ANALYTICAL AND NUMERICAL STUDY OF THE DISPERSION RELATION OF THE INTERACTION BETWEEN A HOT CYLINDRICAL PLASMA AND ELECTRON BEAMS

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Abstract

Based on the kinetyc theory of the plasma, we present a treatment of wake field phenomenon. We developed a mathematical model to calculated the frecuencies in an inhomogenical hot plasma that is interacting with electron beams. We used different inhomogeneous and temperatures for the plasma and electron beams. The system is put in a strong magnetical field. Finally, we examined the relationship between this effects and the instability that appears in this cases.

Introduction

There exists at present experimental evidence of the particles accelerated by the insertion of a beam into the plasma (Argon Advanced Accelerator Test Facility [1]). In respect there are many theorical papers that explain the basic fundamentals of this phenomenon [2,3,4], it considers in general multifluid models for the plasma wake field [5,6]. Besides its confined in wave guides has been studied. We present a theorical model based on the Vlasov - Poisson equations in orden to calculate the dispersion relation of a plasma that interaction with electron beams in cylindrical wave guides in a strong magnetical field along the z-axis. We find the electrostatic wave propagating in this medium. We estimate besides the termical effect and radial inhomogeneous for the plasma and the electron beams. We do not considerate the ion dynamics, i.e, we deal with an electronic plasma. The method is based on expansion of radial function of the inhomogeneous problems in terms of the well know of eigenfuctions for the homogeneous bounded plasma.

Plasma wake field theory

One model of finite system for any radial density profile is that of multispecies plasma inmersed in very strong magnetic field. We try initially the case of two species: plasma and electron beams, they are symmetric along the guide axis. Due to the strong magnetical field we don't considere the electron transversal movement, that is, the electron radio larmor is very small. Under this conditions the axial velocity distributions functions $f(r,\theta,z,v,t)$ and the equilibrium distribution function may be written as: $f^0_\mu(\vec{r},v) = g_\mu(r)\, F^0_\mu(\vec{r},\vec{v},t)$ where μ denotes the kind of particle in the plasma. The Vlasov - Poisson equation system can be written in terms of $f(r,\theta,z,v,t)$

$$\frac{\partial f_\mu(\vec{r},t)}{\partial t} + v\frac{\partial f_\mu(\vec{r},t)}{\partial z} - \frac{e_\mu}{m_\mu}\frac{\partial \Phi(\vec{r},t)}{\partial z}\frac{\partial f_\mu}{\partial v} = 0 \qquad (1)$$

$$\nabla^2\Phi(\vec{r},t) = -4\pi\Sigma e_\mu n_\mu(0)\int dv f_\mu(\vec{r},t) \qquad (2)$$

$\Phi(\vec{r},t)$ is the electrostatic potential, $n_\mu(0)$ the particle density on axis and the sum Σ is carried out over al species present in the system. In order to solve the pass equations systems must use a method of small perturbations [7], even though we suppose that the system is in stationary state, the distributions function is independent of z, axially symetric and $\Phi^0(\vec{r},t) = 0$; then we can write:

$$f^0_\mu(\vec{r},v) = g_\mu(r)\, F^0_\mu(v) \qquad (3)$$

Where $g_\mu(r)$ is the radial electron density profile normalized, $F^0_\mu(v)$ is the axial equilibrium velocity distribution, then we have:

$$f_\mu(\vec{r},t) = f^0_\mu(r,v) + f^1_\mu(\vec{r},\vec{v},t) + \ldots \qquad (4)$$

$$\Phi(\vec{r},t) = \Phi^1(\vec{r},t) + \ldots \qquad (5)$$

We take first terms only, besides in cylindrical coordinates the equations (1) and (2) are:

$$\frac{\partial f^1_\mu}{\partial t} + V\frac{\partial f^1_\mu}{\partial z} - \frac{e_\mu}{m_\mu}\frac{\partial \Phi}{\partial z}g_\mu(r)\frac{\partial F^0_\mu}{\partial v} = 0 \qquad (6)$$

$$\nabla^2\Phi(r,\theta,z,t) = -4\pi\,\Sigma e_\mu n_\mu(0)\int dv f^1_\mu(r,\theta,z,v,t) \qquad (7)$$

