TRANSVERSE STABILITY OF THE HIGH INTENSITY SPS PROTON BEAM

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Introduction

The subject of this paper will be the transverse stability in the SPS of the high intensity fixed target proton beam. We will deal with coupled bunch instabilities. The half bunch length of the injected beam at 14 Gev/c is typically 2 ns in the SPS.It shrinks during the acceleration to a minimum of ≈ 1 ns at transition.This sets the upper limit of the beam frequency spectrum to 700 MHz. The SPS impedance at high frequencies can be approximated by a low Q resonator also known as the broad band model. The resonator frequency is 1.35 GHz.Its impedance is purely inductive below 700 MHz. This inductive impedance is much smaller than the resistive wall impedance at the lowest betatron oscillation frequencies. The resistive wall transverse impedance decreases with the square root of the frequency while the inductive transverse impedance remains constant.For frequencies higher than the cutoff frequency of the transverse feedback system($\approx$ 5 MHz) the inductive impedance is much larger than the resistive wall transverse impedance.The effects on beam stability are important. In the next paragraph we will study the total transverse impedance as a function of frequency.The necessary conditions for stability will be determined and compared with experimental observations. An interesting phenomenon of Landau damping was observed experimentally and will be analysed theoretically.

The SPS impedance between 0 and 700MHz

In an accelerator it is possible to identify two sources of transverse impedance. The first source is the wall resistivity and the second source are the crosssection variations of the vacuum chamber.

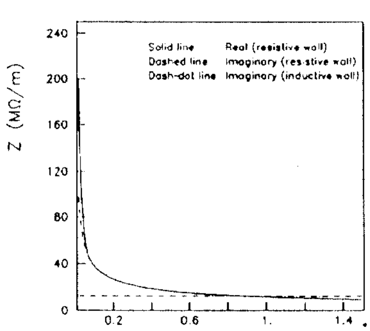
The former is simply the impedance of the smooth inner vacuum chamber wall.This impedance is resistive when the skin depth is larger than the thickness of the wall and this occurs at very low frequency.The skin depth decreases with increasing frequency and the impedance becomes complex when the skin depth is smaller than the wall thickness.

The effect of the cross section variations can conveniently be approximated by a damped resonator, better known as broad band resonator.The components of the broad band model for the SPS are well known [2] .

The expression for the transverse resistive wall impedance is:

$$Z_{\perp rw} = (Rc/b^3)\sqrt{[2\rho\mu/(\Omega(n-Q))]}(1+j)$$

where R is the machine radius,c the speed of light,b is the transverse dimension of the vacuum chamber,ρ the resistivity of the vacuum chamber wall,μ the permeability,Ω the angular revolution frequency,Q the machine tune, $\Omega(n-Q)$ the betatron oscillation frequency with mode number n. For chambers with a circular cross section,b is the chamber radius. For flat elliptical chambers as in the SPS b is the half height when the calculation is done in the vertical plane.For the horizontal plane b is an equivalent radius. [3] In the SPS we find that b^3 (horizontal) = 2 b^3 (vertical).



Vertical transverse impedance in the SPS

Fig 1

The expression for $Z_{\perp rw}$ is valid in the SPS for frequencies above 100 kHz. The lowest betatron frequency is 15 kHz in the SPS.The transverse resistive wall impedance is resistive at that frequency and has following values:

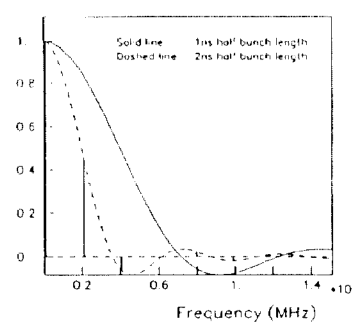
$$Z_{\perp rw} = 120 \text{ M}\Omega/\text{m horizontally}$$
$$Z_{\perp rw} = 240 \text{ M}\Omega/\text{m vertically}$$

The transverse impedances drop inversely with frequency up to 100 kHz.They become more and more complex for higher frequencies and drop with the square root of frequency.This is shown in Figure 1 for the vertical plane.

