TRANSVERSE TWO-BEAM INSTABILITY AT LEP

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Abstract

The problem of the transverse instability of point-like bunches for two counter-rotating beams in a storage ring is formulated and solved within a linear approximation. Only localized cavities are included as source of the coupling impedance. The difference in the coupling impedances of co-moving and counter-moving bunches is pointed out. A secular equation for the transverse rigid bunch dipole instability is presented and is applied to phase I of LEP.

Introduction

Pellegrini and Renieri [1] investigated the problem of the longitudinal coupled rigid bunch instability for two counter-rotating beams in 1974, and their results were recently applied by Pellegrini [2] to LEP. In this note we shall do similar calculations for the corresponding transverse instability. As in the references above, we include only the localized cavities as the source of the coupling impedance [3].

Cavity Resonances

The transverse dipole impedance $Z(\omega)$ of a standing wave cavity coupling the co-moving bunches can be expressed as the sum of the cavity resonance contributions,

$$Z(\omega) = \sum_{\lambda}\rho_{\lambda}\frac{\omega_{\lambda}}{\omega}\frac{1}{1-iq_{\lambda}(\omega/\omega_{\lambda}-\omega_{\lambda}/\omega)},$$
$$= i\sum_{\lambda}\rho_{\lambda}\Gamma_{\lambda}\left[\frac{1}{\omega-\omega_{\lambda}+i\Gamma_{\lambda}}-\frac{1}{\omega+\omega_{\lambda}+i\Gamma_{\lambda}}\right], \quad (1)$$

where λ is the resonance mode number, ω_{λ} and q_{λ} are, respectively, angular resonance frequency and the quality factor of the mode, $\Gamma_{\lambda}=\omega_{\lambda}/2q_{\lambda}$ is the mode damping factor, and ρ_{λ} is the shunt impedance. Causality implies [4] that $\rho_{\lambda}\geq 0$.

The transverse impedance $\overline{Z}(\omega)$ which couples the counter-rotating bunches can similarly be written as

$$\overline{Z}(\omega)= i\sum_{\lambda}\Gamma_{\lambda}\left[\frac{\overline{\rho}_{\lambda}}{\omega-\omega_{\lambda}+i\Gamma_{\lambda}}-\frac{\overline{\rho}_{\lambda}^{*}}{\omega+\omega_{\lambda}+i\Gamma_{\lambda}}\right]. \quad (2)$$

The shunt impedance $\overline{\rho}_{\lambda}$ for the counter-moving beams are generally complex [5]. However, since LEP cavity is symmetric longitudinally about the center of the cavity, $\overline{\rho}_{\lambda}=\pm\rho_{\lambda}$. The sign is determined by the symmetry property of the EM fields corresponding to the resonance mode.

Coherent Modes

We study the coherent transverse rigid dipole motion of counter-rotating electron (e^-) and positron (e^+) beams in the storage ring, each beam composed of B equally spaced identical point-like bunches. The azimuthal position of the k-th bunch, $k=0,1,\ldots,B-1$, is given by $\theta=-(\omega_0 t-2\pi k/B)$ for the e^+ beam and by $\theta=\omega_0 t-2\pi k/B$ for the e^- beam.

We assume that the only source of the transverse beam impedance is the standing wave cavities and that the change of the transverse position of a beam particle during a transit of a cavity is small compared to the betatron wavelength so that the cavity can be treated as a thin lens. The position of the cavities are denoted by $\theta=\Theta^j$, $j=1,2,\ldots,N_c$, where N_c is the total number of cavities around the ring. We also approximate the betatron amplitude function by a constant, $\beta=R/Q_0$, where R is the ring radius and Q_0 is the unperturbed betatron tune.

