The Analytical Lattice Approach for the Ring Design of BESSY II

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Introduction

BESSY II is proposed as a low emittance electron storage ring for the production of high brilliance photon beams /1/. The nominal particle energy is 1.5 GeV. The storage ring lattice presents a tenfold symmetry. Special features of the unit cell are the low natural emittance of about $5 \cdot 10^{-9}$ rad·m, the long, dispersion free straight section of 5 m and the achromatic bend consisting of 3 gradient free dipoles of equal length.

To support the design of the machine lattice we have developed an analytical lattice approach. This tool gives us a fast survey of all possible lattice solutions, which satisfies the required lattice properties. This approach is done in the kick code approximation and considers only the linear lattice behavior. It is supposed to optimize the linear lattice structure as a starting point for further detailed work on the lattice.

A general scheme of the unit cell is given in figure 1. There are two symmetry points for each plane of the beta functions, in the center of the straight section and in the center of the inner dipole. Between the straight section and the outer dipole we have a quadrupole triplet, between the outer dipole and the inner dipole we have a quadrupole doublet.

For the analytical approach we subdivide the lattice in parts and consider each section separately. In this way we need to optimize only two parts, the achromat, which is finally reduced to a single doublet, and the triplet. For each part we distinguish two sets of Twiss parameters. The first type of these parameters are very important to define the required lattice properties. The values of these parameters at the boundary of the lattice sections are discussed in the next two chapters. For each kind of linear lattice solutions these values have to be kept fixed.

The second type of Twiss parameters are less constrained: they are not so strongly involved in defining the desired lattice properties. By changing now the last type of Twiss parameters we will generate different lattice solutions, but in any case with the desired lattice properties.

By applying the method of analytical betatron matching /2/ we are able to calculate positions and strengths of the quadrupoles for a given set of Twiss parameters at the boundaries. The calculation is done in a direct way, without any iteration procedure. Therefore the computer code is very fast and we are able to consider all possible lattice solutions within the kick code approximation.

The Achromat

We define the section of the achromat from center to center of the two outer dipoles. The task of this section is an achromatic bending and an adjustment of the natural emittance to about $5 \cdot 10^{-9}$ m·rad. This implies a special shaping of the horizontal beta function, which is discussed in detail elsewhere /3/. The results are summarized here.

Because of the symmetric lattice structure, the analytic approach considers only the section from the center of the outer dipole to the center of the inner dipole. We only need to design the doublet in between. The doublet is defined by the values of the beta function at the beginning and at the end of this section.

To obtain the achromatic bend, the value of the horizontal beta function at the beginning (b1) and at the end (b2) of this section has to satisfy the relation

$$2\cos\phi = -\sqrt{b2/b1} \ ,$$

where ϕ is the horizontal phase advance across this lattice part. This equation is valid for dipoles of equal length.

The calculation of the minimum natural emittance in the triple bend achromat (TBA) differs slightly from the single dipole calculation, as done by /4/. The emittance like function H

$$H = \gamma \eta^2 + 2\alpha \eta \eta' + \beta \eta'^2$$

is conserved between the dipoles and therefore leads to a coupling between the dipoles. A minimization of the natural emittance in the TBA is dependent on this coupling and gives the following conditions for the horizontal Twiss parameters. We present results of a double series expansion (obtained by the algebraic computer code MACSYMA) with respect to the bending angle ℓ and with respect to the phase advance $d\phi = \phi - 180$.

The minimum of the beta function has to be localized in the first dipole at

$$s = (\ell/2)\cdot[1-1/(10-4\cos^2\phi) + \ldots],$$

where the dipole ranges from s = 0 to s = ℓ. The best choice for the minimum of the beta function at this point is given as

$$b_o = \ell \cdot \sqrt{170}[1/120 - d\phi^2/2448 + \ldots].$$

With this result the expression $\langle H \rangle/\rho$ takes the minimum value

$$\langle H \rangle/\rho = (\ell/\rho)^3 [\sqrt{170}/180 - d\phi/36 + 7\sqrt{170}d\phi^2/2040 + \ldots].$$

The expression $\langle H \rangle / \rho = k\,(\ell/\rho)^3$ is proportional to the natural emittance ε_n for a zero gradient dipole.

$$\varepsilon_n = k\,(\ell/\rho)^3\,\gamma^2 \cdot 3.823\cdot 10^{-13}\ [\mathrm{m\cdot rad}]\ .$$

Finally, k takes the minimized value

$$k = 0.0675$$

for the triple bend achromat with zero gradient. Compared to a symmetric two dipole achromat arrangement ("Chasman-Green") our k value is only 5% larger /4/. Therefore, the same limit in the natural emittance is achievable by a TBA or by a "Chasman-Green" achromat if we distribute the same number of dipoles around the ring. For a fixed natural emittance the TBA results in a more compact machine than a "Chasman-Green" achromat solution.

