BEAM-BEAM INTERACTIONS AND LUMINOSITY CONSIDERATIONS IN RHIC*

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Abstract

The performance of the RHIC machine is largely determined by the intrabeam scattering process. However, beam-beam interactions due to both nuclear scattering and intrinsic Coulomb excitation of the heavy ions will result in beam loss and luminosity reductions. The transverse electric field, responsible for Coulomb excitation, is multiplied by a Lorentz Gamma factor at relativistic energies. For ^{197}Au beams in RHIC (Lab. Frame) this factor is 108. Once Coulomb excitation has occurred, the nucleus may fragment or emit a proton, and thus be lost from the beam. Estimates of the beam lifetime will be presented, based on perturbative calculations for the Coulomb excitation process. In addition, di-lepton production from the enormous transverse electric field will be discussed and related to the more general luminosity questions.

Introduction

The Relativistic Heavy Ion Collider (RHIC) at Brookhaven will accelerate[1] heavy ions up to ^{197}Au, at an energy/nucleon of 100 GeV. At this top energy, the two beams of heavy ions will collide in up to six available experimental halls situated around the ring. Of particular interest to the physics community, is the possibility of both forming and detecting the new and exotic form of matter known as a quark-gluon plasma. It is understood that this new form of matter has the highest probability of being formed in a central collision.

In addition to this so-called nuclear collision, which of course depletes particles from the beam, it is also important to understand the components of the beam-beam interaction due to the strong, long range forces between charged heavy ions. Of course, these processes are mainly peripheral in nature.

While it is understood that the long-term stability of the heavy ion beams in RHIC are determined by intra-beam scattering,[2] it is also important for long-term luminosity considerations to accurately estimate beam loss due to the unique electric fields generated at RHIC energies during the beam-beam crossing of the heaviest ions. These electric fields may excite internal nuclear multiphonon resonance states,[3] which subsequently particle decay, or generate di-lepton pairs[4] from the Quantum Electrodynamics vacuum, which are subsequently captured by ions in the beam. Both mechanisms contribute to beam-loss mechanisms.

In this paper, I calculate the perpendicular electric field at top RHIC energies for ^{197}Au and compare with the critical field for di-lepton production. The standard perturbative capture formalism is also briefly discussed. I also show at what energy (assuming forward scattering only) the simple perturbative formula for the dipole cross-section[3] (phonon resonance, multipole = 1) may be considered inaccurate. In particular, current and near future research efforts are discussed where these should provide more accurate cross sections for these peripheral processes. I note, that relative changes in the magnitude of these peripheral cross sections may be readily accommodated through the magnitude of the mini-insertion parameter β^*.[5]

Di-Lepton Production

The perpendicular component of the electric field for a charge Ze, at a distance R from a fixed target, is given by

$$E_{\perp}(R,t) = \frac{Ze}{R^2}\,\frac{\gamma_{EQ}}{\left(1+\left(\frac{t}{\Delta t}\right)^2\right)^{3/2}}$$

where γ_{EQ} is the Lorentz gamma and $2\Delta t$ is the duration of pulse ($2\Delta t = 2R/\gamma_{EQ}\beta c$). For ^{197}Au at 100 GeV/A in the collider, the equivalent Lorentz factor for a fixed target is $\gamma_{EQ} = 23{,}327$, and for R = 25 fm, we find $E_{\perp}(\max) = 4.25 \times 10^3$ MV/fm and $2\Delta t = 2.14 \times 10^{-3}$ fm/c.

In order to understand the significance of these values, we compare $E_{\perp}$ with the critical field E_{crit} for di-lepton production, where

$$E_{crit} = \left(2m_L c^2\right)^2/e(hc).$$

Table 1. Values of Critical Electric Field for di-lepton Production at RHIC Energies.

Probability of di-lepton production with this $E_{crit} = e^{-1}$.

Lepton Pair	Lepton Mass (MeV/c^2)	E_{crit} (MV/fm)
e^+e^-	2×.511	5.29×10^{-3}
$\mu^+\mu^-$	2×105.6	226.4
$\tau^+\tau^-$	2×1784.0	64.52×10^3

Perturbative Calculations for Di-Leptons

Obviously, comparing the values of Table 1 with $E_{\perp}(\max)$, we expect copious electron-positron production and some di-muon production during beam crossing. Of particular importance to the RHIC beam lifetime is the probability for capture of these electrons into the atomic shells. Di-muon capture may be safely ignored relative to electron capture. At the present time all theoretical estimates for these processes are pertubative in nature, and typically based on the well known Weizacker-Williams[6] (W-W) formalism. In this method, the electromagnetic field of the relativistic ion with charge Ze is replaced by an equivalent field of photons $\eta_{\pi\ell}(\omega)$, and the capture process cross section σ_c is given by

$$\sigma_c = \sum_{\pi\ell} \int \frac{d\omega}{\omega}\, \eta_{\pi\ell}(\omega)\, \sigma_\gamma^{\pi\ell}(\omega)$$

where $\sigma_\gamma^{\pi\ell}(\omega)$ is the corresponding photonuclear cross section, calculated in the first order perturbative limit.