A coherent mode of the two-beam instability is characterized by the cyclic mode number S, $S=0,1,\ldots B-1$, and for each value of S, there are two coherent modes with the coherent tunes Q given by the following dispersion relations:

$$Q-Q_0=-i\frac{I_{av}}{\omega_0}\chi\left[\Xi_0\pm\sqrt{\Xi_+\Xi_-}\right], \quad (3)$$

where I_{av} is the average current per beam, $\chi=ec/(4\pi E_0Q_0)$, E_0 is the beam enegy, and the effective impedances Ξ's are given by

$$\Xi_{0,+,-}=\sum_{j=1}^{N_c}\Xi^j_{0,+,-},$$

with

$$\Xi^j_0=\sum_{n=-\infty}^{\infty}Z^j_{nB+S};\quad \Xi^j_{\pm}=\sum\overline{Z}^j_{nB+S}\,exp\,[\pm i2\Theta^j(nB+S)],$$
$$Z^j_n\equiv Z^j[(n-Q_0)\omega_0];\quad \overline{Z}^j_n\equiv\overline{Z}^j[(n-Q_0)\omega_0].$$

We shall refer to the first term on RHS of (3) as the co-moving contribution to the coherent tune shift and the second term, the counter-moving contribution.

From Eq.(3), the total growth rate per unit average current, α_{tot}, can also be split into co-moving and counter-moving parts, $\alpha_{tot}=\alpha\pm\overline{\alpha}$, with

$$\alpha=\chi\ \mathbf{Real}(\Xi_0);\quad \overline{\alpha}=\chi\left|\mathbf{Real}\left[\sqrt{\Xi_+\Xi_-}\right]\right| \quad (4)$$

It is convenient to scale the cavity resonance parameters in terms of the bunch frequency : $\psi_{\lambda}\equiv\omega_{\lambda}T_B$ and $\gamma_{\lambda}\equiv\Gamma_{\lambda}T_B$, where $T_B\equiv 2\pi/\omega_0 B$ is the bunch period. Also, we set $Q_S\equiv 2\pi(Q_0-S)/B$. In terms of these variables, the effective impedances become

$$\Xi^j_0=-\frac{i}{2}\sum_{\lambda}\rho^j_{\lambda}\gamma^j_{\lambda}\,[cot\frac{1}{2}(\psi^j_{\lambda}+Q_S-i\gamma^j_{\lambda})-(\psi^j_{\lambda}\rightarrow-\psi^j_{\lambda})\,], \quad (5)$$

$$\Xi^j_{\pm}=-\frac{i}{2}\sum_{\lambda}\overline{\rho}^j_{\lambda}\gamma^j_{\lambda}\,\Phi_{\lambda\pm}(\psi^j_{\lambda})\,[\Upsilon_{\pm}(\psi^j_{\lambda})-(\psi^j_{\lambda}\rightarrow-\psi^j_{\lambda})\,], \quad (6)$$

where

$$\Phi_{\pm}(\psi^j)=exp\,(\pm i2Q_0\Theta^j\pm\gamma^jB\Theta^j/\pi-i\mu^j_{\pm}(Q_S-i\gamma^j),$$
$$\Upsilon_{\pm}(\psi^j_{\lambda})=exp\,[i\psi^j(\pm B\Theta^j/\pi-\mu^j_{\pm})][cot\frac{1}{2}(\psi^j+Q_S-i\gamma^j)+i\,],$$

and $\mu^j_{\pm}$ is the smallest integer greater than $\pm B\Theta^j/\pi$. Obviously, $\mu^j_{+}+\mu^j_{-}=1$. We allowed in these equations the possibility of the resonance parameters corresponding to the same resonance varying from cavity to cavity.

The co-moving part of the growth rate per current can now be written as

$$\alpha=\frac{\chi}{2}\sum_{j,\lambda}\rho^j_{\lambda}\gamma^j_{\lambda}A(\omega^j_{\lambda},q^j_{\lambda}), \quad (7)$$

where

$$A(\omega,q) \equiv \frac{\sinh\gamma}{\cosh\gamma - \cos(\psi + Q_S)} - (\psi \rightarrow -\psi)$$

We see that the two terms in α are of opposite signs; this is a manifestation of the well-known fact that for a beam which consists only of co-moving particles, the slow waves make the beam unstable, while the fast waves damp the instability.

It is more cumbersome to express the counter-moving contribution $\overline{\alpha}$ in terms of the resonance parameters because of those phase factors in $\Xi_{\pm}$. We shall find a simplified expression for $\overline{\alpha}$ in the next section by using an approximation which is appropriate for LEP.

