# EFFECTS OF INSERTION DEVICES ON BEAM DYNAMICS

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## Abstract

The introduction of insertion devices in storage rings leads to a significant reduction in beam dynamics, which is due to the non-linearities of the devices and to the linear distortion. These three causes are investigated for undulators and wigglers by computer simulation.

## Introduction

The reduction of dynamic aperture verified by the introduction of an insertion device in the lattice is essentially due to:

1) non-linear fields of the device;

2) breaking of the optics symmetry;

3) the phase shift produced may drive the working point on a resonance.

In order to investigate these effects, four particles with different initial phases have been tracked over 100 revolutions using the program RACETRACK, which searches the maximum stable amplitudes with the following assumptions:

1) for the cases investigated, the scanning through the amplitudes was done with a fixed step of 2 mm;

2) the non-linearities are due only to sextupoles and to insertion devices;

3) the field components in the device are given by [1]:

$$\begin{aligned} B_x &= B_o\,(k_x/k_y)\,\mathrm{sh}(k_x x)\mathrm{sh}(k_y y)\cos(kz)\\ B_y &= B_o\,\mathrm{ch}(k_x x)\mathrm{ch}(k_y y)\cos(kz)\\ B_z &= -\,B_o\,(k/k_y)\,\mathrm{ch}(k_x x)\mathrm{sh}(k_y y)\sin(kz) \end{aligned} \tag{1}$$

where $B_o$ is the maximum transverse field on axis, $\lambda_o$ is the device period and $k_x^2 + k_y^2 = \; = k^2 = (2\pi/\lambda_o)^2$. Here, z is assumed to be the direction along the device axis, y is the vertical direction and x the horizontal one. For $k_x$, $k_y$ not zero, the field increases in a non-linear way, giving thus rise to intrinsic non-linear effects.

The equations of motion with respect to the reference orbit may be derived as [2]:

$$\begin{aligned} x'' = &-\,(k_x/2k^2\rho^2)\,[k_x x + k_x^3x^3/6 + k_x k_y^2 xy^2/2]\\ &-\,(\cos(ks)/\rho)\,[k_x^2x^2/2 + k_y^2y^2/2 +\\ &\qquad + k_x^2k_y^2x^2y^2/4 + k_x^4x^4/24 + k_y^4y^4/24]\\ &-\,(y'yk\sin(ks)/\rho)\,[1 + k_y^2y^2/6 + k_x^2x^2/2]\\[6pt] y'' = &-\,(k_y/2k^2\rho^2)\,[k_y y + k_y^3y^3/6 + k_y k_x^2 yx^2/2]\\ &+\,(\cos(ks)/\rho)\,[k_x^2xy \; + \; k_y^2y^2/2 \; +\\ &\qquad + k_x^2k_y^2xy^3/6 + k_x^4x^3y/6]\\ &+\,(x'yk\sin(ks)/\rho)\,[1 + k_y^2y^2/6 + k_x^2x^2/2] \end{aligned} \tag{2}$$

where $1/\rho \;= B_o/(3.3\ E)$ is the bending curvature. Thus, the linear effects of a device are those of a quadrupole focussing both in the vertical and in the horizontal plane.

If we assume that the field is uniform in the x-direction, i.e. $k_x$ is null, the equations of motion become:

$$\begin{aligned} x'' = &-\,(\cos(ks)/\rho)\,[k_y^2y^2/2 + k_y^4y^4/24]\\ &-\,(y'yk\sin(ks)/\rho)\,[1 + k_y^2y^2/6]\\[6pt] y'' = &-\,(1/2\rho^2)\,[y + k_y^2y^3/6] +\\ &+\,(x'yk\sin(ks)/\rho)\,[1 + k_y^2y^2/6] \end{aligned} \tag{3}$$

Thus, it focusses only in the vertical plane with strength $k_v =$ $= 1/2\ (1/\rho)^2 \approx (B_o/E)^2$. The non-linear terms, instead, increase with the particle transverse elongation and are proportional to $(B_o/E)^p$ $(2p/\lambda_o)^q$.

Wigglers, therefore, are expected to cause predominantly linear optics distortion due to their higher fields. However, the symmetry break combined with the strong lattice sextupoles may excite additional resonances. Undulators, instead, cause particle distortion due to their non-linear effects (low $\lambda_o$) and to the symmetry break (in case of high fields). Of these two effects, one may be predominant with respect to the other or they may both occur.

