RECENT PROGRESS OF PERMANENT MAGNETS

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Abstract

New types of permanent magnets are discussed in this paper. These magnets are composed of radially magnetized(multipole magnets) or vertically magnetized (linear periodic magnets) segments. The required magnetic field are produced by modulating the widths (MW type) or thicknesses (MT type) of the segments. The performance of these magnets is discussed.

1. Introduction

Multipole permanent magnets are widely used in electrical machines, accelerators, beam transport systems NMR CT and other places. Linear periodic magnets are widely used in photon factory and free electron laser. The commonly used permanent magnets are the modulating magnetization direction type(MMD)[1,2]. These magnets are composed of permanent magnet segments with various magnetization directions.

In the new permanent multipole magnets only radially (centrifugally and certripetally) magnetized magnet segments are used. By modulating the widths (MW type)[3,4,5] or modulating the thicknesses (MT type)[3,6] of the segments, the required magnetic field can be formed. And in the new linear periodic magnets (wiggler and undulator)[3,7] the magnet segments are all magnetized parallely with the required magnetic field which is formed by modulating the widths of the segments.

2. The structure of the new magnets

For multipole magnets in order to establish a 2N pole magnetic field the cylindrical magnet core is divided into 2N equal parts. Each part of the magnet core is divided into n equal sectors. The central angle of each sector is δ, and

$$\delta = \pi/Nn \quad (1)$$

In each sector, there is a permanent magnet segment. In the MW type the widths of the segments are modulated according to the angular coordinates of their central lines and the required magnetic field distribution. If the angular coordinate of the central line of a segment is $j\delta$, its width or its central angle δ_j is :

$$\delta_j = \delta\,|\mathrm{Sin}Nj\delta| . \quad (2)$$

And we call it the jth segment.

The magnetization directions of the segments are arranged as follows:

$$M_j = |M|, \text{if } \mathrm{Sin}Nj\delta \geqslant 0;\ M_j = -|M|, \text{if } \mathrm{Sin}Nj\delta \leqslant 0. \quad (3)$$

The inner and outer edges of the magnet core are circles with radius R_1 and R_2 respectively. Figure 1 is a sketch of the cross section of the magnet core with N=2, n=6. In the sketch, the arrows indicate the magnetization directions of the segments.

For MT type multipole magnets $\delta_j = \delta$, but the inner and outer edges of the magnet core are* :

$$R_1 = R_o - A\mathrm{Sin}N\theta ,\ R_2 = R_o . \quad (4)$$

A and the magnetization of the magnet segments M_r fullfil the following relations:

$$A = |A|,\ M_r = |M|, \text{ if } \mathrm{Sin}N\theta \geqslant 0; \quad (5a)$$

$$A = -|A|,\ M_r = -|M|, \text{ if } \mathrm{Sin}N\theta \leqslant 0. \quad (5b)$$

Where θ is the angular coordinate. Figure 2 is a sketch of the upper half of the cross section of the MT type magnet.

Figure 3 is a sketch of the cross section of the new type of linear periodic magnets. The symmetrical plane of the magnet gap is the xz-plane. The required magnetic field is along the y-axis. The magnet segments are periodically placed along the x-axis and the magnetic field periodically distributions along the x-axis also. The length of each period is λ. In each period , there are 2n equal sections and the length of each section is $\delta = \lambda/2n$. In each section a segment is placed at its center. The widths of the segments are modulated according to the x-coordinates of the central line of these segments. If the x-coordinate of the central line of a segment is $(2j-1)\delta/2$, its thickness is δ_j and δ_j is described by:

$$\delta_j = \delta\,|\mathrm{Cos}(2j-1)\,\pi/2n| . \quad (6)$$

* They can be $R_1 = R_o, R_2 = R_o + A\mathrm{Sin}N\theta$ or $R_1 = R_o - A\mathrm{Sin}N\theta$, $R_2 = R_o + A\mathrm{Sin}N\theta$ also.

