SPIN MATCHING CONDITIONS IN LARGE ELECTRON STORAGE RINGS WITH PURELY HORIZONTAL BEAM POLARIZATION

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Introduction

In the last few years the self-polarization of electrons and positrons in storage rings (Sokolov-Ternov effect[1]) was observed in many storage rings: ACO[2], VEPP2[3], VEPP4[4], SPEAR[5], DORIS[6], PETRA,[7] and CESR.[8] In all these machines the beam was polarized by synchrotron radiation emission in the bending magnets of the arcs. As a consequence the spin axis in the arc must be parallel or antiparallel to the bending field, depending on the particle species.

At LEP (at least in phase I) the situation is different. The beam is polarized by so-called asymmetric wigglers at a point or points of the machine.[9] In the very beginning these wigglers will produce transversal polarization and the arcs act only as a spin transport system. Many years ago it was discussed whether it is possible to rotate the wigglers by 90 degrees and produce horizontally polarized beams.[13] It was shown experimentally at low energies[14] that such a system could work. It is also well known that a horizontal polarization can be maintained by so-called Siberian Snakes[10] in proton machines, as has been experimentally shown recently.[11] A Siberian Snake rotates the spin around the momentum axis by 180 degrees. As a result most of the perturbations which would otherwise add up over many revolutions compensate each other after two revolutions.

It is interesting at least from an intellectual point of view whether the Siberian Snake can be applied to electron machines with an asymmetric wiggler as a polarizer. It is assumed that the wiggler polarizes the beam in the horizontal direction and that the Siberian Snake is opposite to the wiggler (fig. 1). The only difference from a proton machine is the emission of synchrotron radiation in the arcs. The calculations are interesting for two reasons:

a.) the configuration shown in fig. 1 is less perturbing for the whole machine compared to spin rotators at each experiment.

b.) it is the present understanding that the degree of polarization is limited in big storage rings by the so-called nonlinear spin acceptance[16]: at higher energies the nonlinear effects contribute significantly to depolarizing effects. A horizontal spin could push this fundamental limit to higher values.

It was first shown by A. Chao and later by C. Prescott in unpublished contributions[12] that for LEP a purely horizontal polarization would have a depolarization time of only a few seconds. The reason is that the particles emit photons along the arcs. As a result the particle spins precess with different angular velocities around the field axis. Since the emission point can be anywhere in the ring the perturbation does not repeat after one revolution and the stability condition of a Siberian Snake is not fulfilled.

The aim of this paper is to define spin matching conditions that compensate this effect.

Depolarization of a purely horizontal polarization without Snakes

A purely horizontal spin precesses around the magnetic bending field. When the particle is deflected by the angle α the spin is deflected by an angle ϕ. The angles are related to each other by

$$\phi = a\gamma\alpha, \qquad (1)$$

where $a = (g-2)/2$ and γ is the Lorentz factor. After a full circulation α is 2π. Assume two particles: one at the nominal energy and one with an energy deviation $\Delta\gamma_0$ caused by the emission of a photon. The spins rotate with different angular velocities. Particles emitting synchroton photons perform synchrotron oscillations. As a result $\Delta\gamma$ varies with time.

$$\Delta\gamma = (\Delta\gamma_0)cos(\omega_s t)e^{-t/\tau_s}, \qquad (2)$$

where ω_s is synchrotron frequency and τ_s is the damping time for synchrotron oscillations. After damping the spins of the particles deviate slightly from each other

$$\Delta\phi = 2\pi\int_0^\infty a(\Delta\gamma)dt = 2\pi a(\Delta\gamma_0)\left(\frac{b}{b^2+\omega_s^2}\right), \qquad (3)$$

where b is $1/\tau_s$. Note that after the damping period the deviation of the two spin directions is small but not zero. A spin tune of 0.08, a revolution frequency of 10 MHz and a damping time of 34 msec (LEP at 45 GeV) leads to a deviation of 10^{-9} radian when the emitted photon changes the energy of the electron by 10^{-3}.

In order to compensate this effect a spin shift proportional to $-\Delta\gamma$ with the proportinal constants defined in equation (3) has to be applied. This can be done by beam bumps. In the following the principal concept is explained and in the appendix the beam bump concept is discussed in more detail.

Starting point for the proof of principle is equation (1). Assume a section of the machine where the particle is deflected by an angle α. Assume further that before and after this section the spin is rotated into the vertical direction. As a result the spin motion of a particle with the energy γ is retarded by $a\gamma\alpha$. The spin motion of a particle with the energy $\gamma+\Delta\gamma$ is retarded by $a\gamma\alpha + a(\Delta\gamma)\alpha$. The difference in the spin motion is $a(\Delta\gamma)\alpha$. By choosing the angle α the spin motion can be influenced proportional to $\Delta\gamma$ with an adjustable proportionality constant. Note that a negative angle α leads to a negative spin shift.