## Effects of Wigglers and Linear Distortion

The linear effects on the ring, felt predominantly with wigglers, are:

1) perturbation of the vertical betatron amplitude function $\beta_y$ around the ring;

2) shift of the vertical betatron oscillation frequency $\nu_y$;

3) opening of stopbands around half-integral values of $\nu_y$ within which the ring cannot operate.

Hence, there is the necessity to reset the initial betatron function around the ring leaving a local perturbation only near the insertion point and to correct the phase shift.

Three ways of achieving the matching have been investigated for the Sincrotrone Trieste, whose lattice structure near an insertion point is illustrated in fig.1 :

1) characteristic betatron matching : change strengths of the quadrupole triplet so as to tune the $\beta$ in the device to its characteristic betatron value $\beta^* = \rho\sqrt{2}$. This matching leads for high fields to a minimum beta function in the device.

2) alfa-matching : change strengths of a quadrupole duplet so as to give a zero betatron slope at the symmetry point. The beta value cannot be influenced and it is larger than the one achieved with characteristic beta matching. However, it gives a less phase shift.

A comparison between the dynamic apertures resulting from these two types of matching has been achieved for the device W1 reported in Table 1.

Since, the non-linear kick sampled by the particle in a device increases with its transverse elongation, the characteristic beta matching, giving a reduction of the beta function in the device, should weaken the non-linear effects. In fact, the maximum vertical stable amplitude for the beta-matching has been found to be 110 mm against the 90 mm for the alfa-matching.

This is true,however, if only the non-linearities of the device are taken into account. If sextupoles are added, additional driving

| | | | |
|---|---|---|---|
| | Table 1. | | |
| Device | W1 | W2 | |
| Energy | 2 | 2 | GeV |
| Field | 5.0 | 5.0 | T |
| Period length | 0.30 | 0.50 | m |
| Period Number | 10 | 2 | |

resonances are excited which lead to a decrease in dynamic aperture.

Figures 2 and 3 report the dynamic apertures resulting from the two matchings when also the sextupoles are taken into account. As a comparison, also those with only sextupoles in the lattice and with the wiggler treated as a quadrupole (showing thus the symmetry break intro- duced) are reported. Clearly, with the beta-matching, the break in symmetry leads to stronger reduction, since the phase distortion is larger.

The third type of matching investigated is to change the strengths of a quadrupole quadruplet so as to match the betatron slope and phase distortions. In this way, the optical symmetry break for the sextupoles is avoided and no additional resonances are excited. This is valid if no sextupole is located inside the matching region, which is not true for the Sincrotrone Trieste. However, we anticipated that, if only one wiggler is considered, the single pair of sextupoles in the matching region, compared to the other 70 around the ring, should not deteriorate much the non-linear behaviour.

An extra quadrupole and an extra straight section has been inserted near the insertion point and several runs have been achieved for the device W2 in table 1 for various values of the field. During the computations, it became quite evident that the compensable fields increase with the length of the straight section.

For the structure of figure 4, in which the quadrupole Q4 and a straight section of 0.5 m have been added (total length of the ring is increased of 20 m), it resulted that the maximum phase distortion which could be compensated is the one corresponding to a field of 3.65 T. Figure 5 illustrates the resulting dynamic aperture, but comparing this with figures 2 and 3 and considering the total length of the ring increase and the limited magnitude of the compensable fields, there is no overall benefit from this type of matching.

Effects of Undulators

Since undulators present smaller period lengths, they are expected to present higher intrinsic non-linear effects with respect to wigglers. Thus, a matching which leads to a small beta value in the device should be achieved. In case of low fields, the alfa-matching is the most adequate for this purpose. Thus, all the computations for undulators have been achieved with alfa-matching and we imply tacitly that all the optics have been readjusted by this means, without having to repeat it for each device.

