DESCRIPTION OF THE DIRECTIONAL COUPLER POSITION MONITOR AS N-PORT

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Abstract: *For the design and description of the beam position monitors for the HERA proton ring a complete n-port model was used. The model is valid for beams with transverse electromagnetic fields and directional couplers with constant impedances over their entire length. The n-port description is compared with the simplified lumped circuit model, the wall current model, and measurements made using HERA beam position monitors.*

Introduction

Directional couplers are widely used as position monitors in large proton accelerators. Their frequency response is well adapted to the bunch lengths of several tenths of a meter to several meters, and their directivity allows the separation of counter-rotating bunches without complicated timing considerations.

Directional coupler position monitors consist of one or two pairs of quarter wavelength antennas within the beam pipe. Each antenna forms a stripline of constant impedance (typically 50 Ω) with the beam pipe wall (see Fig. 1). The closer the beam to one antenna of the pair, the higher will be the induced electrical signal on this antenna relative to the other. This fact is exploited for the position measurement. For the thorough understanding and design of the pickup a model is necessary. It should provide a quantitative understanding of the absolute signal strength on the antennas and of the dependence of the signal strength on the beam position and the Fourier component of the beam current, and must describe the directivity. A rather general approach is the description as an n-port.

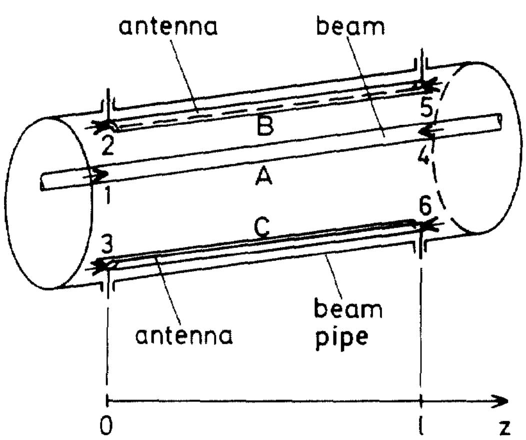


Fig. 1 Directional coupler pickup with two antennas (6 port)

N-Port Model

In this section we want to relate the input currents I_i and the input voltages U_i of the directional coupler pickup with a simple function $Y(\vec{U}) = \vec{I}$. For given termination admittances, all electrical properties of the pickup can be calculated as a function of the beam properties. For the case of only transverse electromagnetic fields, lossless antenna signal transmission at the speed of light and a highly relativistic beam, we have a linear device and the function Y is simply the following admittance matrix:

$$\mathbf{Y} = j \cdot c \cdot \begin{pmatrix} C/\tan\theta & C/\sin\theta \\ C/\sin\theta & C/\tan\theta \end{pmatrix} \qquad (1)$$

with c .... speed of light
C .... capacitance matrix of the antennas and beam in the vacuum pipe
$\theta = 2\pi\nu l/c$ ... electrical length of the antennas
ν .... frequency
l .... physical length of the antennas.

Derivation of the admittance matrix

For the purpose of illustration it is sufficient to restrict oneself to the case of a beam with one pair of antennas, i.e. a 6-port. The conventions are shown in figs. 1, 2 and 4. First we excite the device with 6 different superpositions of voltages and calculate the corresponding currents. The currents may then be calculated for any voltage combination $U=(U_1,\dots,U_6)$; this leads directly to the coefficients of the admittance matrix.

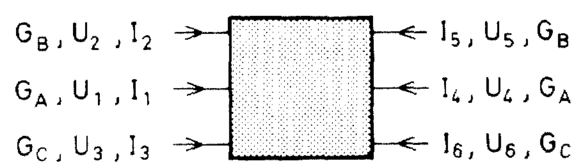


Fig. 2 Conventions for the 6 port description of the two antenna pickup

A short review of the lossless twin conductor problem is helpful. Here one calculates the influence of two conductors which are electromagnetically coupled to each other (see Fig. 3). Using Maxwell's equations and some conventions one finds:

$$U(z) = U(l)\cdot\cos k(l-z) + j\, I(l)/Y_o \cdot \sin k(l-z)$$

$$I(z) = I(l)\cdot\cos k(l-z) + j\, U(l)\cdot Y_o \cdot \sin k(l-z) \qquad (2)$$

$$Y_o = \sqrt{C/L} = c\cdot C$$

with C and L the capacitance and inductance of the line (e.g. [8]).

