ANALYSIS OF THE TRIPLE BEND ACHROMAT AS A LATTICE FOR BESSY II

B. Simon, R. Maier*, G. Wüstefeld
BESSY GmbH, Berlin, FRG
* now at KFA Jülich, Jülich, FRG

1 Introduction

Lattices of low emittance storage rings for synchrotron radiation are usually very compact with a relatively low phase advance per superperiod. Therefore it is difficult to adapt the usual techniques of introducing additional sextupoles and octupoles or phase matching used in other machines, to compensate the non-linear effects caused by the strong chromaticity compensating sextupoles. Using an analytic approach, we analyse the linear lattice in order to choose linear parameters that create as little as possible non-linearities instead of compensating for them afterwards. The analysis was performed for the Triple Bend Achromat (TBA) lattice, but can easily be adapted to other lattice types. We introduce this way of lattice design using the parameters of the 1.5 to 2.0 GeV synchrotron light source BESSY II planned in Berlin as listed in [1].

2 The Method

A summerized description of the analytic approach that we have developed can be found in [2]. Here we shall only give a short review of the central ideas and present our results. For the analysis the lattice has been subdivided. Only two simple structures, the achromat and the triplet, are left to be analysed, optimized and matched in order to construct the complete lattice. We use the beta functions at certain points of the lattice as input parameters and calculate the position and the strength of the magnets in the kick approximation. Some of those input parameters are set to yield certain properties of the lattice, e.g. the low emittance or the achromatic characteristic. The other input parameters are varied to scan the field of all possible solutions. These solutions, that already ensure that some of the lattice requirements are fullfilled, can be reduced further by adding new criteria, e.g. to provide enough space in a certain drift region or to yield low chromaticities. After a promising achromat and triplet have been found, the resulting working point, the chromaticity, the sextupole strengths necessary for chromaticity compensation and the amplitude dependent tune shift of the whole lattice are calculated in order to get a first insight in the non-linear properties of the lattice, all still in the kick approximation. The thin lense data are then adjusted to thick lenses using fitting procedures.

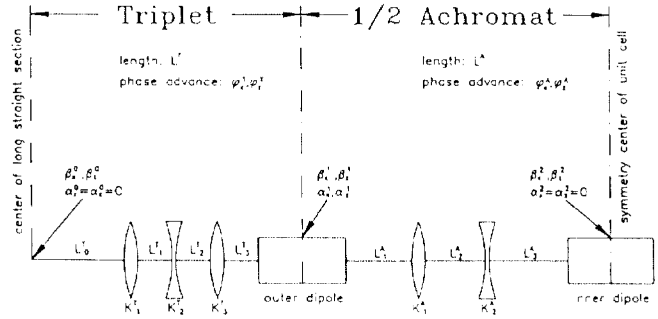

Figure 1: Sketch of half of the symmetric unit cell showing the devision of the lattice and the important parameters as used in the analytic approach.

3 The Achromat

To calculate the strengths and positions of the quadrupoles in the achromat it is sufficient, due to the symmetry of the unit cell, to confine oneself to half the achromat, defined from the center of the outer dipole (superscript 1) to the center of the inner dipole (superscript 2) as shown in Figure [1].

The achromatic property of this structure is achieved by fulfilling a certain relation between the horizontal beta function at the center of the dipoles, β_x^1, β_x^2, and the horizontal phase advance, ϕ_x^A, between them [2]. In our programs β_x^1 and ϕ_x^A are input parameters, while β_x^2 is calculated to guarantee the suppressed dispersion function in the straight sections.

The natural emittance created by the achromat can be expressed in terms of β_x^1, α_x^1, and ϕ_x^A [3] and is basically fixed by the size of the minimum of β_x in the outer dipole, while the variation of α_x^1 and ϕ_x^A, within certain bounds, have less influence. To reach the desired natural emittance of $\approx 4.5 \cdot 10^{-9} \cdot \pi \cdot m \cdot rad$, we obtain:

$$\beta_x^1 = 0.2m$$
$$-0.1 \leq \alpha_x^1 \leq 0.4$$
$$140^\circ \leq \phi_x^A \leq 180^\circ.$$

Two further input parameters for the achromat program are the overall length of the structure, $L^A = 3.8$ m, and the vertical beta function in the inner dipole, $\beta_z^2 = 10.0$ m (insertion device requirement).

