CALCULATIONS OF EDDY-CURRENT EFFECTS IN A RAMPED DIPOLE FIELD

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Abstract

A two-dimensional computer program has been developed, aiming at a rapid evaluation of the eddy-current-induced magnetic field in a ramped dipole magnet with a conducting beam tube. The program is based on a closed-form sum of the infinite number of mirror currents representing the additional influence of the iron in the poles. Various tube profiles can be put in. — The results of the program have been compared with calculations using the POISSON code. The agreement is very good, even for beam pipes extending out near to the pole edges.

Calculations of the temperature distribution in the walls of a rectangular beam-tube, cooled by a pattern of water-pipes, have been done, by the POISSON code also, in order to minimize the outgassing from the walls. A typical result is given.

Calculation of magnetic effects

Underlying concepts

Assume a vast homogeneous magnetic field B_y which is ramped at a rate of $\dot{B}$ Tesla/sec. Perpendicular to this magnetic field there are two parallel conducting filaments, very long (l_{fil}) compared to their mutual distance $2x$. At their distant ends, they are connected to each other, forming a closed loop of area $S = 2x \cdot l_{fil}$. The law of induction then gives the induced electric field strength in the filament from

$$E_z \cdot 2 \cdot l_{fil} = \dot{B} \cdot S = \dot{B} \cdot 2x \cdot l_{fil}$$

whence $E_z = \dot{B} \cdot x$ Volts/m. As a result, there will flow a current in the filaments, in opposite directions, and with a strength $I_z = E_z/R$, where R is in Ω/m. These currents will give rise to a secondary magnetic field, which in most practical cases is small compared to the driving ramped field, and given by Biot-Savart's law in any point which is far from the ends of the filaments.

Next step is to introduce semi-infinite ferromagnetic pole plates of large permeability above and below the filaments, and with surfaces perpendicular to the original field. The secondary field will then be considerably modified, and it can be shown that the additional induced field is identical to what would be obtained from two infinite sets of mirror filaments originating from each of the original filaments. The currents in these fictive filaments all have the same strength and direction as those in the real ones. To obtain the total field in a given point a vector summation of the fields from all these filaments must be be performed. It simplifies matters considerably that a closed expression has been derived by Hague [1] for this mirror summation. His formulae for one filament read (converted to the SI system):

$$B_x = \frac{\mu_0}{4}\frac{I_z}{g}\left[\frac{\sin\frac{\pi}{g}(y+h)}{\cosh\frac{\pi}{g}x - \cos\frac{\pi}{g}(y+h)} + \frac{\sin\frac{\pi}{g}(y-h)}{\cosh\frac{\pi}{g}x - \cos\frac{\pi}{g}(y-h)}\right]$$

$$B_y = \frac{\mu_0}{4}\frac{I_z}{g}\left[\frac{\sinh\frac{\pi}{g}x}{\cosh\frac{\pi}{g}x - \cos\frac{\pi}{g}(y+h)} + \frac{\sinh\frac{\pi}{g}x}{\cosh\frac{\pi}{g}x - \cos\frac{\pi}{g}(y-h)}\right]$$

where g is the gap height between the pole plates, and h the height of the filament above the bottom pole plate.

The fundamental two-fold symmetric arrangement is obtained by introducing two pairs of filaments, such that the two pairs lie symmetrically with respect to the median plane of the pole plates. Finally, to form a real beam-tube, the (infinitely thin) filaments have to be substituted for thin slices of metal sheet adjacent to each other, each slice of width ds carrying a current which can be written as

$$I(x) \cdot ds = \sigma \cdot \dot{B} \cdot D \cdot x \cdot ds = A \cdot x \cdot ds,$$

where σ is the conductivity of the sheet and D its thickness. For "horizontal" parts of the beam-tube (i.e. those which are parallel to the poles) $ds = dx$, and for vertical parts $ds = dy$, while for possible sloping parts $ds(x)$ has to be calculated from the shape of the tube.

Computer code

A two-dimensional computer code, called GENEDDY, and based on the formulae of Hague, has been developed to provide the integration over the whole beam-tube profile of the x- and y-components of the induced field at an arbitrary point between the poles. The core of the program is dealing with only two (dimensionless) variables, viz. the width of the beam tube (P), and the smallest spacing (ISOL) between beam tube and pole-shoe, both in units of the beam-tube height. The GENEDDY code can handle various quasi-elliptical beam-tube shapes (i.e. also super- and sub-ellipses with exponent $\neq 2$), as well as rectangular shapes (including walls of a thickness different from that of the floor and roof), and corner-cut rectangles. The field-point density in the x-direction can be chosen at will, whereas that in the y-direction is limited by the width of line-printer paper.

Tests of the program

Various features of the program have been checked against more or less hand-made calculations and also against calculations performed by other computer codes. The simplest check was done by (almost) removing the iron effect by giving a big value (50) to the parameter ISOL, and computing the field at the origin from two symmetrically placed pairs of filaments. The result agreed with a Biot-Savart (BS) calculation. — To check the summation formula of Hague in a sufficiently simple case, BS was applied to a single filament placed in the median plane, together with all its mirror currents. The evaluation involves the summation of the series $1/(a^2 + N^2)$, where N goes from 1 to infinity; this is achievable by Cauchy's method of residues [2].

The notion of pole shoes that are semi-infinite in both the x- and the y-direction could in principle give rise to errors when applied to a real magnet. To check that, POISSON-code [3] runs on a configuration with four symmetrically placed filaments in the gap of an ordinary C-magnet were done. The agreement with the GENEDDY output was better than 0.2% even for filament positions as close as 1 cm from the edge of a gap that was 8 by 37 cm.