However, it is not reasonable to build a machine with these optimized numbers. The strong focussing necessary to get such a small beta function would produce too many chromatic effects. Therefore we slightly increase the beta function value b_0 to 20 cm instead of 8 cm and adjust the phase advance to 160 degrees to get a natural emittance which is about 2 to 3 times the minimum value /5/. Also, we displace the minimum of the beta function by few degrees from its optimized location to meet the requirements of the 160 degrees phase advance in a more relaxed way.

The study of the natural emittance in the TBA gives us a clear picture of the constraints on the horizontal Twiss parameters. In the vertical plane there is no such restriction. We fix the vertical beta function to a value of 10 m in the symmetry point in order to reduce vertically the angular divergence of the synchrotron radiation emitted in the central dipole.

The task of the doublet is now well defined at the beginning and the end of this lattice section. In the analytical calculation of the doublet we are now able to produce directly solutions which are both achromatic and of a low natural emittance. There is still some freedom for the beta functions, mainly in the vertical plane. Changing the values at the boundaries or the phase advance will result in different beta functions between the dipoles. Here we try to find optimal solutions for placing the sextupoles.

The Triplet

The lattice section which we define as the triplet starts at the middle of the straight section and ends at the center of the first dipole. The full length of the straight section is 5 m supplying ample space for placing the insertion devices. The value of the beta function at the beginning of this part is set to 2.5 m for the vertical plane and to 10 m for the horizontal plane, in order to minimize interactions between the wiggler and the beam.

The section starts with a symmetry point, the derivatives of the beta functions are zero in both planes. At the end of this section the Twiss parameters are defined by the starting values of the achromat section. Both solutions are combined at this point. In the triplet section the phase advance in both planes is free of choice and it is used to adjust the working point.

In our analytical computer code the triplet is treated as a doublet plus one quadrupole. The single quadrupole has a fixed position at the end of the straight section but its field strength is variable. Again, the main task of the analytical approach is the calculation of a doublet.

The Matching of the Beta Function

A part of the Twiss parameters (β, α, ϕ) at the boundaries of our sections are well defined by the lattice requirements. We are now interested to find all solutions of the placements and the field strengths of the magnets which match the values of the beta functions. Changing those beta functions which are unconstrained will produce different solutions.

The analytical tool was developed for a quadrupole doublet in the thin lens approximation /2/. In this approximation the doublet matrix is written as

$$\begin{pmatrix} 1 & \ell_3 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ d_2 & 1 \end{pmatrix} \begin{pmatrix} 1 & \ell_2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ d_1 & 1 \end{pmatrix} \begin{pmatrix} 1 & \ell_1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}$$

where the ℓ_i are drift lengths and the d_i are kick strengths.

The matrix m_{ij} can be expressed in terms of the Twiss parameters

$$m_{11} = \sqrt{\beta_2/\beta_1}\ (\cos\Delta\phi + \alpha_1 \sin\Delta\phi)$$

$$m_{12} = \sqrt{\beta_1\beta_2}\ \sin\Delta\phi$$

$$m_{21} = -\left[(1 + \alpha_1\alpha_2)\ \sin\Delta\phi + (\alpha_2 - \alpha_1)\ \cos\Delta\phi\right]/\sqrt{\beta_1\beta_2}$$

$$m_{22} = \sqrt{\beta_1/\beta_2}\ (\cos\Delta\phi - \alpha_2 \sin\Delta\phi)$$

(if we apply this to a specific plane we add an index v or h). Additionally we introduce a further abbreviation $m_{ijh} + m_{ijv} = 2\,a_{ij}$, $m_{ijh} - m_{ijv} = 2\,s_{ij}$. The three lengths characterizing the doublet can be computed in terms of the Twiss parameters by

