to a response other than at the excitation frequency. In order to explain the frequency coupling, we follow the analysis of three-wave coupling in ref. 3 and introduce a modulation of the instantaneous energy of the form

$$\varepsilon = \varepsilon_o + eV_o\sin\Omega_o t$$

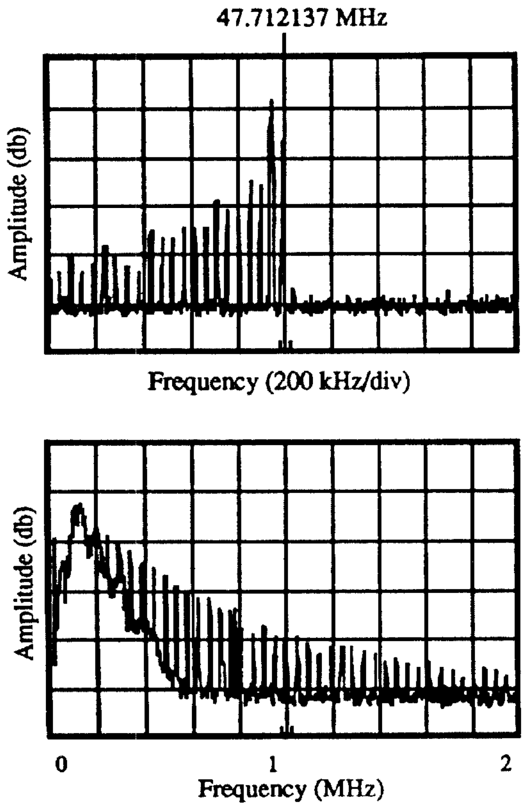

Fig. 1(a) Frequency spectrum near pump frequency $\Omega_o = 1000\omega_o$ (b) low frequency spectrum. Lines are spaced by one revolution frequency.

We have assumed that the excitation is spatially uniform. The resulting equation for g becomes

$$\frac{\partial g}{\partial t} + i\, n(\omega_o + k_o[\varepsilon_o + eV_o \sin\Omega_o t])g = -\frac{e\omega_o}{2\pi} U_n \frac{\partial g_o}{\partial \varepsilon}$$

where $k_o = -\eta\,\omega_o/\beta^2 E_o$ is the frequency dispersion and U_n is the wake potential of the n^{th} harmonic. The solution for $g(t,\varepsilon)$ may be expressed as a doubly infinite series of Bessel functions. We use the Fourier representation of the wake potential and integrate g over ε to finally arrive at the modulated current

$$I(\Omega) = \frac{(e\omega_o)^2}{2\pi i} \sum_{k,l} J_k(\mu) J_l(-\mu)\, I(\Omega + [k+l]\Omega_o) \cdot Z(\Omega + [k+l]\Omega_o) \int_{-\infty}^{\infty} d\varepsilon \frac{\frac{\partial g_o}{\partial \varepsilon}(\varepsilon)}{n(\omega_o + k_o\varepsilon) - k\,\Omega_o - \Omega}$$

where $\mu = \dfrac{nk_o eV_o}{\Omega_o}$.

This equation represents the coupling of adjacent longitudinal modes due to the modulation at Ω_o. We consider two distinct modes of index n_1 and n_2, and we keep only the lowest order terms in the Bessel function expansion. Retaining the contribution of the wake field of mode n_1 on mode n_2 and vice versa leads to the following coupled set of equations

$$I_1(\Omega)\, D_1^{(0)}(\Omega) = \frac{\mu}{2} I_2(\Omega-\Omega_o) Z(\Omega-\Omega_o)(\chi_1^0[\Omega] - \chi_1^1[\Omega]) + \frac{\mu}{2} I_2(\Omega+\Omega_o) Z(\Omega+\Omega_o)(\chi_1^0[\Omega] - \chi_1^1[\Omega])$$

$$I_2(\Omega)\, D_2^{(0)}(\Omega) = \frac{\mu}{2} I_1(\Omega-\Omega_o) Z(\Omega-\Omega_o)(\chi_2^0[\Omega] - \chi_2^1[\Omega]) + \frac{\mu}{2} I_1(\Omega+\Omega_o) Z(\Omega+\Omega_o)(\chi_2^0[\Omega] - \chi_2^1[\Omega])$$

where

$$D_j^{(k)}(\Omega) = 1 - \chi_j^{(k)}(\Omega) Z(\Omega)$$

$$\chi_j^{(k)}(\Omega) = \frac{(e\omega_o)^2}{2\pi i} \int_{-\infty}^{\infty} \frac{\frac{\partial f_o}{\partial \varepsilon} d\varepsilon}{\Omega + k\Omega_o - j(\omega_o + k_o\varepsilon)}$$