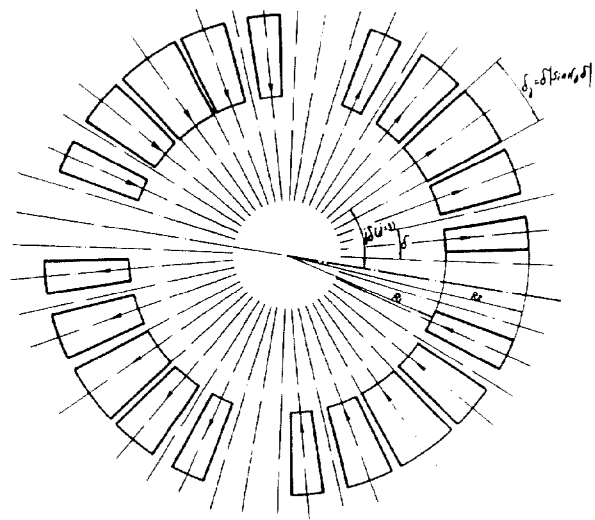

Figure 1. A sketch of the cross section of a MMD magnet

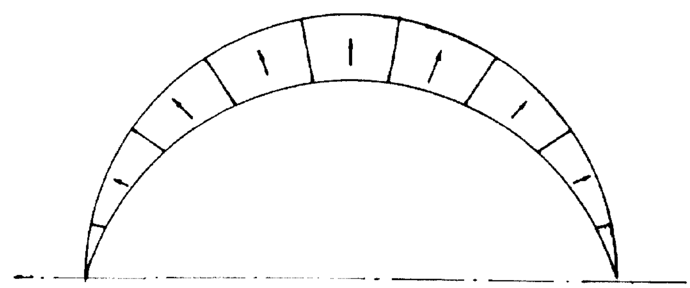

Figure 2. A sketch of the upper half of the cross section of a MT magnet

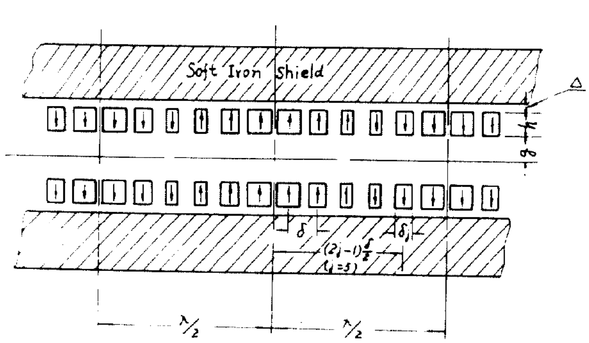

Figure 3. A sketch of a MW type linear periodic magnet with 2n=12

The segments are all magnetized in the y-direction. The magnetization directions of the segments are arranged as follows:

$$M_{y'j} = |M|, \text{ if } \cos(2j-1)\pi/2n \geqslant 0; \qquad (7a)$$

$$M_{y'j} = -|M|, \text{ if } \cos(2j-1)\pi/2n \leq 0. \qquad (7b)$$

In figure 3 only one whole period is shown. The arrows indicate the magnetization directions.

3. The scalar potential and the magnetic field

The scalar potential produced by multipole magnet is:

$$\phi(r,\theta,z) = \int M_{r'}\frac{\partial}{\partial r'}\left(\frac{1}{\rho}\right)dV' + M_{\theta'}\frac{1}{r'}\frac{\partial}{\partial\theta'}\left(\frac{1}{\rho}\right)dV' + M_{z'}\frac{\partial}{\partial z'}\left(\frac{1}{\rho}\right)dV' . \qquad (8)$$

Here $\rho^2 = r^2 + r'^2 - 2rr'\cos(\theta - \theta') + (z-z')^2$. (9)

The integral is taken over the whole volume of all the magnet segments. Generally in 2-dimensional ideal case for MW magnets:[4,5]

$$\phi(r,\theta) = -2\pi|M|r\ln\frac{R_2}{R_1}\sin\theta , \text{ for } N = 1; \qquad (10a)$$

$$\phi(r,\theta) = -2\pi|M|\frac{r^N}{1-N}\left(\frac{1}{R^{N-1}} - \frac{1}{R^{N-1}}\right)\sin N\theta, \quad \text{for } N \neq 1. \qquad (10b)$$

The above equations indicate that in the 2-dimensional ideal case, the MW magnets produce a pure 2N-pole magnetic field.

Putting equations (4,5) in equation (8) and neglecting high order terms of A/R_o, one gets the spatial expression of the potential produced by MT magnets:[6]

$$\phi(r,\theta) = -2\pi|MA|\left(\frac{r}{R_o}\right)^N \sin N\theta . \qquad (11)$$

This is a potential of pure 2N-pole magnetic field also

In analysing the linear periodic magnets we use the Cartesian coordinate system. Because of the segments are all magnetized along y-direction only, the potential at point P(x,y,z) is as follows:

$$\phi(x,y,z) = \int M_{y'}\frac{\partial}{\partial y'}\left(\frac{1}{\rho}\right)dV' . \qquad (12)$$

Here $\rho^2 = (x-x')^2 + (y-y')^2 + (z-z')^2$.

