DISCRETE HELICAL SPIN ROTATORS

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Abstract

A family of multi-twist transverse-field spin rotators using tilted bending magnets is described that is useful for Siberian snakes as well as other spin rotators. The device has less orbit excursions than other designs found in the literature and, in applications in electron rings, exhibits less radiative depolarization for a given length of the spin rotator. A 3-twist snake and a 1-twist 90° rotator are presented, the first for use in the proposed TRIUMF KAON factory and the second for possible use in an electron storage ring to achieve longitudinal polarization at the interaction point. Orbit errors and their correction are discussed.

Transverse field spin rotators

Motivation for this work arose from the desire to be able to use Siberian snakes in proton accelerators at kinetic energies as low as 3 GeV, the injection energy of the 30 GeV Driver synchrotron of the proposed TRIUMF KAON factory. Since solenoid rotators need too high a field integral at 30 GeV, transverse-field rotators are to be used, giving spin precession angles larger by a factor $\gamma G/(1+G)$ for the same field integral (G is the gyromagnetic anomaly, 1.7928 for protons). These are problematic at low injection energies, however, due to the large orbit excursions necessitating correspondingly large magnet apertures. Keeping the orbit excursions small is therefore a strong concern in the design of Siberian snakes.

So far the most effective way of achieving small orbit excursions is the multi-twist helical snake proposed by E.D. Courant.[1] A transverse helical field is used to rotate the spin. Courant showed that in such a magnet the orbit excursions decrease roughly with $1/n$, while the length of the device increases only with $\sqrt{n}$ (n is the number of twists).

From a constructional point of view a helical magnet with several twists, 3 T field, and between 5 and 10 m long appears to be not a simple device. Also, once built the correction of field errors invariably present seems difficult. Finally, in order to create a straight-through device the orbit restoration necessary at both ends implies that the beam passes through the helix at an angle other than 90° to the field, creating longitudinal field components that will disturb the spin motion and also give rise to additional twisting of the planes.

We therefore analyzed an approximation to the helix using a number of discrete bending magnets. If successful, these magnets can be positioned along the orbit such that longitudinal field components are minimized. Also, the magnets can be aligned and adjusted in place in order to correct any orbit errors. The helix is approximated in much the same way as a circle is approximated by a polygon.

To analyze a discrete-magnet helix, we replace the helix with, say, M twists by a series of N magnets tilted about the longitudinal axis by an equal angle against each other such that exactly M periods are created. The effect on the spin of this array can be described by its spinor transfer matrix:

$$M_{\text{helix}} = \prod_{n=1}^{N} e^{i\pi\frac{M(2n+1)}{2N}\sigma_y}\, e^{i\frac{\alpha}{2}\sigma_z}\, e^{-i\pi\frac{M(2n+1)}{2N}\sigma_y}\,, \tag{1}$$

where α is the spin precession angle of each magnet, and $\pi M(2n+1)/2N$ is the tilt angle of each magnet. The index n counts the magnets in the helix.

This product can be simplified by combining the rotations of successive elements about the longitudinal axis to yield

$$M_{\text{helix}} = e^{i\pi\frac{M}{2N}\sigma_y}\left(e^{i\frac{\alpha}{2}\sigma_z}\, e^{i\pi\frac{M}{N}\sigma_y}\right)^N e^{-i\pi\frac{M}{2N}\sigma_y}\,. \tag{2}$$

If the expression in braces represents a rotation by an angle of π/N, M_{helix} is traceless and represents a snake, since we can always rotate the whole array about the longitudinal axis to move the axis of precession into the horizontal plane. This gives and expression for the angle α:

$$\alpha = 2\arccos\left(\frac{\cos\pi\left(\frac{M}{N}+\frac{1}{2}\right)}{\cos\pi\frac{M}{N}}\right)\,. \tag{3}$$

For α to be real, the number of magnets has to be more than twice the number of twists.

The tilt angle of the first magnet has to be

$$\theta_1 = \pi\left(\frac{M}{N}+\frac{1}{2}\right) \tag{4}$$

for the axis of rotation to be in the horizontal plane, and the angle between the axis of rotation and the longitudinal axis is

$$\theta_{\text{ax}} = \arccos\left(\frac{\tan\pi\frac{M}{N}}{\tan\pi\left(\frac{M}{N}+\frac{1}{2}\right)}\right)\;; \tag{5}$$

this angle, however, will be changed when the necessary orbit restoration is included.