We consider solutions where $\Omega \sim \omega_o$ and as before retain only resonant terms, leading to

$$I_2(\Omega+\Omega_o)\, D_2^{(0)}(\Omega+\Omega_o) \approx \frac{\mu}{2} I_1(\Omega) Z(\Omega) \chi_2^0(\Omega+\Omega_o)$$

$$I_2(\Omega-\Omega_o)\, D_2^{(0)}(\Omega-\Omega_o) \approx \frac{\mu}{2} I_1(\Omega) Z(\Omega) \chi_2^0(\Omega-\Omega_o)$$

The final dispersion relation is found by eliminating the unknown currents from the coupled set of equations.

$$D_1^{(0)}(\Omega) = \frac{\mu^2}{4} \frac{Z(\Omega-\Omega_o) Z(\Omega) \chi_2^0(\Omega-\Omega_o)(\chi_1^0[\Omega] - \chi_1^1[\Omega])}{D_2^{(0)}(\Omega-\Omega_o)} + \frac{\mu^2}{4} \frac{Z(\Omega+\Omega_o) Z(\Omega) \chi_2^0(\Omega+\Omega_o)(\chi_1^0[\Omega] - \chi_1^1[\Omega])}{D_2^{(0)}(\Omega+\Omega_o)}$$

The strongest coupling occurs when the denominator of one of the terms on the right hand side goes to zero, namely at the resonance condition of the shifted frequency of mode n_2. The growth rate can be found by solving the above dispersion relation for the frequency Ω. The solution is shown for the case when $n_1 \ll n_2$ as a function of the drive frequency Ω_o in Fig. 2. The maximum growth $(\mathrm{Im}(\Omega) > 0)$

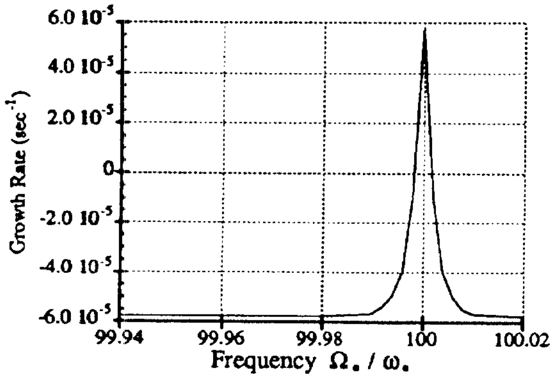

Fig. 2. Growth rate vs. drive frequency Ω_0.

occurs when $(n_1+n_2)\omega_0 = \Omega_0$. In addition, this model indicates an intensity threshold for the coupling to occur, as shown in Fig. 3. The process becomes unstable when the

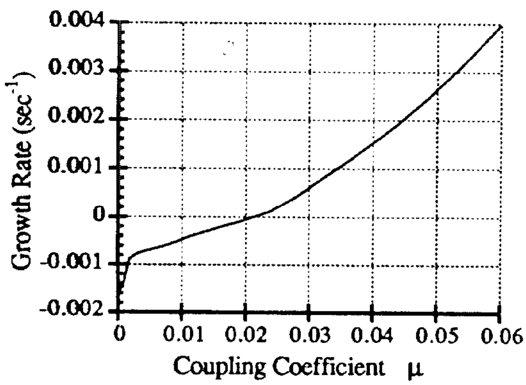

Fig. 3 Growth rate as a function of the coupling

coupling strength exceeds the power lost by Landau damping.

4 COMPARISON BETWEEN THE MODEL AND EXPERIMENT

The above model corresponds to the observations in two important ways. First, the coupling is observed to have an intensity threshold which depends on the strength of Landau damping. Second, the single-sided nature of the coupling is ensured by the frequency matching condition described above. While we have only modelled the coupling of adjacent modes, it is conceivable that, once generated, each decay mode in turn can decay into other lower frequency components, thus explaining the multiple line spectra observed. However, it is also possible that coupling across multiple harmonics can occur.

Other phenomena remain unexplained by our model, such as the lack of coupling with a stationary excitation frequency. It is possible, that changes in the equilibrium distribution can occur, which over long time scales will cause Landau damping to increase, thereby increasing the threshold for the coupling.

5 DISCUSSION

We conclude that that our model based on weak nonlinear coupling is in qualitative agreement with the observations. In partucular, the single-sided character of the coupling as well as the intensity threshold are explained by the theory. The coupling can be viewed as a scattering of one longitudinal mode from the pump wave oscillations into another longitudinal mode. The primary condition for coupling to occur is the fact that the pump frequency must equal the sum of the two decay frequencies.

The implication of these results is the fact that conventional beam transfer function measurements may be in question when either the exciting voltage or the beam intensity is sufficiently high. However, the observation of coupling can, in principle, be used to determine the ring impedance if details of the distribution function are known. Another consequence of these results is the fact that the overall beam stability is altered by the presence of the nonlinear coupling, though the modification is typically small.

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