The resonant frequency of the SPS broadband resonator is 1.35 GHz.The half bunch length of the high intensity proton beam is around 2 ns during the 14 Gev/c injection.The bunch shortens to $\approx$ 1 ns.Figure 2 shows the envelopes of the bunch spectra assuming parabolic bunches of 1 and 2 ns half lengths. It is clear that the resonator frequency is much larger than the bunch spectrum even for the shortest bunch in this mode of the SPS operation.Therefore the complex resonator impedance reduces to a pure inductive impedance. In [2] following values where calculated for the constant inductive wall transverse impedance:

$$Z_{\perp iw} = 5.2 \text{ M}\Omega/\text{m horizontally}$$
$$Z_{\perp iw} = 12.5 \text{ M}\Omega/\text{m vertically}$$

The transverse inductive wall impedance and the transverse resistive wall can be compared in Figure 1 for the vertical plane.The horizontal plane is similar.



Bunch spectrum in the SPS

Fig 2

The driving force of the instability

The driving force of the instability is proportional to the convolution product of the transverse impedance and beam intensity spectrum.

The RF frequency in the SPS is 200 MHz.When the machine is handling high intensity proton beams nearly all buckets are filled. The beam intensity spectrum contains only a few spectral lines within the spectrum envelope (Figure 2).

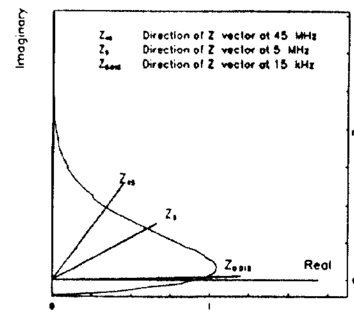
The resistive wall impedance will only couple with one line at the lowest frequency.The frequency of the next line will be 200 MHz higher and the resistive wall impedance there is negligable. The lowest frequency spectral line is proportional to the average beam current.

The inductive wall impedance,however couples with all the spectral lines.The sum of the spectral lines is proportional to the peak circulating beam current,or,in other words,proportional to the average current divided by the bunching factor B_f. In order to compare the various comwe list the transverse impedances for vertical plane in the SPS.The horizontal impedances are half the vertical ones.

Table 1: *Vertical transverse impedance in the SPS*

Frequency	Resistive wall impedance	Inductive wall impedance/B_f
Mhz	MΩ/m	MΩ/m
0.015	240	25j
5.0	5.3(1+j)	25j
45	1.8(1+j)	25j

Bunching factor $B_f = 0.5$



Stability diagram with octupoles

Fig 4

The stabilising forces

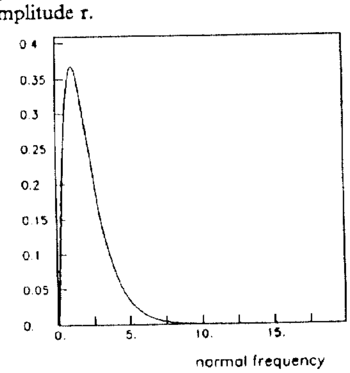
The first stabilising force is generated by a transverse feedback system.The SPS system has a bandwidth of 5 MHz and prevents coupled bunch instabilities within that frequency range.

The second source of stabilisation is Landau damping generated by a spread in betatron oscillation frequencies. This spread is created by the direct space charge tune shift(Laslett tune shift) and by octupolar fields. It turns out ,in the case of the SPS, that the necessary octupolar tune spreads are larger than the tune spread available from the direct space charge tune shift even at the lowest beam energy.We will therefore neglect the space charge effect.It simplifies the arguments without loss of the essentials.

Landau damping with a set of non linear oscillators has been treated in detail in [1] . In particular it was found that a frequency distribution $h(\omega)$ created by octupolar fields can be treated as an equivalent set of linear oscillators but with following frequency distribution:

$$F(\omega) = -k\omega(dh/d\omega)$$

where the octupolar field creates a frequency shift $\omega = r^2/k$ on a particle with betatron amplitude r.



Frequency distribution with octupoles

Fig 3

If we assume the normal Gaussian amplitude distribution then $F(\omega)$ will have the typical form given in Figure 3.The corresponding stability diagram [1] is shown in Figure 4 as well as the directions of the impedance vectors at 15 kHz,5 MHz and 45 MHz. The importance of the inductive wall impedance at the higher frequencies may be clear from that figure.At 15 kHz the resistive wall is dominant.In practice it is tamed by the transverse feedback system.At 5 MHz the impedance is mainly inductive.The feedback is no longer efficient and stability can only be provided by Landau damping.The resistive component is small. Nevertheless it can cause an instability.The necessary Landau damping is determined by the amplitude of the impedance vector which is significantly larger than the sole resistive component.The situation is very similar at 45 MHz. Stability without Landau damping could be achieved by a hypothetical superdamper which can resolve the transverse motion of 2 neighbouring bunches.In the case of the SPS this would require a bandwidth of 100 MHz.