Several authors[1,4] have estimated σ_c using the perturbative formula and predict $\sigma_c \sim 100$ barns. However, at top RHIC energies, we may find that the perturbative limit is not sufficient. For instance, a simple estimate for the di-lepton creation probability, p_{pair}, is given by

$$p_{pair} \;\alpha\; \exp\left[\frac{-4m_L^2c^3}{heE_{\perp}}\right]$$

At top RHIC energies and R=25fm, the formula gives $p_{pair}=1$. Of course, the amplitude a_{fi} associated with this process must also be near unity, and thus violate the required perturbative limit, $a_{fi} \ll 1$.[7]

It is well known from general quantum-mechanical studies, that calculations using perturbative formalisms in non-perturbative domains are not bounded by unitarity or flux conservation requirements, and the amplitudes often increase with increasing potential strength. On the contrary, probability amplitudes calculated in a self-consistent, non-perturbative theory, tend to saturate[7] as the potential strength increases, because of unitarity bounds.

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I note for the di-lepton problem that the field or interaction of interest, $E_{\perp}(R,t)$, increases linearly with the Lorentz γ factor and decreases as R^{-2}. Indeed, model calculations[8] for di-lepton production from U+U at top RHIC energies ($\gamma_{EQ}=2\times10^4$) predict a breakdown in first order perturbation theory for γ_{EQ} as small as 500 and an R as large as 386 fm.

Non-Perturbative Calculations

In view of the importance of di-lepton production and capture for RHIC luminosity considerations, we (C. Bottcher, M. Strayer + author) are calculating production and capture at RHIC energies using so-called B-spline techniques.[9] The technique has been developed and applied to non-perturbative, strong-field problems. Our approach is to use a time-dependent scattering formalism based on the Quantum Electrodynamics Language density L_{QED}, where

$$L_{QED} = \hat{\bar{\psi}} \left[\gamma_\mu(i\partial^\mu + eA^\mu) - m\right] \hat{\psi} - \frac{1}{4} F_{\mu\nu}F^{\mu\nu} - J_\mu A^\mu.$$

$A\mu$ and $F\mu\nu$ are approximated to classical electromagnetic fields, γ_μ is the usual Dirac matrix, J_μ the external nuclear current and $\psi(x)$ the lepton field of the quantum field theory. In these calculations, for each distance R and time t of the collision, the full perturbative series is effectively summed. The W-W perturbative method is contained as a subset within this non-perturbative theory. Although calculations are incomplete at this time, general physics arguments[7] and exploratory B-spline calculations suggest that the overall capture cross section will be smaller than the W-W result.

Electron capture, following beam crossing, is expected to be the largest single loss mechanism for RHIC when the nuclear charge state $\geq$ 60. A cross section of 100 barns for this process (from W-W theory) corresponds to a beam loss rate of 0.032/hour. This is not prohibitively large, considering an expected 10 hour beam lifetime.[1]

Coulomb Disassociation

Of primary importance to the beam loss of medium mass nuclei (i.e. ^{63}Cu) and of secondary importance to the beam loss of ^{179}Au, is Coulomb disassociation of nuclei from excited dipole resonance states.

Perturbative Estimates for Disassociation

This subject has been studied in the literature by several authors,[3] but once again the theory exists only for excitation probability of a single dipole resonance, in the perturbative limit. Applying these formulas to top RHIC energies the dipole cross section σ_D is 50.0 barns for ^{197}Au+^{197}Au. This corresponds to a beam loss rate of .0162/hour. Here, I critically address the accuracy of this best available nuclear theory to RHIC luminosity problems, and introduce a new "rule of thumb" to help judge the validity of this approach for future applications. Assuming that only forward scattering (scattering angle $\approx$ 0) is of interest to beam crossing, the probability amplitude a_{fi} of exciting the nuclear dipole resonance is given by,[3,10]

$$a_{fi} = \frac{8\pi}{\gamma_{EQ}} \sqrt{\frac{\pi}{3}} \frac{Z}{R} \times K_1\left(\frac{E_f R}{\gamma_{EQ}\ \beta\hbar c}\right) \times \sqrt{BE1}$$

where $K_1(x)$ is a Bessel function, E_f is the excitation energy of the nuclear dipole resonance, BE1 is the strength of the resonance and γ_{EQ} is the equivalent Lorentz factor for a fixed target ($\gamma_{EQ} = 2\gamma^2_{coll} - 1$). For values of the nuclear seperation $r < R$, the dipole potential is ignored. Using the dipole sum rules;[3]

$$E_f BE1 = 14.8\ NZA^{-1}$$
$$E_f = 78\ A^{-1/3}$$

where $A = N + Z$ and N is the neutron number, the transition amplitude becomes

$$a_{fi} = \frac{1.16}{\gamma_{EQ} R} \sqrt{\frac{\pi^3 Z^3 N}{A^{2/3}}} \times K_1\left(\frac{78R}{\gamma_{EQ}\beta\hbar c A^{1/3}}\right).$$

This quantity is plotted in Figures 1 and 2 for ^{197}Au+^{197}Au and ^{63}Cu+^{63}Cu respectively, as a function of γ_{coll}.