The low emittance and the achromatic condition leave practically no choice for the shaping of the horizontal beta function. The vertical beta function however is almost free of restrictions. The vertical phase advance, ϕ_z^A, is varied from 0° to 200° and the β_z^1 and α_z^1 are output parameters of the program, together with the length of the drifts, L_1^A, L_2^A, L_3^A, the strength of the quadrupoles, K_1^A, K_2^A, and the two figures $chro_{x,z}^A$ which are proportional to the chromaticity contribution of this section:

$$chro^A_{x,z} = \sum_i K^A_i \cdot \beta_{x,z}.$$

Another requirement for the achromat is that the chromaticity compensating sextupoles must be inserted here. As expected from the strong restrictions on the horizontal beta function the distance between the outer dipole and the first horizontally focussing quadrupole is almost constant, $L^A_1 = 1.65$ m. The position of the second quadrupole can range from being close to the inner dipole to sitting next to the first quadrupole, which causes different developments of the vertical beta function. Under the assumptions made above, the two beta functions will always intersect between the two quadrupoles. No solution exists that allows one to place a sextupole there. Nor can both sextupoles be placed in the comparably long section, L^A_1. Thus, there exists only one possible sextupole configuration for our low emittance TBA lattice. Due to the necessarily large vertical and small horizontal beta function in the inner dipole, the defocussing sextupole has to be placed in L^A_3, the focussing one in L^A_1. Consequently, additional restrictions for the achromat are $L^A_3 > 1.25$ m and $beta_x > beta_z$ in L^A_1.

Plotting $chro^A_z$ versus $chro^A_x$ we again find the difference in flexibility between the two dimensions:

$$-17 < chro^A_x < -13$$
$$-11 < chro^A_z < +1.$$

The extreme values of $chro_z$ correlate with extreme values of the vertical phase advance, ϕ^A_z, which then implies large values of β^1_z and α^1_z, which are forbidden by the demand $\beta_x > \beta_z$ in L^A_1. For relaxed values, $60° < \phi^A_z < 140°$ we get $chro^A_z \approx -6$, $\beta^1_z < 4.$ m and $\alpha^1_z < 2.5$. The amount of chromaticity created in the achromat section of the lattice thus is almost the same for all solutions of interest. As there are no other favourable characterictics of a certain type of achromat, the remaining freedom of solutions can be used to meet possible constraints imposed by the triplet and to adjust the working point of the lattice.

4 The Triplet

The triplet is defined from the center of the outer dipole to the center of the straight section, Fig.[1]. The length of the triplet must be ≈ 5.3 m to reach the fixed circumference of 182.4 m. We must provide 2.8 m to the center of the first quadrupole for (half) the insertion devices, and the beta functions at the middle of the straight section are set to $\beta^0_x = 10.0$ m and $\beta^0_z = 2.5$ m.

In the case of the triplet the input parameters for the analytic program are $\beta^1_x, \alpha^1_x, \beta^1_z, \alpha^1_z$ in the center of the outer dipole, $\beta^0_x, \beta^0_z, (\alpha^0_x = \alpha^0_z = 0.)$, in the middle of the straight section, and the length of the drift for the insertion devices, L^T_0. The horizontal phase advance, ϕ^T_x, and the vertical phase advance, ϕ^T_z, are varied. The output parameters are again the strength of the kicks, K^T_1, K^T_2, K^T_3, and the length of the drifts, L^T_1, L^T_2, L^T_3, the total length of the triplet, L^T, and $chro^T_{x,z}$. All parameters are shown in Figure [1].