Experimental observations

The octupolar field was carefully tuned out during a machine experiment in one transverse plane at a time during the 14 Gev/c injection coast.The transverse feedback was operational. The beam was unstable in both planes.The octupoles were then powered until stability was achieved. The beam emittances were 1.7 and 0.85 π μradm in the horizontal and vertical planes respectively.The observations were done either with half a machine circumference filled with 1.75 10^{13} particles or a full machine with 3.5 10 ^{13} particles. The instability develops at a frequency around 4 to 5 MHz. When the machine is only partially filled additional spectral lines appear around the main line with a spacing equal to the revolution frequency.However they will couple with the same resistive wall impedance as the main line since this impedance will hardly change for these frequency differences at 5 MHz. The sum of the spectral lines around this frequency for the partially filled machine is equal to the single spectral line of the full machine.Both situations are equivalent from the point of view of the instability. It may be more physical to remark that the beam intensity contained in one wave length of the instability is strictly the same in both cases.

- *horizontal plane:* Growing transverse signals were observed but the rate was so low that it could not be determined with enough accuracy.No beam loss occurred.A clear indication of beam instability showed up on the wire scanner profiles. Figure 5 shows 2 scans at an interval of 1.1 seconds.The emittance growth is obvious.The absence of beam loss can be explained by the large horizontal aperture in the SPS.
  By powering the horizontal Landau octupoles to a normalised integrated field of 2 m^{-3} the instability disappeared and the emittance was conserved.
- *vertical plane:* Clear transverse signals were observed.The instability was accompanied by beam loss.(The vertical aperture is much more restricted than the horizontal aperture.)The maximum signal level was observed around 5 MHz (Figure 6). The growth rate was between 5 and 10 msec (Figure 7). The instability could be stabilised by powering the vertical Landau octupoles to a normalised integrated field of either 5 m^{-3} or -19 m^{-3}.

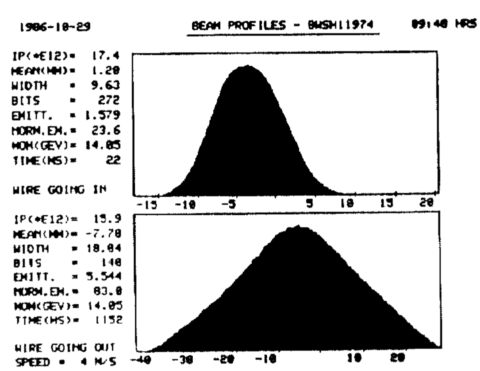
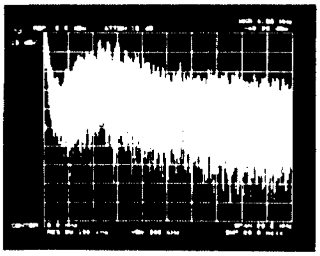
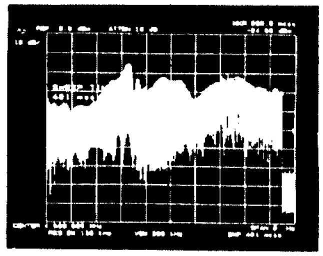


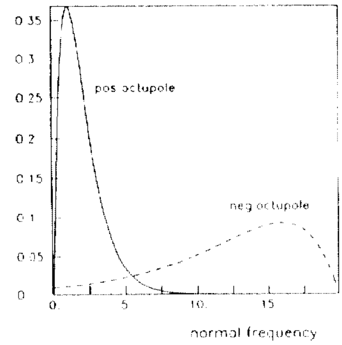
Fig 5



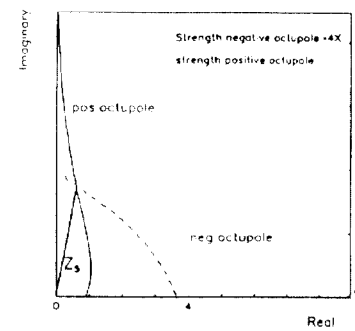
Spectrum of instability
Fig 6



Signal growth at 4.6 MHz
Fig 7



Frequency distribution with octupoles
Fig 8



Stability diagram with octupoles
Fig 9

Analysis

We will do two things.First we will verify that the stability provided by the octupolar fields is consistent with the information on transverse impedance.Secondly we will clarify the importance of the sign of the octupolar field for beam stability.