In order to solve this equations we use Fourier-Bessel expansions for the radial coordinate, the Fourier series for the angle θ and the Fourier integral for the z coordinate. We assume that the next orthogonal set is complete:

$$Y^{(k)}_{me}(r,\theta,z) = \frac{J_m(P_{ml}r)}{\sqrt{2}\,\pi\, a\, J_{m+1}(X_{ml})}\exp(im\theta + ikz) \qquad (8)$$

J_m is the Bessel function, $P_{ml}a$ are the radial waves number that can be determined by $P_{ml}a = X_{ml}$ where X_{ml} are the zeros of $J_m(X_{ml}) = 0$. On the other hand, we utilized the lapalce transform for the temporal variable. Finally, we get the following dispersion relation:

$$\left[\frac{Ka}{X^2_{me} + ka}\,\Sigma\, I_\mu(k,\omega)\, C^{ll'}_{\mu m} - II\right] A^n_m(\vec{k}) = 0 \qquad (9)$$

Where K is the wave number, a is the radio wave guide, II is the unity matrix and A_m are the eigenvectors that are associated with electrical field, then:

$$I_\mu(k,\omega) = \frac{\omega_{p\mu}^2(0)}{k^2} \int \frac{dv\ dF_\mu^0/dv}{(v - \omega/k)}$$

$$\omega_{p\mu}^2(0) = \frac{4\pi l^2 n_\mu(0)}{m_\mu} \tag{10}$$

Fu is the Maxwellian distributions and,

$$C_{\mu m}^{ll'} = \frac{2}{a^2 J_{m+1}(X_{ml}) J_{m+1}(X_{ml'})} \ldots$$

$$\ldots \int dr\ r\ J_m(P_{ml} r)\ J_m(P_{ml'} r)\ g_\mu(r) \tag{11}$$

The coefficient $\mu = 1,2$ corresponds to the plasma and electron beams, $g_\mu(r)$ gives the radial inhomogeneity of the μ species. The equation (9) is solved by using the Serizawa method [8] that for numerical facilities, in dimensionless units is:

$$\frac{\bar{K}^2}{X_{ml}^2 + \bar{K}^2} \left\{ \frac{C_{mp}^{ll'} Z'(\beta_p)}{2\lambda_{dp}^2 \bar{K}^2} + \frac{C_{mb}^{ll'} Z'(\alpha)}{2\bar{\lambda}_{db}^2 \bar{K}^2} - \ldots \right.$$

$$\left. \ldots - \delta_{ll'} \right\} A_m^n(k) = 0 \tag{12}$$

the P subindex means plasma and B electron beams, furthermore:

$$\bar{K} = Ka \qquad \lambda_{d\mu}^2 = \frac{K_B T_\mu}{m_l \omega_{p\mu}^2(0) a^2}$$

$$\beta_p = \frac{\bar{\omega}_p^2}{\bar{K}^2} \frac{1}{2\bar{\lambda}_{Dp}^2} \qquad \alpha = \frac{\bar{\omega}_B^2}{\bar{K}^2} \frac{1}{2\bar{\lambda}_{DB}^2} - \left(\frac{1}{K_{BT}}\right)^{1/2}$$

E is the energy of the beam and Z' is the derivate of the dispersion function.

Results

The matrix equation is solved to find its eigenvalues (frecuencies) using a numerical routine developed in the Universidad Nacional de Colombia-Manizales-which it permits:

1) Evaluate the equation (11) for any density profile g(r).

2) Evaluate the dispersion function Z and their derivates.

3) Calculate the eigenvalues of equation (12).