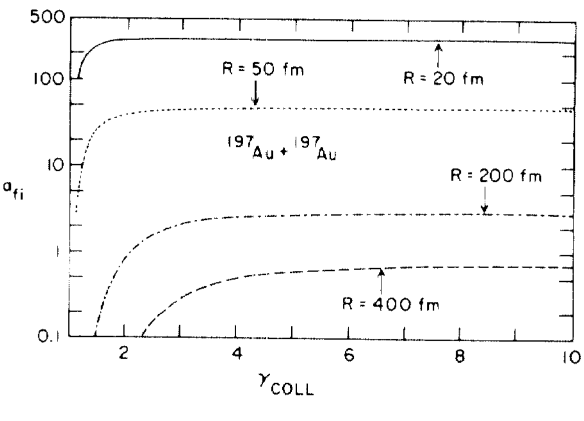

Figure 1. Graph of the probability amplitude a_{fi} for exiting the first dipole resonance in ^{197}Au, and a function of γ_{coll}. The magnitude of a_{fi} is shown for a range of R values.

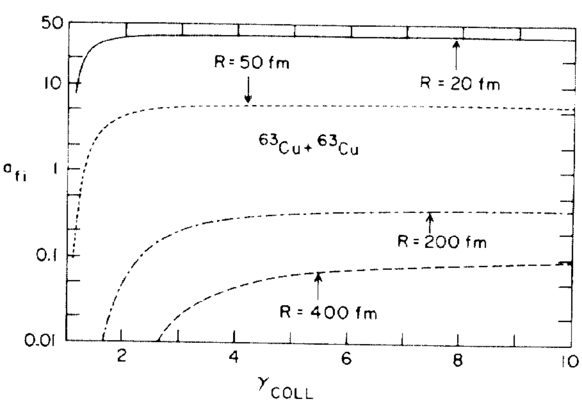

Figure 2. As figure 1 but for ^{63}Cu.

From these figures, it can be seen that a_{fi} becomes a constant at quite small values of γ_{coll}. In particular, for R = 20 fm, 50 fm I find $a_{fi} > 1$. Hence for these smaller distances the perturbative formalism for a_{fi} may not be considered accurate. Obviously, as R increases or the nuclear charge decreases the amplitude a_{fi} becomes smaller.

Rule of Thumb: Using a property of Bessel functions ($K_1(x) = x^{-1}$ $|x| < 1$) it is possible to derive a useful "rule of thumb" for the validity of the perturbative approach. In this way the value of the radius for which $a_{fi} \leq 1$ is

$$R \geq 4\ Z^{3/4}\ N^{1/4}$$

The value is independent of γ_{coll}, as figures 1 and 2 show clearly. For ^{197}Au+^{197}Au this value is R > 350 fm. For ^{63}Cu+^{63}Cu we find R > 120 fm. Typical perturbative calculations for ^{197}Au+^{197}Au start at R > 10 fm.

The consequences of these results on RHIC beam-beam interactions are very interesting. As with W-W for di-lepton production, the perturbative estimate σ_D = 50.0 barns for the first dipole resonance in ^{197}Au is expected to be an upper limit. However, for small R values the value of $2\Delta t$ is such that the electromagnetic pulse resembles a delta function, composed of an infinite number of harmonics. The result for $2\Delta t$, together with the breakdown of first order perturbation theory for σ_D, implies the strong possibility exists of exciting several dipole resonances of higher excitation energy. This is a new physical effect and has not been studied in a consistent non-perturbative formalism. However, the width of these new resonances is expected to be quite broad, and so the overall dipole cross section to the first and subsequent dipole resonances (if they exist), is not expected to be much larger than the upper limit given by the simple perturbative formalism.

Discussion

In this paper, I have critically addressed the best available formalisms for estimating loss mechanisms in RHIC during beam crossing. A brief outline of current research which will improve these estimates was also given. In addition, the inaccuracy of simple perturbative approaches at RHIC energies was emphasized, and a new "rule of thumb" was introduced to help guide the accuracy of simple perturbative formalisms. The estimates for beam loss, which are based on perturbative formalisms, are most probably an upper limit. These numbers are not considered prohibitively large for a ten hour beam lifetime.

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