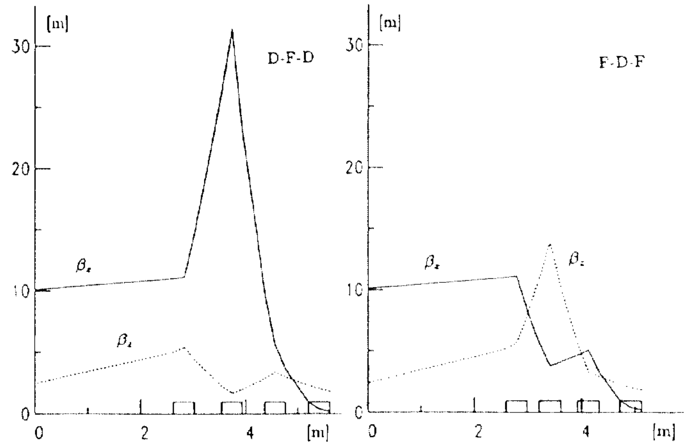

Figure 2: The two possible types of triplets: the D-F-D triplet (left) and the F-D-F triplet (right).

In general two distinct types of triplets exsist (Fig. [2]). In the D-F-D triplet the horizontal beta function is defocussed in the first quadrupole. The resulting large β_x values have to be focussed back to 0.2 m in the dipole in order to match the achromat. The chromaticity created by this type of triplet is always almost twice as large as in the F-D-F triplets. In the F-D-F triplets, where the outer quadrupoles are horizontally focussing, the maximum values of both beta functions are small, and the chromaticities are moderate. Figure [3] shows the chromaticities of all the triplets that where calculated under the restrictions made above. The low horizontal chromaticity achieved by the

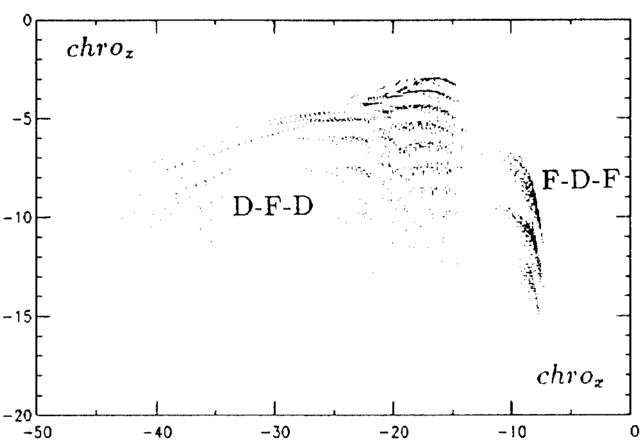

Figure 3: The contributions to the chromaticity of the two types of triplets are clearly different when plotted agaist each other.

F-D-F triplets is very pronounced and the vertical chromaticity is only slightly larger than in the D-F-D triplets. In both cases the contributions to the chromaticity, $chro_{x,z}$, depend strongly on the settings of the vertical beta function and is independent of the overall length of the triplet. The stronger vertical focussing in the D-F-D triplet leads to vertical phase advances up to 200°, compared to $\phi^T_z \leq 115°$ for F-D-F triplets. The horizontal phase advance is $80° \leq \phi^T_x \leq 115°(85° - 130°$ for F-D-F). An additional difference between the F-D-F and the D-F-D triplets concerns the values of β^1_z and α^1_z. No F-D-F triplets exist for $\beta^1_z \geq 5$ m and $\alpha^1_z \geq 1$.

The only critical parameter of the triplet is the overall length. Only few solutions (F-D-F and D-F-D) are as short as needed.

5 The Working Point

The TBA lattice permits a wide range of possible working points, while providing a small emittance, no dispersion in the straight sections, sufficient space for the sextupoles, and the desired overall length of the lattice. Adding up the phase advances in the achromat and the triplet, we can estimate that the horizontal tune of the lattice will lie between 13.7 and 15.4. The weak horizontal focussing of D-F-D triplets restricts the horizontal tune of the whole lattice to 13.7 - 14.2, unconveniently centered around the integer resonance. The range of horizontal tunes of lattices using a F-D-F triplet is wider and lies slightly higher with $14.1 < Q_x < 15.4$.