Let us go back to Figure 3 and 4.Figure 3 shows a frequency distribution of linear oscillators.It has the same stabilising properties as the frequency distribution caused by octupolar fields on a beam with a Gaussian amplitude distribution. The tuneshift as a function of amplitude for an octupole is:

$$\Delta Q_4 = (-3/16\pi)\beta S_4 r^2$$

where r is the amplitude and S_4 is the normalised octupole strength. The frequency scale of Figure 3 is normalised with the frequency shift of the particles with amplitude 1 σ.The stability diagram derived from the distribution of Figure 3 is shown in Figure 4. From [1] we know that a tune spread δQ can damp an instability with a coherent tune shift ΔQ smaller than $\delta Q d^{-1}(\xi)$ .The stability diagram is the contour that the vector $d^{-1}(\xi)$ describes in the complex plane when scanning the normalised frequency ξ of the distribution. In order to determine the threshold condition $d^{-1}(\xi)$ is read from the stability diagram in the direction of the driving impedance.

A transverse impedance $Z_\perp$ causes the following tune shift:

$$\delta Q_\perp = neZ_\perp / [8\pi^2 Q(E/ec)]$$

We select in Figure 4 the impedance vector at 5 MHz and we find that $d^{-1}(\xi) = 3.5$.Applying the formulae given above we can compute the necessary octupole strength for stability.The calculated strenghts are 4.1 m^{-3} in the vertical plane and 1 m^{-3} in the horizontal plane. Experimentally we found 5 and 2 m^{-3}. The agreement is very good taken into account that the experimental values include a stability margin.

Let us now turn to the sign of the octupoles. Already from Figures 3 and 4 we notice a strong asymmetry in the distribution and also in the stability diagram. We will concentrate on the slow wave which is at the origin of instabilities. A positive octupole produces a negative tune shift.A negative tune shift corresponds to a frequency increase in the slow wave. Therefore the distribution function in frequency shown in Figure 3 for a slow wave is consistent with a positive octupole. Rather then repeat the stability calculation for negative octupoles we prefer to compare in the same figure the stability situation for positive and negative octupolar fields.We copy the first quadrant of Figure 4 valid for positive octupoles and inductive impedances.Then we choose a distribution equivalent to a negative octupolar field but 4 times as strong (Figure 8). Remember that we require approximately 4 times more negative octupolar field than positive octupolar field in the experiment (-19 versus 5 m^{-3}). The corresponding stability diagram is multiplied by 4 as well and is shown on the same plot (Figure 9). This has the advantage that the impedance vectors are identical for the two cases so that direct comparison becomes possible.We have drawn the impedance vector at 5 MHz,that is the frequency where the instability grows. From Figure 9 it may be clear that the large negative octupolar excitation provides much more stability than the small positive excitation for impedances which are more resistive than inductive. For more inductive impedances however,as in our case,the same stability is achieved by both the large negative and small positive octupole .

Conclusions

It has been shown that with the present transverse feedback system of the SPS octupolar fields are necessary to stabilise the high intensity proton beams at frequencies beyond the cutoff of the system. The strength of the necessary octupolar fields are determined by the wall inductance.Hence octupoles remain necessary at the same excitation level even when the feedback bandwidth is increased,although the instability growth rate decreases as the square root of the increased bandwidth due to the reduction of the resistive wall effect. Only a bandwidth which covers all the coupled bunch modes,i.e., 100 MHz in the SPS,would allow a stable operation without octupoles. The tune spread induced by octupolar fields may have adverse effects on the beam.Large tune spreads make it more and more difficult to avoid non linear resonances.Therefore the sign of the necessary octupole becomes extremely important.Indeed it has been demonstrated that in the case of a dominant inductive impedance one needs about 4 times more negative octupolar strength in the SPS than positive octupolar strength.

The tuneshift caused by the transverse impedance and the emittance are inversely proportional to the beam momentum. Since the tune spread is proportional to the beam emittance it follows that a constant normalised octupolar field would stabilise the beam during acceleration.The bunching factor however increases by about a factor2 between injection(14 Gev/c) and extraction(450 Gev/c) in the SPS.This requires a doubling of the normalised octupolar fields to keep the beam stable.

Acknowledgments

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References

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