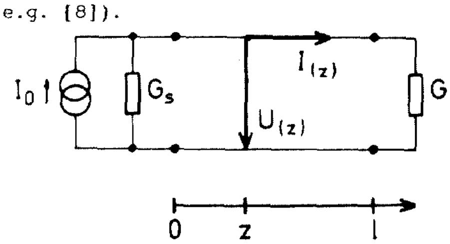


Fig. 3 Lossless twin conductors with TEM coupling ($Y_o = G_s = G_l$)

Next let us excite the 6 port in a particularly simple way: $U = (-u_1, u_1, u_1, 0, 0, 0)$, keeping in mind that we cannot set both ends of a conductor to zero. Using (2) we have for conductor A, i.e. the beam:

$$0 = U_A(z=l)$$

$$-u_1 = U_A(z=0) = j\, I_A(z=l)/Y_{1A}\cdot\sin kl$$

with $Y_{1A} = c\,(C_A + 2C_{AB} + 2C_{AC})$

$$i_{1A}(z) = I_A(z=l)\cdot\cos k(l-z)$$

Eliminating $I_A(z=l)$ leads to:

$$i_{1A}(z) = j\cdot u_1\cdot Y_{1A}\,\cos k(l-z)/\sin kl.$$

For conductor B, i.e. the first antenna, there follows similarly

$$i_{1B}(z) = -j\, u_1\, Y_{1B}\, \frac{\cos k(l-z)}{\sin kl}$$

$$Y_{1B} = c\,(C_B + 2C_{AB})$$

and for the third conductor accordingly. So we know the currents on each part for the first voltage vector. We do the same for the other five voltage vectors $(u_2,-u_2,u_2,0,0,0)$, $(u_3,u_3,-u_3,0,0,0)$, $(0,0,0,-u_4,u_4,u_4)$, $(0,0,0,u_5,-u_5,u_5)$ and $(0,0,0,u_6,u_6,-u_6)$. Finally we get the admittance matrix Y as in equation (1) by combining the applied voltages and derived currents, where the capacitance matrix **C** is given by:

$$\mathbf{C} = \begin{pmatrix} C_A+C_{AB}+C_{AC} & -C_{AB} & -C_{AC} \\ -C_{AB} & C_B+C_{AB}+C_{BC} & -C_{BC} \\ -C_{AC} & -C_{BC} & C_C+C_{AC}+C_{BC} \end{pmatrix}$$

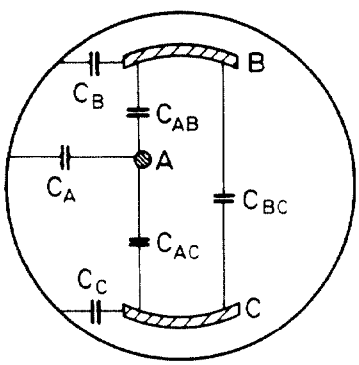


Fig. 4 Conventions for the capacitance matrix of a two antenna pickup

Application to the directional coupler

For the application of a directional coupler in an accelerator one is usually not interested in the admittance matrix Y as in (1) itself, but in the response of the pickup to a given beam, i.e. a current. So let us terminate the beam with an admittance G_A and the antennas with admittances G_B and G_C. Then for a voltage U_2 at the beginning of the antenna B we have a current $I_2 = -G_B \cdot U_2$, etc. This permits elimination of all the currents of the r.h.s. of $\mathbf{Y}\cdot\vec{U} = \vec{I}$, except the beam input current.
Next this complex system of linear equations has to be solved for the voltages. In general this has to be done numerically. There is a simple closed solution only for the case of a single antenna and the beam. It was originally derived by Cristal [3]. Here it is even possible both to terminate the device properly and to get (theoretically) full directivity. The terminating admittances G_A, G_B have to be chosen in the following way:

$$G_A^2 = c^2 \cdot C_A/C_B \cdot (C_A \cdot C_B - C_{AB}^2)$$

$$G_B^2 = c^2 \cdot C_B/C_A \cdot (C_A \cdot C_B - C_{AB}^2)$$

One finds then an antenna voltage U_2 of

$$U_2 = \frac{k}{\sqrt{G_A \cdot G_B}} \cdot \frac{1}{\sqrt{(1-k^2/tg^2\theta)+1}} \cdot \exp\left(i \cdot \mathrm{arctg}\sqrt{\frac{1-k^2}{tg^2\theta}}\right) \quad (3)$$

with $k = C_{AB}/\sqrt{C_A \cdot C_B}$ the coupling, and $U_5 = 0$ (directivity).