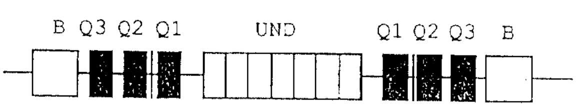


**Fig.1.** Lattice structure near insertion device

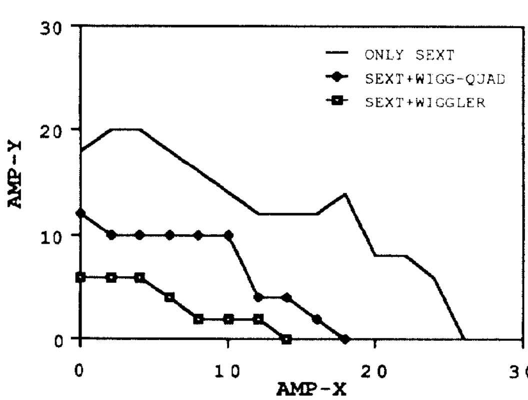


**Fig.2.** Dynamic aperture with alfa-matching

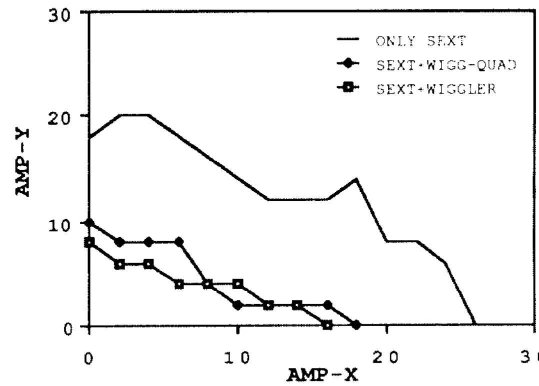


**Fig.3.** Dynamic aperture with beta-matching

Symmetry break and non-linear effects

To see to what extent the symmetry break and the non-linear field influence the dynamic aperture of the beam, computations have been done in three configurations:

1) only one undulator in the lattice;

2) six undulators in the lattice arranged in a symmetric configuration, six times a free straight section - an occupied one;

3) six undulators arranged in a non-symmetric configuration, six times a free straight section - six times an occupied one.

The plane undulators that have been investigated are those reported in Table 2 and the resulting dynamic apertures for the three above configurations are illustrated in figures from 6 to 9. From a comparison between the two undulators in the various configurations, the limitation in dynamic aperture resulting from the undulator U2 having the greatest period length seems to be essentially due to the symmetry breaking. Instead, for undulator U1, having the smallest period length, it seems that the non- linearities are the main cause of the dynamic reduction.

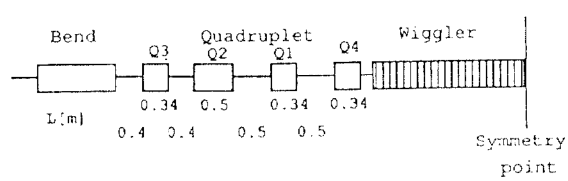


**Fig.4.** Structure for quadruplet matching

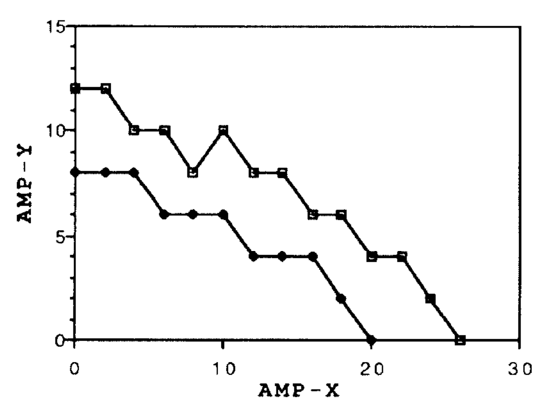


**Fig.5.** Dynamic aperture with quadruplet matching

Table 2

| Undulator | U1 | U2 | |
|---|---|---|---|
| Energy | 2 | 2 | GeV |
| Field | 1.2 | 1.2 | T |
| Period | 0.055 | 0.20 | m |
| Period Number | 100 | 27 | |
| Length | 5.5 | 5.4 | m |

Effects of period length on beam dynamics

In order to see clearly the influence of the period length on the dynamics, after having readjusted the optic distortion, a comparison between devices having the same optics (i.e. same field, energy and total length) must be done.

Several runs have been made for the device reported in table 3 halfing each time the period length $\lambda_o$ and doupling the number of periods $N_p$, in order to keep the optics unchanged. The dynamic apertures for the different values of $\lambda_o$ in figure 10 show that as the period length decreases the reduction increases.