The field integral needed for the helix is given in the limit of $\beta = v/c = 1$ to be

$$BL = N\alpha\frac{m_p}{0.3G}\,. \tag{6}$$

The simplest device possible is a one-twist rotator with three magnets, each of them with a spin rotation of π. This is the same device proposed some time ago by Derbenev and Kondratenko,[2] which has thus been identified as the first member of the helical-snake family. Unfortunately orbit excursion and field integral needed are too large to make this device attractive for all except the highest energies.

The above equations can be graphed in a diagram plotting the number of twists versus the number of magnets, with BL as a parameter (Fig. 1). As is evident, the field integral needed for a given number of twists decreases with the number of magnets used as the approximation of the helix gets smoother, but the decrease levels off to asymptotically approach the value for the continuous helix for large N. On the other hand, if a certain number of magnets is used

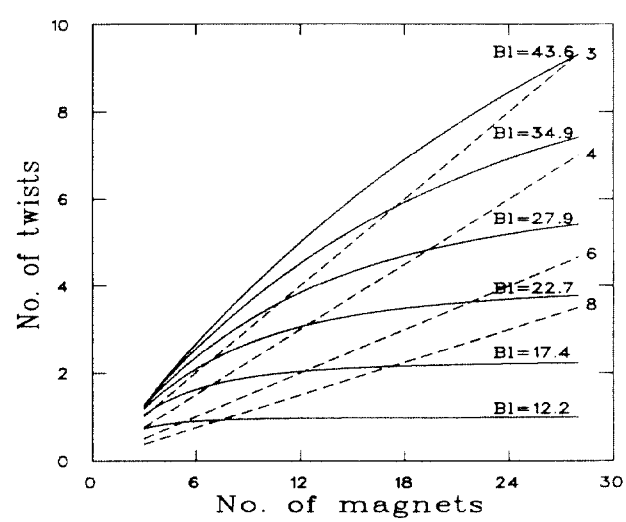

Fig. 1. Number of magnets vs. number of twists for discrete magnet helices, for different values of the field integral BL (Tm). The dashed lines represent helices with a constant number of magnets per twist as labelled.

the field integral rises with the number of twists, at first moderately but more steeply as the approximation of the twists becomes more crude.

In order to find a snake suitable for the Driver ring, we evaluate some helices in more detail. If we have 12 magnets for the helix, 3 T field each, we can construct helices with the number of twists ranging from 1 to 5 with these magnets. In Table I the properties of these helices are listed, neglecting for now the space needed between individual magnets, and also neglecting the orbit restorers. As can be taken from col. 4, the 3- and the 4-twist helix with 4 and 3 magnets per twist, respectively, have the smallest orbit excursions. The 3-twist "rectangular" helix requires about 25% less field integral, however, making it more economical.

The 3-twist "rectangular" helix requires a field integral of 23 Tm. Given this value, we can ask whether or not there are designs with even less orbit excursions but about the same field length. In Table II the snakes with about the same field length are listed. The 3-twist "rectangular" helix is clearly preferable to the 2-twist "triangular" helix, while the 4-twist snake requires such a large number of magnets that it constitutes for all practical purposes a continuous helical magnet. The 3-twist helix will therefore be considered in the rest of the paper. It has the additional advantage of having two orthogonal planes, albeit tilted by 45° against the horizontal/vertical planes, thus making it easier to operate than the other alternatives.

Table I. 180° helices with 12 magnets, 3 T field each.

twists	Magnets / twist	α (°)	Vertical orbit (cm)	Length (m)	Orbit shape
1	12	33.9	18.9	4.1	dodecagonal
2	6	47.3	9.1	5.8	hexagonal
3	4	61.2	6.2	7.4	rectangular
4	3	80.1	5.7	9.8	triangular
5	2.4	119.4	7.0	14.6	subtriangular

No spaces between magnets are considered. Vertical orbit excursion is calculated for 3 GeV protons. Two additional orbit restorers are needed for a snake.

Table II. 180° helices with a field integral of about 23 Tm.

twists	Magnets	α (°)	Vertical orbit (cm)	Length (m)	Orbit shape
2	6	117.7	12.1	7.2	triangular
3	12	61.2	6.2	7.4	rectangular
4	86	8.7	4.7	7.5	21.5 Magnets/twist

No spaces between magnets are considered. Vertical orbit excursion is calculated for 3 GeV protons and 3 T magnets. Two additional restorers are needed for a snake.

In order to create a straight-through device with no orbit offset, as needed for operation of the snake during acceleration, orbit-restoring magnets have to be added to the helix. The simplest of such restoration schemes consists of two horizontally bending orbit-restoring magnets, one before and one after the helix, each of the same but opposite strength. This orbit correction scheme is not optimal, since it displaces the orbit asymmetrically. A more elaborate system such as given by Courant creates orbit excursions symmetrically about the axis, thus halving the displacement in each magnet. For simplification we do not analyze such a scheme in this paper, although it would certainly be used in an actual device.