Application to LEP

We estimate in this section the contribution of the RF system to the two-beam transverse rigid dipole instability at LEP. The storage ring parameters we use are given in Table 1.

Energy	E_0	20	Gev
Ring Radius	R	4243	meter
Vertical Tune	Q_0	78.35	
No. of Bunches per Beam	B	4	
RF Frequency	f_{rf}	352.21	MHz
RF Cavity Cell Length		42	cm
Vert. Rad. Damping Time	τ_D	404.8	msec

Table 1: Beam Parameters

During LEP-Phase I, the RF system consists of 4 RF stations, each station with 32 cavities, and each cavity in turn consists of 5 cells; 640 cells in total.

We construct two models for our purpose. In either of these models, each cell is treated as an independent cavity and the coupling among cells is ignored; hence, $N_c = 640$.

In Model I, all cells are assumed to be identical; therefore, the cavity resonance parameters ω_λ, q_λ, ρ_λ, and $\overline{\rho}_\lambda$ are independent of the cavity number j. The resonance parameters [6] are listed in Table 2.

λ	$\omega_\lambda/2\pi$ (GHz)	ρ_λ $(M\Omega/meter)$	$q_\lambda/10^4$	$\overline{\rho}_\lambda/\rho_\lambda$
1	0.614	18.67	7.08	+1
2	0.762	19.41	5.58	-1
3	1.034	3.27	6.81	-1
4	1.072	12.37	5.01	-1
5	1.240	4.34	7.05	-1
6	1.325	19.38	6.65	+1
7	1.515	3.79	5.57	+1
8	1.583	5.213	6.66	-1

Table 2: RF Dipole Resonance Parameters

The co-moving growth rate per current is then, from Eq.(7),

$$\alpha = (1/2)N_c\chi\sum_\lambda A(\omega_\lambda,q_\lambda).$$

Before calculating the counter-moving contribution $\overline{\alpha}$ from the second equation of Eq.(4), we have to evaluate $\Xi_+\Xi_-$. We recall that $\Xi_\pm$ is a sum of the cavity contributions, $\Xi_\pm^j$, and $\Xi_\pm^j$ is in turn the sum of the resonance contributions. Since the resonance frequencies $\omega_\lambda/2\pi \approx 10^9/sec$ and the closest distance between two cells is about 42 cm, the phase factor in $Y_\pm$ oscillates rapidly with changing j. Therefore, we keep only the Θ^j-independent part in the product $\Xi_+\Xi_-$, and the result is

$$\Xi_+\Xi_- = -\frac{1}{2}N_c\sum_\lambda(\overline{\rho}_\lambda\gamma_\lambda)^2\Lambda(\omega_\lambda,q_\lambda), \tag{8}$$

with

$$\Lambda \equiv \frac{1-\cos(\psi_\lambda + Q_S + i\gamma_\lambda)}{[\cosh\gamma_\lambda - \cos(\psi_\lambda + Q_S)]^2} + (\psi_\lambda \rightarrow -\psi_\lambda).$$

Now, $\overline{\alpha}$ can be calculated by direct substitutions of above equations into the second of Eq.(4).

We emphasize here that ignoring the oscilating terms in the product $\Xi_+\Xi_-$ is quite different from ignoring the oscillating terms in Ξ_+ and Ξ_- seperately, which was done in reference [2]. We would obtain $\overline{\alpha} = 0$ in the latter approximation.

Landau damping is small compared to radiation damping for the coherent mode under consideration, we shall therefore condider only radiation damping. Recall from Table 1 that the vertical damping time $\tau_D = 404.8$ msec.

The results of Model I are given in Table 3 in terms of the threshold currents $1/(\alpha\tau_D)$, $1/(\overline{\alpha}\tau_D)$ and $1/(\alpha_{total}\tau_D)$.

S	$1/(\alpha\tau_D)$	$1/(\overline{\alpha}\tau_D)$	$1/(\alpha_{tot}\tau_D)$
0	0.038	0.023	0.014
1	-0.033	0.020	0.050
2	-0.092	0.018	0.022
3	0.120	0.012	0.011

Table 3: Results of Model I -- Threshold Currents Per Beam (in units of mA.)