For a small coupling k we would have:

$$|U_2| = k/\sqrt{G_A \cdot G_B} \cdot |\sin\theta|(1 + 1/2k^2\cos^2\theta) \quad (4),$$

i.e. even for relatively large couplings one has a simple sinusoidal frequency dependence.

Capacitance Matrix

In the n-port description of the directional coupler all the geometric information is contained in the capacitance matrix and the antennal length. For the determination of the capacitance coefficients we used an SOR code and for a particular monitor we used the method of conformal mapping. Comparisons with measurements show an uncertainty of 5-10% in the coefficients. The other problem is the effort of doing all the necessary capacity calculations for the full matrix.
The relations between the monitor geometry and the pickup response to the beam in the context of the n-port model is summarized in Fig. 5.

$$\left.\begin{array}{l} \left.\begin{array}{l} C_{ij} \\ l \end{array}\right\} Y_{ij} \\ G_{A,B,C} \end{array}\right\} U_{2,3}\,(\nu, i_B) \;\Big\}\; \frac{U_2}{U_3}\,(x,y)$$

Fig. 5 Procedure to calculate the pickup properties

Other models

There are two approaches commonly used for the quantitative description of the behaviour of a directional coupler position monitor (see e.g. [1], [2]): a lumped circuit model, which describes the frequency dependence of the output signal on the beam, and the wall current model, which describes the total output voltage itself and its dependence on the beam position. Both models refer to a directional coupler without cross coupling between the antennas. They will be described for the case of one antenna with the conventions of Figs. 1, 2.

The lumped circuit model

In this model one replaces the ends of the antenna by current sources and the antenna itself by a transmission line (see Fig. 6). No further coupling is assumed. Here we have

$$U_2 = i_o\,|\sin(2\pi\nu l/c)| \quad \text{and } U_5 = 0$$

with i_o a constant current to be determined by another method. This is exactly the n-port model result for the case with only one antenna and zero coupling, $k = C_{AB}/\sqrt{C_A \cdot C_B} = 0$, see Equation (4).

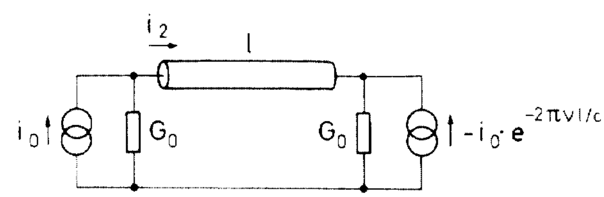
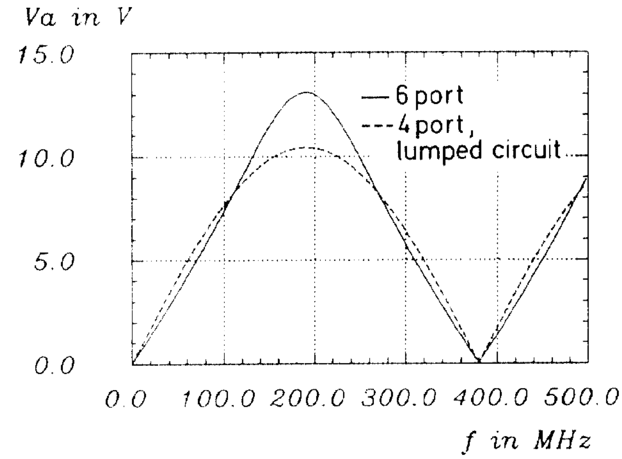


Fig. 6 Pickup in the lumped circuit model

For the HERA position monitors [6], [7] k is 8%, so both the 4-port and the lumped circuit model give virtually the same results, but the presence of the extra antenna has a large influence on the transfer function (see Fig.7 and in [7] Fig.4). Due to critical and imperfect input matching in our measurement setup, we do not include a comparison to laboratory data.
For the case of large coupling k=0.5 (see Fig. 7b) there is less influence of the extra antenna, but, as expected, a larger deviation between the 4 port and the lumped circuit models; see Equation (4).