References

[1] K. Halbach, "Physical and Optical Properties of Rare Earth Cobalt Magnets", Nuclear Instruments and Methods, 187, pp.109-117, 1981

[2] L. Smith, "Effect of Wigglers and Undulators on Beam Dynamics", ESG TECH NOTE-24, Sept. 1986


Table 3

| Undulator | U3 | |
|---|---|---|
| Energy | 1.5 | GeV |
| Field | 1.2 | T |
| Period | 0.25 | m |
| Period Number | 20 | |
| Length | 5.0 | m |

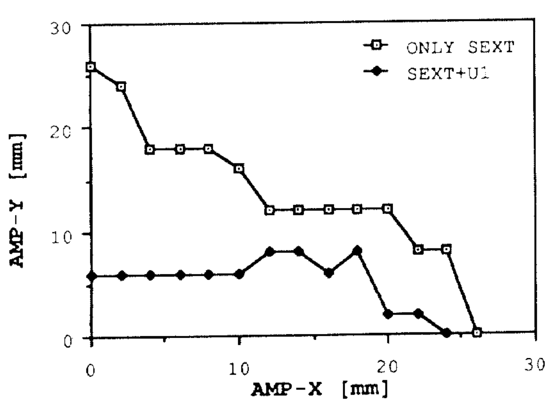


Fig. 6. Undulator U1

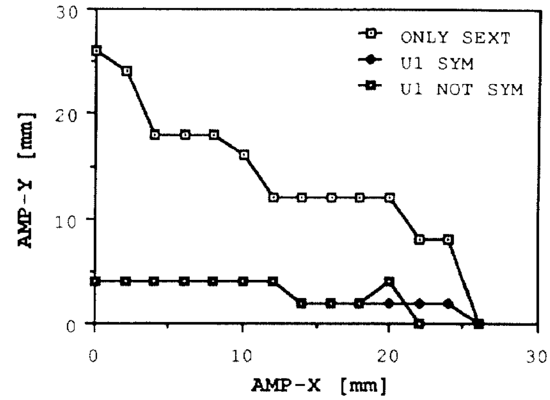


Fig.7. Six Undulators U1

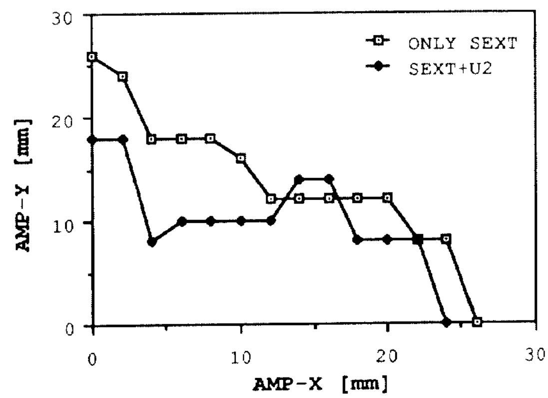


Fig.8. Undulator U2

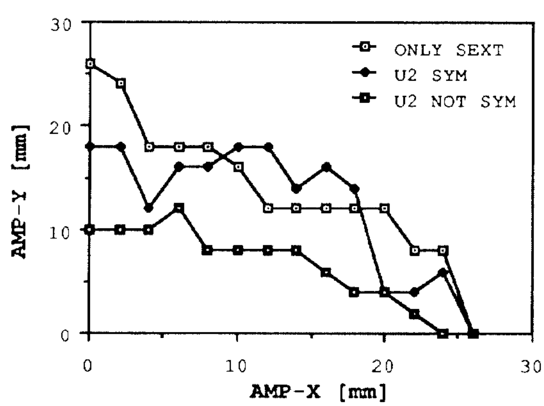


Fig.9. Six Undulators U2

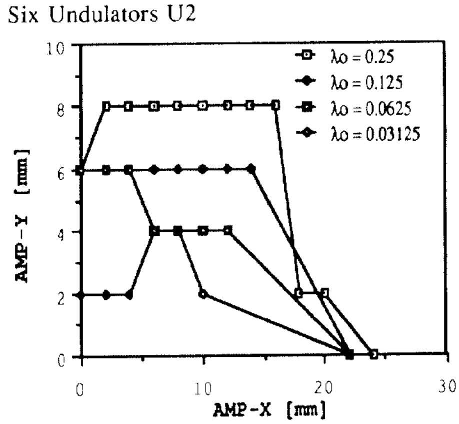


Fig.10 Effects on period length on dynamic aperture