In this paper we just present results to evaluate the interaction between two species, but the result may be generalized for three or more species which ones can have different density profiles and temperatures. The figure 1 ilustrate the result for $(E/KT)^{1/2} = 10^4$, $\lambda^2 = 0.05$, $\lambda^2 = 0.01$. Figure 2 ilustrate the differences that exists between passed values and one electron beams less energetical $(E/KBT)^{1/2} = 10$. The frecuencies have two parts, $w = wr + iwi$, the imaginary part is always negative and gives the Landau damping, it is shawn in the down branches (figures 1,2), they used density profiles for the plasma and beam, respectively the following:

$$\text{a)}\ g_{ep} = \frac{1}{1 + (\alpha r/a)^2} \qquad \alpha = 3 \quad a = 5.\ 2$$

$$\text{b)}\ g_{eB} = \frac{1}{1 + (\exp(\frac{1}{c})(r/b-1)} \qquad Cb = 0.8 \tag{13}$$

Conclusions

Due to the fast convergence of the Bessel functions, we take only fifteen terms (it's the range of the dispersion matrix). The problem could be generalized for any number of species, we could consider the plasma, electron beams and a proton beams. The Landau damping depends strongly on the density profile, it is small for $Ka < 1$ and it is significant for $Ka > 1$. The model may be extended to include finite Larmor darius effects and ion waves.

Acknowledgments

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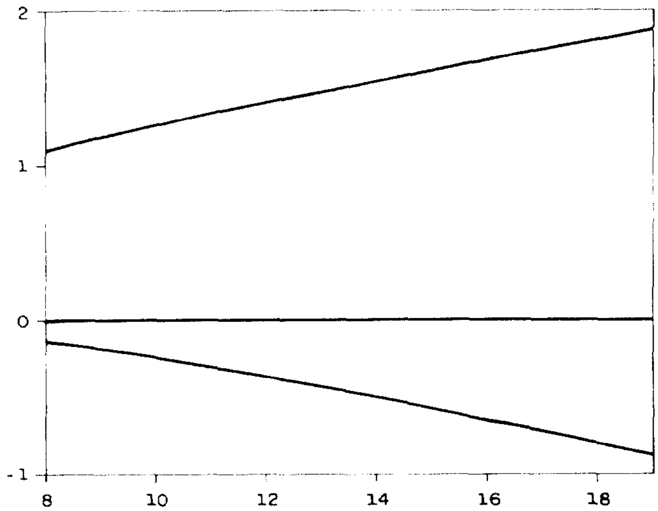

FIG (1): DISPERSION RELATION FOR: $\bar{\lambda}_{DP} = 1 \times 10^{-2}$ $\bar{\lambda}_{DB} = 5 \times 10^{-2}$; $\left(\frac{E}{K_B T}\right)^{1/2} = 1 \times 10^{+4}$

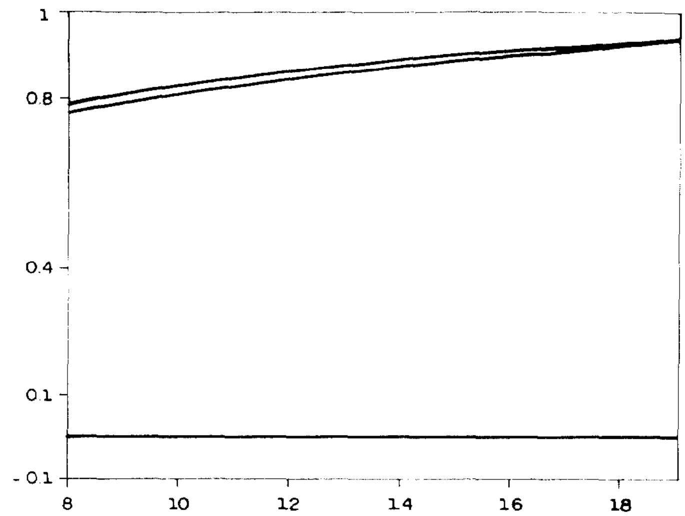

FIG (2): THE UPPER (DOWN) IS DRAW WITH: $\bar{\lambda}_{DP} = 1 \times 10^{-2}$ ($\bar{\lambda}_{DP} = 5 \times 10^{-2}$), $\bar{\lambda}_{DB} = 7 \times 10^{-2}$ ($\bar{\lambda}_{DB} = 7 \times 10^{-2}$)