According to equation (3) the required spin shift is very small. Therefore in a real machine is required not an impractical 90 degree spin rotation but simply a deviation of the spin from the horizontal plane in one or several bending magnets. It will be shown later that this is sufficient.

It has to be taken into account that the spin shift is performed every revolution. Using equation (2) the spin shift has to be summed over many revolutions:

$$\Delta\phi_f = 2\pi a(\Delta\gamma_0)\sum_{n=1}^{\infty}\left(e^{-nt_0/\tau_s}cos(nt_0\omega_s)\right)$$

$$= 2\pi a(\Delta\gamma_0)\frac{1-e^{-t_0/\tau_s}cos(\omega_s t_0)}{1-2e^{-t_0/\tau_s}cos(\omega_s t_0)+2e^{-2t_0/\tau_s}cos^2(\omega_s t_0)}. \quad (4)$$

The spin matching condition for this case is $\Delta\phi = -\Delta\phi_f$.

Spin matching condition with concentrated cavities

Equation (2) is an idealized description of particles in a storage ring. In a real storage ring the cavities are concentrated in one or two sections. The particle energy is changed by emission of a photon in the arc. After the emission the energy is not changed until the particle passes through the cavities. Assume now that a spin shift proportional to the energy is performed in front of and after each set of cavities. The spin is changed in front of the first set of cavities proportional to $-\Delta\gamma_0$ by an angle $\Delta\phi_0$. The cavities change the energy to $\Delta\gamma_1$ and the spin is shifted after the cavities proportional to $\Delta\gamma_1$ by an angle of $-\Delta\phi_1$. The energy is not changed until the spin comes to the next set of cavities. Before the particle enters the next set of cavities the shift is proportional to $-\Delta\gamma_1$, namely $\Delta\phi_1$. As a result the effect of the second shift is canceled. Finally all shifts are cancelled except the first one:

$$\Delta\phi_f = \Delta\phi_0 - \Delta\phi_1 + \Delta\phi_1 - \Delta\phi_2 + \Delta\phi_2 - ..... = \Delta\phi_0. \quad (5)$$

The spin matching condition is $\Delta\phi = -\Delta\phi_0$. The shift is proportional to $-\Delta\gamma_0$. A comparison with formula (5) shows that the spin shift no longer depends on storage ring parameters.

The spin shift proportional to the energy before and after the cavity is equivalent to a spin shift proportional to Δs, where Δs is the deviation of the particle from the center of the bunch. This can be shown in the following way. Δs is expressed as

$$\Delta s \approx (\Delta\gamma)\int_{s_1}^{s_2}\alpha(s)ds, \quad (6)$$

where $\alpha(s)$ is the momentum compaction factor. On the other hand, the energy gain in a cavity is proportional to Δs. When the spin is shifted before and after a cavity proportional to $\Delta\gamma$ with different signs the shift will be therefore proportional to Δs.

Spin matching with concentrated cavities and rotator

The principal layout is shown in fig. 1. Note that there is only one stable condition possible in such a configuration ("stable" means that the spin returns after one revolution into the same position). This stable position or n-axis in the polarizing element points in the radial direction. As a result the spin is aiming in the direction of the n-axis when the particle is polarized in the polarizing element or when a photon is emitted in the polarizing wiggler. Assume now that a photon is emitted somewhere in the arc. The energy is reduced and the spin comes to the rotator with a deviation of

$$\Delta\phi_n = a(\Delta\gamma_0)\left(\pi - \frac{s_2 - s_1}{R}\right) \quad (7)$$

from the n-axis, where s_1 is the position of the rotator and s_2 is the position where the photon is emitted. s is only counted in the bending magnets. R is the bending radius in the magnets. $\Delta\phi_n$ is a deviation from the n-axis and remains a deviation after damping and finally leads to depolarization.

The scheme for compensation for this effect is similar to the scheme shown in the previous chapter. If it is assumed that Δs in equation (6) and $s_2 - s_1$ in equation (7) are proportional to each other (which is true for most storage rings), the effect can be compensated by a spin rotation before and after the cavity in the previously described manner.

This concept is explained in more detail in the following. The deviation of the spin from the n-axis is, according to equation (7),

$$\Delta\phi_n = a(\Delta\gamma_0)\left(\pi - \frac{s_{cavity} - s_{emission}}{R}\right). \quad (8)$$

The change in energy in a cavity according to equation (6) is

$$\Delta\gamma_{cavity} = k(\Delta\gamma_0)\int_{s_{emission}}^{s_{cavity}}\alpha(s)ds,$$

$$\approx k_1(\Delta\gamma_0)\left(\frac{s_{cavity} - s_{emission}}{R}\right), \quad (9)$$

k and k_1 are constants. As mentioned, the spin is shifted proportionally to Δs or $\Delta\gamma_{cavity}$,

$$\Delta\phi_{total} = k_2\Delta\phi_n + k_3\Delta\gamma_{cavity},$$

$$= k_2(a\Delta\gamma_0)\left(\pi - \frac{s_{cavity} - s_{emission}}{R}\right) + k_3\Delta\gamma_0\left(\frac{s_{cavity} - s_{emission}}{R}\right) \quad (10)$$

If k_2 and k_3 are chosen in an appropriate way,

$$\Delta\phi_{total} \approx a(\Delta\gamma_0)\pi \quad (11)$$

is independent of the point of emission.