Besides straightening the orbit in the horizontal plane, these magnets also rotate the axis of spin rotation of the snake, to coincide with the longitudinal axis. The device therefore constitutes a snake of the first kind. Unfortunately there appears to be no way of constructing a multiturn snake with a precession axis significantly different from the longitudinal axis.[3,4]

The helix together with the orbit restorers still produces some orbit deviation in the vertical plane due to higher-order effects. These can be corrected by small vertical orbit correctors at each end, of the same strength and polarity. Since their axis of rotation is orthogonal to the spin rotation axis of the snake array, their spin rotations cancel and the total spin rotation will remain 180°.

Using this scheme we modelled a three-twist rectangular snake using a spin and orbit tracking program. Assuming 3 T magnets, each of the magnets that make up the helix is 0.62 m long with 13 cm aperture, and 15 cm of space is left between the magnets. The aperture is filled at injection by 8 cm of orbit excursion and a full-intensity beam size of 5 cm. Since the polarized beam will have typically about 1/6 of the emittance of the full-intensity beam, a margin of 3.6 cm is contained in the beam size. Together with the orbit restoration the full length of the snake is 10.71 m. Figure 2 shows the array of magnets.

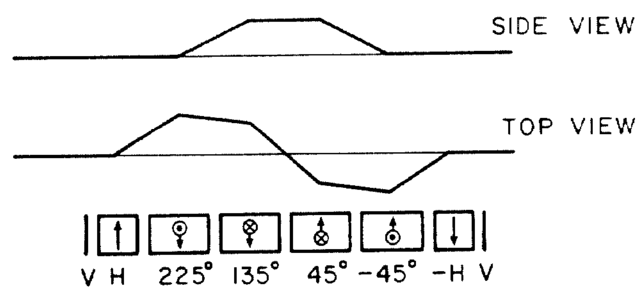

Fig. 2. Rectangular helical snake. Only 1 first out of three is shown. The angles indicate the tilt of the magnets about the axis.

The parameters of the helical snake can be compared with those of a snake of 1. kind designed by K. Steffen.[5] This snake, probably the best known design, needs a total field integral of 19 Tm, and has, at 3 T field, a length of 11.3 m since there have to be a few straight pieces in order to allow for a straight-through device. The orbit excursions are 15 cm at 3 GeV. The Steffen snake needs 10 magnets, 6 short ones and 4 long ones, compared to 12 magnets plus two shorter orbit restorers for the 3-twist rectangular helical snake. Already with the simple orbit restoration scheme used here, the helical snake outperforms the Steffen snake, while a more elaborate orbit correction would reduce the orbit excursions by another factor of 2, to about 4 cm.

A snake of this type was modelled and included in a straight section of the new race-track lattice for the Driver ring[6] using the program DIMAD. The quadrupole at the symmetry point of the straight section is replaced by the snake array; a quadrupole doublet at each end provides matching to the lattice. The beta functions (shown in Fig. 3) have been kept as small as reasonable in order to have a small beam size and also to minimize the effect of edge focusing and field errors in the snake magnets.

90° Spin Rotator

Since the helical rotator has obvious advantages for Siberian snakes we investigated its usefulness for 90° spin rotators. Of particular importance is the application in electron storage rings in order to achieve longitudinal polarization at the interaction point.

For reasons of symmetry, a helix with M twists rotating the spin by 90° is equal to one-half of a 180° helix with $2M$ twists, and the axis of rotation is in the horizontal plane if Eq. (4) is obeyed. One additional horizontal bending magnet is needed to rotate the rotation

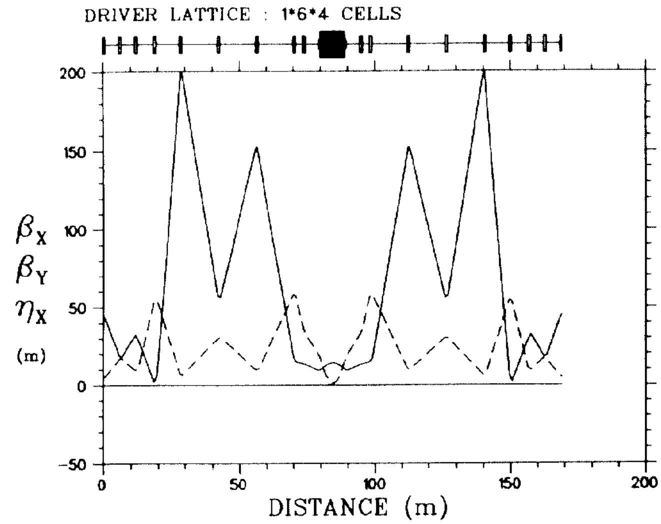

Fig. 3. Lattice functions of the straight section of the TRIUMF KAON Factory Driver, with 3-twist rectangular helical snake.

axis into the radial direction. Since the rotator has to rotate by exactly 90° only for one energy, there is no need for orbit restoration to form an overall straight-through device; the rotator can replace some of the bending magnets.