We see from this table that $\overline{\alpha}$ tends to be greater than α. A notable example is that $\overline{\alpha}/\alpha \approx 10$ for the mode $S = 3$. This can be understood by comparing the roles played by the fast and the slow waves in these two contributions. We have seen in the last section that in the co-moving contribution, the slow waves enhance while the fast waves damp the counter-moving contribution to the instability. On the other hand, the fast waves as well as the slow waves may in fact excite the beam instability in the counter-moving contribution.

Now the more realistic Model II: We assume here, as we did in Model I, that ρ_λ, $\overline{\rho}_\lambda$ and q_λ are independent of the cavity number j and that their values are as given in Table 2. However, the angular resonance frequency ω_λ is assumed to vary from cavity to cavity due to the construction errors [7]. Specifically, we assume that the value of ω is distributed among the cavities according to a Gaussian probability function,

$$P_\lambda(\omega_\lambda) = exp[-(\omega_\lambda - \omega_{\lambda 0})^2/2\sigma_\lambda^2]/(\sqrt{2\pi}\sigma_\lambda),$$

where $\sigma_\lambda = 10^{-4}\omega_{\lambda 0}$, and that different cavity resonances are not correlated. We take the value for ω_λ as given in Table 2 as the value for $\omega_{\lambda 0}$.

We adopt the approach of reference [8] in estimating α; namely we take the rms value of Eq.(7):

$$\alpha = \frac{\chi}{2}\sqrt{N_c\sum_\lambda(\rho_\lambda\gamma_\lambda)^2\int d\omega_\lambda P_\lambda(\omega_\lambda)[A(\omega_\lambda,q_\lambda)]^2}.$$

An estimation of $\overline{\alpha}$ can be obtained by a slight modification of the Model I calculations. Instead of Eq.(8), we take here

$$\Xi_+\Xi_- = -\frac{1}{2}N_c\sum_\lambda(\overline{\rho}_\lambda\gamma_\lambda)^2\int d\omega_\lambda P_\lambda(\omega_\lambda)\Lambda(\omega_\lambda,q_\lambda). \tag{9}$$

This together with the second equation of Eq.(4) give $\bar{\alpha}$ of this model.

The results of Model II for LEP are given in Table 4.

S	$1/(\alpha\tau_D)$	$1/(\bar{\alpha}\tau_D)$	$1/(\alpha_{tot}\tau_D)$
0	0.553	77.1	0.549
1	0.390	92.2	0.388
2	0.553	336.	0.552
3	0.390	153.	0.389

Table 4: Results of Model II -- Threshold Currents Per Beam (in units of mA.)

Comparing Tables 3 and 4, it can be noted that the co-moving contributin α is lower in Model II than in Model I by a factor of $\approx 1/\sqrt{N_c}$, as expected. Secondly, the counter-moving contribution $\bar{\alpha}$ is completely negligible in Model II, even though it was quite important in Model I. This is due to a strong cancellation in the integral in Eq.(9); the integrand chages sign quite rapidly with changing ω_λ.

Conclutions

There is a qualitative difference between the ways the counter-moving and the co-moving beams contribute to the two-beam rigid coupled bunch oscillations. The main difference is that while the effects of the slow and the fast waves tend to cancel each other in the co-moving contribution, they may reinforce each other in the counter-moving contribution.

We treat each cell in LEP as an independent cavity; therefore, the effective number of cavities $N_c = 640$. This big N_c introduces two sources of fast oscillation and hence a reduction in the counter-moving contribution. The first souce is the time delay factor $exp(\pm iB\,\psi\Theta^j/\pi)$ between the opposing bunches. This factor oscillates rapidly with the changing cavity position Θ^j. The second source is the inevitable spread of the resonance frequencies from cavity to cavity due to construction errors.

Assuming that all cavities have equal resonance parameters, the couter-moving contribution to the growth rate is greater than the co-moving contribution even after the oscillating tems due to the time delay factor have been averaged out. The counter-moving contribution becomes negligible only when the variation of the resonance frequencies is taken into accout.

For storage rings where the effective number of cavities N_c is small, the effect of neither of these oscillations is important. Depending on the placement of these cavities, we can expect a dramatic enhancement of the two beam relative to the single-beam instabilities.

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