Generally the dimension of the magnet along the z-axis is much larger than the magnet gap. We can approximately evaluate the integration over the z-axis from ∞ to $-\infty$ at first and get:

$$\phi(x,y) = 2\int M_{y'}\frac{y-y'}{(x-x')^2+(y-y')^2}dx'dy' .$$

And H_y — the required magnetic field can be expressed as follows:

$$H_y = 2\int M_{y'}\left[\frac{y-h-g}{(x-x')^2+(y-h-g)^2} - \frac{y+h+g}{(x-x')^2+(y+h+g)^2} - \frac{y-g}{(x-x')^2+(y-g)^2} + \frac{y+g}{(x-x')^2+(y+h)^2}\right]dx' . \qquad (13)$$

Considering the structure of the magnet,we get:

$$H_{y'} = \sum_{q=-p}^{p}\ \sum_{j=-(n-1)}^{n}\int_{x'_-}^{x'_+} 2M_{y'j}\,I\,dx', \qquad (14)$$

where

$$x'_+ = (2j-1)\frac{\delta}{2} + \frac{1}{2}\delta_j , \qquad (15a)$$

$$x'_- = (2j-1)\frac{\delta}{2} - \frac{1}{2}\delta_j . \qquad (15b)$$

And

$$I = \frac{y-h-g}{(x-x'-q\lambda)^2+(y-h-g)^2} - \frac{y+h+g}{(x-x'-q\lambda)^2+(y+h+g)^2} - \frac{y-g}{(x-x'-q\lambda)^2+(y-g)^2} + \frac{y+g}{(x-x'-q\lambda)^2+(y+g)^2} . \qquad (16)$$

The totle number of periods of the magnet is 2p+1 and the y-axis is the symmetrical line of the magnet.

Considering equation(7), for case $n \gg 1$ H_y can, to a good approximation, be described by:

$$H_y = 2|M|\sum_{q=-p}^{p}\ \sum_{j=-(n-1)}^{n}\left[\frac{y-h-g}{(y-h-g)^2+(j\delta+q\lambda-x)^2} - \frac{y-g}{(y-g)^2+(j\delta+q\lambda-x)^2} + \frac{y+g}{(y+g)^2+(j\delta+q\lambda-x)^2} - \frac{y+h+g}{(y+h+g)^2+(j\delta+q\lambda-x)^2}\right].$$

Generally $p \gg 1$, we can to a good approximation, put $p=\infty$ and get the field expression in the 2-dimensional ideal case($n \gg 1, p \gg 1$).[5]

$$H_y = 4\pi|M|\cosh\frac{2\pi y}{\lambda}\, e^{-2\pi g/\lambda}\left(1-e^{-2\pi h/\lambda}\right)\cos\frac{2\pi x}{\lambda} . \qquad (17)$$

So in the 2-dimensional ideal case, the field produced by this new type magnet is a pure sinusoidal curve with period along the x-axis.

4. The field strength and quality

The soft iron shield can be used in both the new types of multipole magnets and MW linear periodic magnets. For the linear periodic magnets the y-coordinates of the ends of the images of the permanent magnet segments in the soft iron shield are $\pm[g+2s(\Delta+h+g)]$, $\pm[g+h+2s(\Delta+h+g)]$, $\pm[g+h+2\Delta+2t(\Delta+h+g)]$ and $[g+2h+2\Delta+2t(\Delta+h+g)]$ respectively. In the above expressions s and t are integers, s runs from one to infinity and t runs from zero to infinity. Considering the positions of the images from equation (12) we get the total field of the magnet with soft iron shield.

$$H_y = 4\pi|M|\cos\frac{2\pi x}{\lambda}\cosh\frac{2\pi y}{\lambda}\, e^{-2\pi g/\lambda}\left(1-e^{-2\pi h/\lambda}\right)\left[\frac{1+e^{-2\pi(h+2\Delta)/\lambda}}{1-e^{-4\pi(h+g+\Delta)/\lambda}}\right]. \qquad (18)$$

Comparing equations (17) and (18), the strengthening factor F of the soft iron shield is as follows:[7]

$$F = \frac{1+e^{-2\pi(h+2\Delta)/\lambda}}{1-e^{-4\pi(h+g+\Delta)/\lambda}} . \qquad (19)$$

Figure 4 shows the dependence of F on h/λ for $g/\lambda=0.05$, o.10 and 0.20. For smaller h/λ, F is greater than two.