In electron rings there is, however, a new restraint due to radiative depolarization in the rotator magnets, where the spin is not parallel to the field. This depolarization is given by[7]

$$d = \sum_{n=1}^{N} \frac{\rho_0^2}{\rho_n^2} \frac{\alpha_n}{2\pi} \left(\vec{s}\cdot\vec{b} - 1\right) , \tag{7}$$

where ρ_0 is the bending radius of the ring magnets, ρ_n is the bending radius of the rotator magnets, and $\vec{s}\cdot\vec{b}$ represents the cosine of the angle enclosed by the polarization vector and the field.

The depolarization d scales with the squared inverse of the length, $1/L^2$, of the rotator and therefore the quantity L^2d is a constant of the design and constitutes a figure of (de)merit, that one wants to minimize. L^2d determines the field strength and length of the rotator if a certain depolarization is not to be exceeded.

In Table III, the parameters of a 1-twist rotator suitable for insertion into the PEP ring have been summarized for 3-8 magnets in the helix. Since the L^2d value decreases quite rapidly with the number of magnets, the fields allowed become larger and the whole array shortens for constant depolarization (5% in Table III). There appears to be quite noticeable gain in increasing the number of magnets, at least up to eight. But already the rectangular rotator is superior to the HERA "mini" rotator,[8] one of the more common designs. Figure 4 shows a possible layout of the rotator, together with the part of the PEP IR it replaces.

Table III. 1-twist 90° helical rotators for electron rings.

Magnets helix	α (°)	L^2d (m^2)	$L(d{=}5\%)$ (m)	B (kG)	Orbit shape
3	117.7	25.4	22.5	4.3	triangular
4	76.4	16.4	18.3	4.8	rectangular
6	47.3	10.23	14.3	5.9	hexagonal
8	34.7	6.48	11.4	7.3	octogonal

No spaces between magnets are considered. L^2d is calculated for $\rho_0 = 165.5$ m and 14.4 GeV electrons. One additional bending magnet is needed for longitudinal polarization.

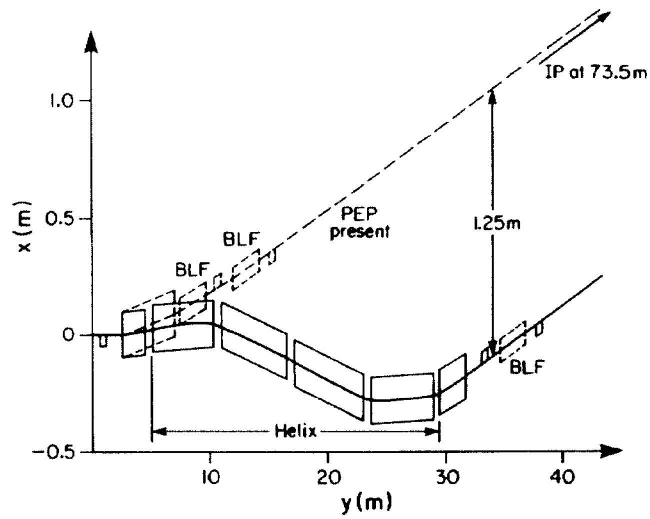

Fig. 4. Geometry for a 90° spin rotator for the PEP electron storage ring. A field of 0.38 T was used for the rotator magnets. BLF are low-field bending magnets for synchrotron radiation shielding.

References

[1] E.D. Courant, Report EHF-87-9, European Hadron Facility, March 1987.

[2] Y.S. Derbenev and K.M. Kondratenko, Preprint 82-28, Inst. of Nuclear Physics, Novoribisk, 1982.

[3] K.F. Steffen, DESY-HERA 87-11, DESY, Hamburg, April 1987.

[4] S.R. Mane, UM-HE-87-10, University of Michigan, Ann Arbor, MI 48109, 1987.

[5] K. Steffen, DESY 83-058 and 83-124, DESY, Hamburg, 1983.

[6] R.V. Servranckx, TRIUMF internal report, TRI-DN-88-3 (1988).

[7] B.W. Montague, Phys. Rep. **113**, 1 (1984).

[8] J. Buon and K. Steffen, Nucl. Instrum. Methods **A245**, 248 (1986).