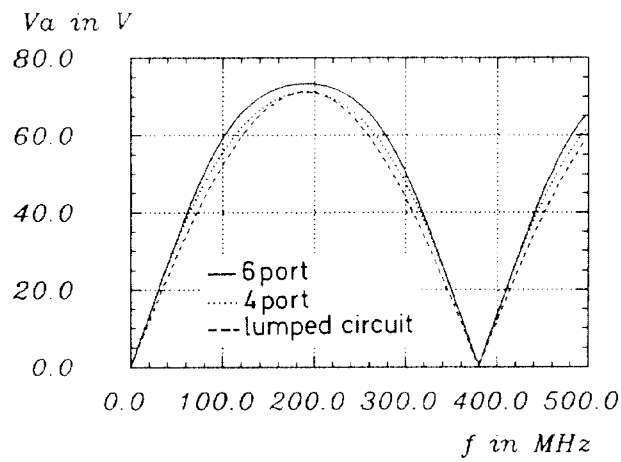


Fig. 7 Pickup response as a function of frequency (transfer function) - a) HERA monitor [6] b) large coupling k=50%

<u>The wall current model</u>
For this model one assumes a highly relativistic beam, transverse electromagnetic fields and a completely shielding vacuum chamber. Then one calculates the current in the vacuum chamber, which shields the oppositely flowing beam current, both absolute and as a function of the beam position. Next one assumes the antennas are only a small perturbation of the vacuum chamber and the angle equivalent amount of current is flowing on them.

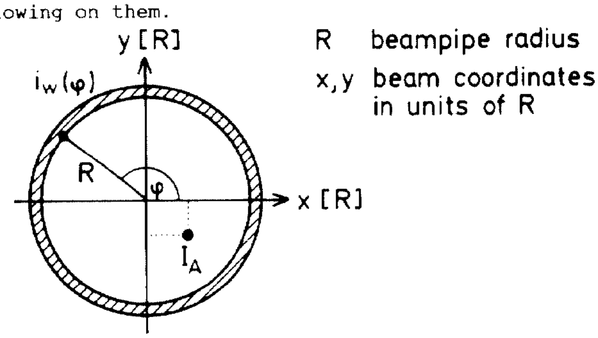


Fig. 8 Conventions for the wall current model

Current distributions in round vacuum chambers are calculated by Cupérus [4] and Regenstreif [5] with different assumptions. Both lead to a current density

$$i_w(\phi) = \frac{I_A}{2\pi R} \cdot \frac{1-x^2-y^2}{1+x^2+y^2-2x\cos\phi-2y\sin\phi} \quad .$$

The comparison of this result integrated over the 36° antenna width with the 4 and 6-port calculations and the measurement is shown in Fig.9 and in [7], Fig. 5.

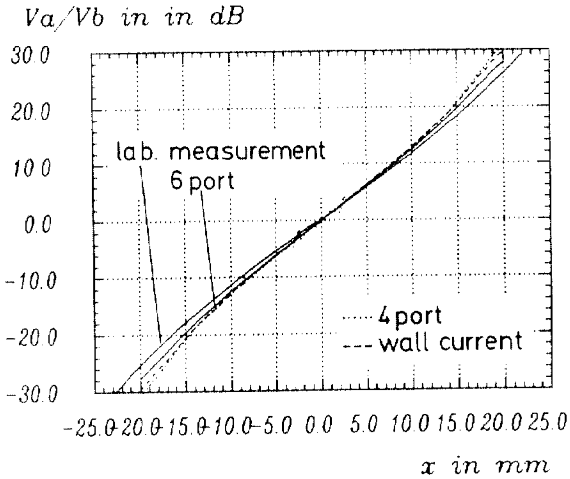


Fig. 9 Pickup response as a function of beam position for HERA monitor [6]

All approaches give similar results, especially in the most important central pickup region. Here the following slopes are found: 6 port 1.20 dB/mm, 4 port 1.24 dB/mm, wall current 1.24 dB/mm, laboratory measurement 1.143 dB/mm, measurement in a small electron accelerator 1.2 dB/mm. One finds the largest differences between the models at beam positions close to the antennas, where the exact antenna geometry is important. For the warm position monitors [7] the differences between the models are larger due to the antennas not bulged into the vacuum chamber. The best description is given by the n port model.

Conclusion

A full n-port description of a directional coupler pickup for a highly relativistic beam is given in the present paper. This is the only way to calculate cross coupling effects between antennas. For a description of a monitor with low coupling and antennas, which hardly disturb the vacuum chamber profile, the lumped circuit model together with the wall current model is sufficient and practical, though less accurate. From a theoretical point of view the coherent description with the n-port model is more satisfying.

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