The procedure of summing the contributions from elements of the beam tube was checked against POISSON runs, in which several tens of line current regions were introduced to simulate the tube (in GENEDDY the number of elements can be many hundred without undue CPU-time consumption). Both rectangular and elliptical tubes were calculated, both giving better than 1% agreement.

The idea of using the induction law combined with the POISSON code was adopted from Hemmie [4][5]. Our calculated field shape for the DESY II beam tube is in excellent agreement with that of ref. [5], and for the maximal sextupole strength we find within 15% the same number as that which was given in [5], and also experimentally verified there.

The simplifying assumtion of well-behaved return paths at infinity for the eddy currents should often be approximately valid thanks to the flanges, generally situated outside the field, in which the return currents can flow rather freely.

Thermal effects

In a machine like CRYRING [6], intended for very highly charged heavy ions, the vacuum requirements are at least two orders of magnitude stronger than in a proton or electron ring. All possible steps must therefore be taken to avoid unnecessary heating of the beam-tube during running, while the tube must be well isolated thermally during bake-out. For the fairly fast cycling envisioned in CRYRING, nearly 2 Hz, and the rather wide beam-tubes necessary to provide inlets for merging particle or laser beams, even a free-air cooled tube would attain several tens of degrees above ambient temperature. Also, it would be unduly cumbersome to mount and dismount the bake-out jackets every time the system has to be pumped down. It was thus decided to include cooling-water pipes in the beam-tube layout. The heat conduction to the water pipes takes place only laterally through the beam-tube wall, and the heat generation in that wall is proportional to I^2/D, while the thermal conductance is proportional to D. Since I is also proportional to D, it follows from the heat flow equation that all temperature differences along the beam-tube surface are independent of the thickness of the tube. Provided that the eddy-current-induced magnetic fields can be coped with, there is then no reason to aim at extremely thin walls (<1 mm). — This was the reason to abandon early ideas on composite beam tubes; such tubes would also be difficult to join to cooling-pipes. — Since the eddy currents increase linearly with the distance x from the tube median, the power density will vary as x^2. Thus, if the cooling-tubes are parallel to the beam, they should be placed rather far out to have the best efficiency.

A sufficient mechanical rigidity of a wide rectangular beam-tube may be obtained by attaching transversal reinforcement bars to the floor and roof. Savings in mechanical complexity may be achieved by combining these bars with the cooling-pipes. The heat flow can then no longer be treated one-dimensionally, but "fortunately" the POISSON code lends itself also to this type of calculations. The cooling pipes are then held at a fixed temperature, and the power density is discretized over the surface. The vertical wall should of course also be included – and can be, just by "straightening out the corner".

The vacuum in the beam-tube (in the absence of real leaks!) is determined by the outgassing rate of its surfaces, and for the high-grade stainless steel (316LN) to be used this rate is in turn essentially proportional to the diffusion rate for hydrogen in the walls. The activation energy for diffusion in 316 LN (15100 cal/mole) implies that the outgassing rate doubles for a temperature rise of 7.3°. It is evidently worth-while trying to avoid large temperature excesses in substantial parts of the beam-tube. Thus, for a given acceptable mechanical complexity, the outgassing rate should be integrated over the beam-tube surface, and the integral minimized by varying the positions of the pipes.

Preliminary calculations show that the surface temperature at max. cycling rate in CRYRING will nowhere exceed 27°, assuming a beam-tube width of 22 cm, a cooling-pipe spacing of 6 cm at floor and roof, and two optimally placed pipes at each side wall (which is 6 cm high). This is valid for a cooling-water temperature of 12°, which is about as low as is practical with regard to the risk of condensation from moist air. The mean temperature of the dipole beam-tube will then be in the neighbourhood of 20°, i.e. the same as the main part of the beam-tube outside the dipole magnets. An illustration is given in Fig 1.

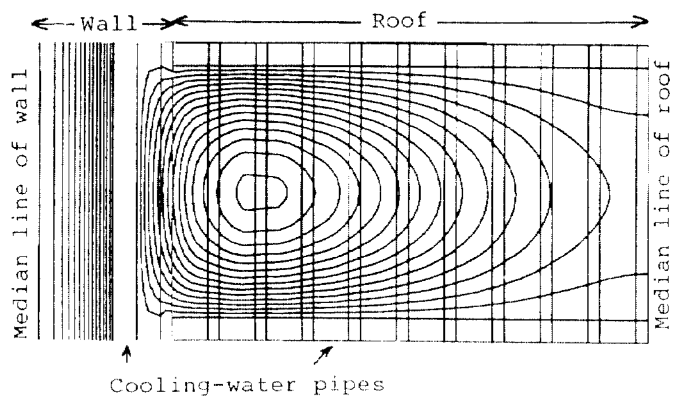

Fig. 1 Typical temperature distribution in a segment of a quarter of a rectangular beam-tube of total width 200 mm and total height 60 mm, equipped with transversal cooling-pipes at every 60 mm on roof and floor, and longitudinal pipes 10 mm from the corners of the walls (which are here shown folded up into the plane of the paper). Ramping rate $\dot{B} = 7$ T/s during 2/3 of the cycle, electrical conductivity $1.3 \cdot 10^6\ \Omega^{-1}\ m^{-1}$, thermal conductivity 13.5 W m^{-1} °C^{-1}. Isotherms are drawn for every degree Celsius.

References

[1] B. Hague, Electromagnetic Problems in Electrical Engineering, Oxford Univ. Press 1929.
[2] E.G. Phillips, Functions of a complex variable, Oliver & Boyd, Edinburgh 1951, p. 134
[3] C. Iselin, POISCR, CERN program library, 1984.
[4] G. Hemmie, priv. comm.
[5] G. Hemmie and J. Roßbach, DESY M-84-05, April 1984
[6] C.J. Herrlander et al., These Proceedings