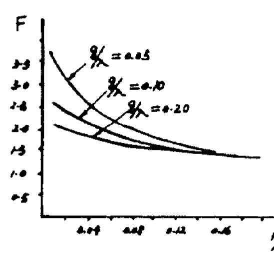

Figure 4. The dependence of F On h/λ

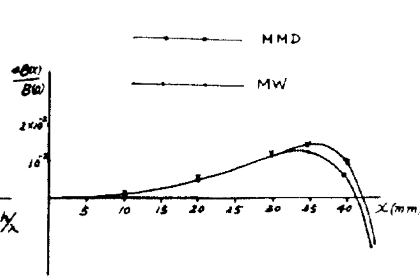

Figure 5. The field distributions of MMD and MW magnet

For the multipole magnets we have calculated the magnetic field of MMD and MW magnets with different aperatures. The datas are shown in table 1. The outer radius of MW magnets is selected so that the net weight of permanent magnet materials of both types with the same inner aperature are the same. From that table it is evident that, the field strength of MW type magnet is higher than MMD magnets.[8]

Table 1.

Type	MMD	MW	MMD	MW	MW	MMD	MW
R_1[cm]	3	3	5	5	5	15	15
R_2[cm]	5	5.87	7	7.9	7.9	25	29.3
B[KG]	4.24	4.45	2.79	3.22	1.32 without shield	4.26	4.42

For MT magnets, we have calculated the field distribution also. The datas are shown in figure 5. The

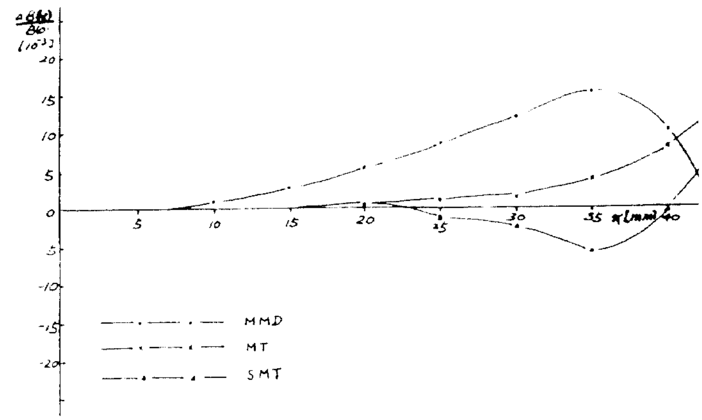

Figure 5. The field distribution of MMD,MT and SMT magnets

geometrical datas of MT magnet are $R_2=R_0=7$cm, and A=2cm so, the vertical half aperature is 5cm. For comparision, the datas of MMD magnet with half aperature of 5cm are shown in figure 5 also. In order to simply the technology, we have studied the simplified MT magnet.[9] The SMT magnets are composed of fan shaped segments, whose outer and inner edges are arcs with radius R_2 and R_{1i} respectively. R_2equals 7cm and R_{1i} are modulated. For one quadrent of the magnet, R_{1i} are 6.6cm, 6.2cm, 5.38cm 5cm and 5cm. Figure 6 is a sketch of the upper half of the cross section of the SMT magnet and the datas of this magnet are shown in figure 5 also. The ratios of

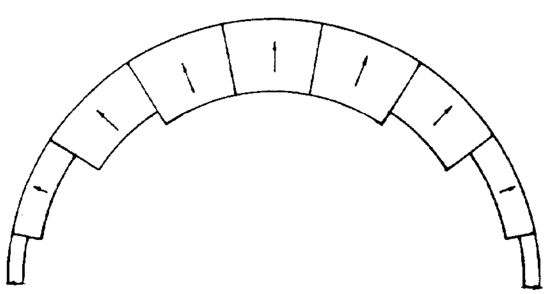

Figure 6. A sketch of the cross section of a SMT magnet

of the field strength and the weight of the required permanent magnet material of SMT magnet to those of MMD magnet are shown in table 2.

Table 2.

Type	MT	SMT
$[B(0)/B(0)_{MMD}]$	0.891	0.917
$[Weight/Weight_{MMD}]$	0.704	0.723

The above results show that, the field distributions of MW,MT and SMT magnets are about the same or even better than that of MMD magnet and the required quantity of permanent magnet material for producing the same field strength is smaller for MW,MT and SMT magnet than that for MMD magnet with the same vertical aperature.

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