Equation (11) is valid only when cavities and spin rotator are not separated by elements with strong synchrotron emission. If there is a significant synchrotron radiation emission between cavity and spin rotator the polarization will be destroyed. In order to avoid this an additional spin shift proportional to Δs has to be installed. This can be done by using a cavity in front and after the spin rotator. The cavities have opposite phase and therefore do not act on the beam.

Appendix: spin matching by closed orbit bumps

As shown in previous papers transverse polarization in high energy electron-positron storage rings can only be obtained by applying closed orbit deviations in an intelligent manner to the existing orbit. The method is based more or less on trial and error; no beam position monitors are able to detect the small orbit variations between a low and a high degree of polarization. In the following it is shown that a similar technique can be applied for pure horizontal polarization.

Assume a closed orbit kick at a certain position in the machine. Let us assume that the spin is rotated as a result of this kick around the momentum axis by an angle β_1. Afterwards the spin is rotated in a bending magnet by Δ around the vertical axis and afterwards by another kick or set of kicks around the momentum axis by β_2. The spin transfer matrix for this case is

$$\begin{pmatrix} \cos\Delta\cos\beta_1\cos\beta_2 - \sin\beta_1\sin\beta_2 & -\sin\Delta\cos\beta_2 & -\cos\Delta\sin\beta_1\cos\beta_2 - \cos\beta_1\sin\beta_2 \\ \sin\Delta\cos\beta_1 & \cos\Delta & -\sin\Delta\sin\beta_1 \\ \cos\Delta\cos\beta_1\cos\beta_2 + \sin\beta_1\cos\beta_2 & -\sin\Delta\sin\beta_2 & -\cos\Delta\sin\beta_1\sin\beta_2 + \cos\beta_1\cos\beta_2 \end{pmatrix} \quad (12)$$

If the starting point (where β_1 is applied) is (x_0, y_0, z_0), and the final point after three deflections is (x_3, y_3, z_3), and z_0 and z_3 are close to zero (purely horizontal spin), the total spin deflection angle can be written in the form

$$\Psi_m = f(\frac{y_0}{x_0}, \gamma) = \sum_{n=0}^{\infty} a_n \left(\frac{y_0}{x_0}\right) \gamma^n. \quad (13)$$

Applying several bumps in a proper way leads to

$$\Psi = \sum_m \Psi_m = \sum_m \sum_{n=0}^{\infty} a_{n,m} \left(\frac{y_0}{x_0}\right) .\gamma^n \approx k\gamma + \text{higher orders.} \quad (14)$$

k can be chosen by selecting a series of suitable bumps. As a result the spin matching in the horizontal case is similar to the correction scheme which has to be applied for the vertical spin direction.[15]

Summary

In a storage ring with a purely horizontal spin and a Siberian Snake, the spin matching conditions are similar to the spin matching conditions for vertical polarization; a combination of beam bumps has to be found which compensate the depolarizing effects. These bumps compensate the random emission of synchrotron emission on the spin.

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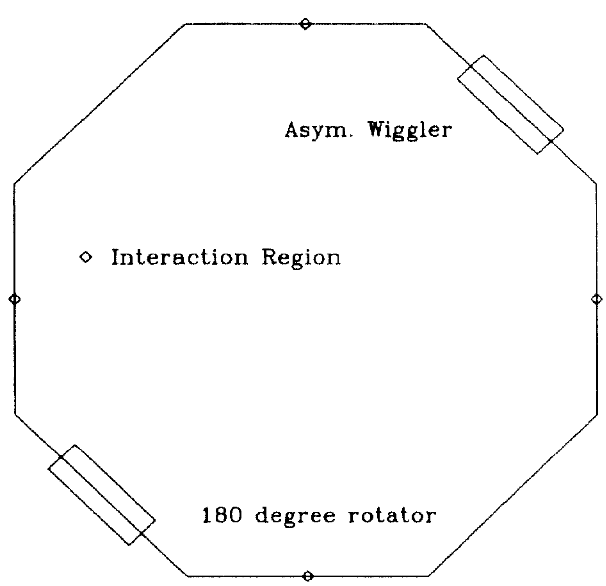

Figure 1. Schematic diagram of storage ring