$$\ell_1 = (a_{22} - 1)/a_{21}$$

$$\ell_3 = (a_{11} - 1)/a_{21}$$

$$\ell_1 + \ell_2 + \ell_3 = (s_{11} + s_{22})/s_{21}$$

and for the two kick strengths we find

$$d_1 = (s_{11} - \ell_3 s_{21})/\ell_2$$

$$d_2 = (s_{22} - \ell_1 s_{21})/\ell_2 .$$

Additionally, the choice of the Twiss parameters is not completely free. They have to satisfy the boundary condition

$$s_{12}a_{21}a_{21} + s_{21}(a_{11} - 1)(a_{22} - 1) = s_{22}a_{21}(a_{11} - 1) + s_{11}a_{21}(a_{22} - 1)\ .$$

This set of basic equations can be further manipulated to keep some more appropriate parameters fixed, for example the overall length of the doublet. In exchange we have to give up the free choice of one of the Twiss parameters.

A lattice solution is discarded by a simple IF statement in the analytic computer code, if it is not of interest. For example, if the distance between two magnetic elements becomes too small, or if the beta function along the lattice section becomes too large, the solution will be skipped. Additionally, some more information can be calculated by the program, for example the chromatic effects of a lattice section. Finally, we come up with two programs, one for the achromatic section and one for the triplet. Effects of the dipoles on the beta function are included.

With this analytic code we have a very fast tool for a parameter scan of the defined achromat and triplet solutions. We now can concentrate on other lattice properties, as sextupole positions, chromatic effects, maxima of beta functions, space for diagnostics and so on. For these last judgements a simple graphic display of the calculated doublet and triplet and its beta functions is very helpful.

To transfer these solutions to standard lattice codes a thick lens optic is required. This is done by a fitting procedure which fits a thick lens optic to our boundary conditions and uses the results of the analytic parameter survey as starting values.

Nonlinear aspects

The linear lattice solution needs to be corrected for chromatic effects. This is done by inserting sextupoles into the achromat section. The sextupoles will reduce the stable, transverse acceptance of the machine, the dynamic aperture. One of the main aspects of a good lattice solution is a large size of the dynamic aperture. The dynamic aperture is normally checked by a tracking routine. This is a rather time consuming procedure.

As a fast tool to distinguish between more or less promising lattice solutions with respect to the dynamic aperture, we apply the method of the distortion functions as proposed by Collins /6/. This analytical approximation calculates the tune shift as a function of the square of the betatron oscillation amplitude. Only if this dependance is weak a good dynamic aperture is feasible. Those lattice solutions need a further careful check by a tracking routine.

The formulas given by Collins are applied to our sextupole arrangement and combined with our inverted kick code program to give a fast, relative comparison of the amplitude dependent tune shift.

Conclusions

The analytical approach proposed here is tailored to a special light source lattice, but a variation of this method can also be applied to study other lattice configurations. This approach provides a very fast, first parameter survey of possible lattice arrangements. Additionally, for the given boundary conditions it is possible to get a complete scan of all possible lattice solutions. Each result calculated in this way satisfies some of the most important lattice goals, and one can concentrate on other aspects of the solution. A systematic characterization of the most important features of the linear TBA lattice is given in /5/.

References

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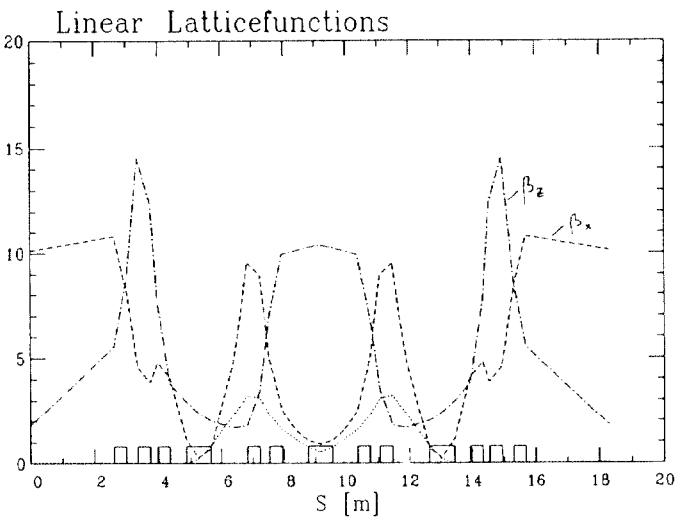

Figure 1: Scheme of the BESSY II unit cell.