For both types of triplets the vertical working point is extremely flexible. It can be varied between ≈ 5 and ≈ 12 while maintaining all the features mentioned above. Looking at the structure resonance diagram, vertical tunes larger than 8.5 seem favourable in order to avoid the resonances $Q_x \pm 2Q_z$ and $3Q_x$ which would drive the tune dependence on large amplitudes, one of the major problems in seeking a good dynamic aperture.

6 The Lattice

A critical question of this analysis is how much of the results and insights obtained in the kick approximation is left when we transfer them to the final thick lenses. The translation to thick lenses is done in two steps. In the first step the resulting kick parameters are varied separately to adjust each element of the thin lense transfer matrix of the achromat and the triplet to the corresponding thick lense transfer matrix. The solution of this fit is by no means unique, and it might take several tries until a good approximation is reached. The resulting thick lense data are used as the input for the fitting procedure implemented in the MAD program [[4]]. Here the total length, and other boundary conditions can be restated if they were lost during the first fit. Again there are many solutions and the constraints must be carefully chosen.

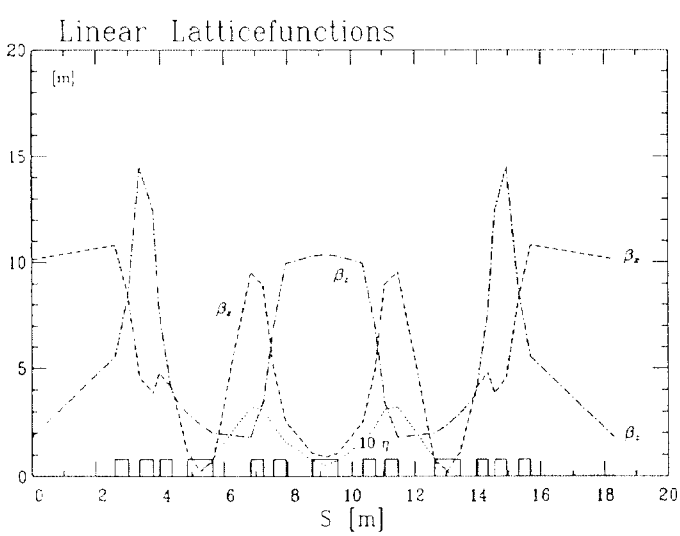

Figure 4: A possible lattice reflecting the above analysis.

In Figure[4] we show one of the possible lattices that have been constructed according to the above analysis. We chose a F-D-F triplet, with chromaticities close to the achievable minimum, $chro_x = -8.4$ and $chro_z = -10.4$. The achromat was chosen to suit the boundary conditions of the triplet and to provide enough phase advance to reach $Q_z > 8.5$. Table [1] shows the important lattice parameters that were calculated in the kick approximation, after fitting the transfer matrices, and for two different results of the MAD fitting procedure.

parameter	kick-approximation	transfer matrices fit only	MAD-fit I	MAD-fit II
Q_x	14.71	14.41	14.18	14.64
Q_z	8.83	9.01	8.69	9.16
ϵ_0 $[10^{-9} mrad]$	≈ 4.5	6.60	4.73	3.66
ξ_x	-37.69	-25.86	-30.53	-36.26
ξ_z	-23.95	-27.46	-25.93	-28.84
SF $[m^{-2}]$	7.76	6.05	7.54	6.
SD $[m^{-2}]$	-8.043	-5.89	-9.12	-10.
η^0 [m]	0.00	-0.018	0.006	0.019
β_x^0 [m]	10.00	6.06	10.13	10.02
β_z^0 [m]	2.50	2.50	1.82	1.66

7 Conclusion

The analysis of the TBA lattice under the boundary conditions of the BESSY II storage ring showed that all requirements can be met with a variety of solutions. The F-D-F triplet has clear advantages over the D-F-D type due to the lower chromaticity and the better working points. Only one type of sextupole configuration exists and the resulting strength of the sextupoles is moderate using an F-D-F triplet. For this lattice type the dynamic apertur seems satisfactory but has not yet been fully investigated. The analytic approach should be extended to include the influence of gradients in the dipoles and of using different dipole lengths.

References

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