CORRECTION OF RANDOM MULTIPOLE ERRORS WITH LUMPED CORRECTORS

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Abstract

A lumped-corrector scheme proposed for reducing the effects due to systematic multipole components in the dipole magnets of the SSC is studied for its effectiveness with respect to random errors in the dipoles. We have developed a simple but general and powerful method for powering the correctors for any multipole. The method applies to any beam effect that is linear in the multipole strength. This method was applied to reducing the "smear" (beam emittance distortion) by a scheme using two correctors per half cell, one adjacent to the quadrupole and one at the middle of the half-cell. The correction algorithm shows that the mid-cell corrector is generally more effective than the corrector at the quadrupole for both systematic and random errors. The smear was computed analytically for a ring made up of 320 cells, assuming the random skew and normal sextupole and octupole errors expected in the SSC dipole magnets. The effects of errors and coarseness in the corrector strengths were evaluated also. Particle tracking computations were used to check the analytic results.

1. Introduction

The lumped-corrector scheme proposed by Neuffer[1] was shown to be very effective for correcting the systematic multipole errors in the dipoles of the Superconducting Super Collider. We shall show that this corrector scheme can also be used effectively to correct the random multipole errors. The criterion for systematic multipole errors is the amplitude- and momentum-dependent tune shift that they produce, whereas the limitation due to random errors is the "smear." In spite of the different physical effects involved, it will be seen that the correction algorithms for the two types of correction are directly related.

2. The Correction Scheme

Neuffer's scheme conceptually used three lumped correctors per half cell, one inboard of each quadrupole and one in the middle of the half cell, which in the SSC contains six dipole bend magnets (Fig. 1):

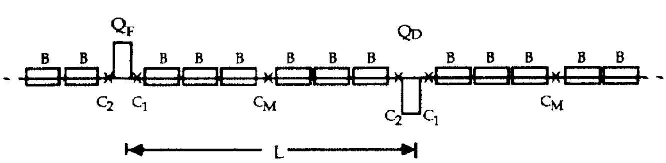

Fig. 1. The conceptual "three-lumped-correctors-per-half-cell" correction scheme. B = bend, Q = quad, C_M = mid-cell corrector, C_1 and C_2 = correctors before and after each quad.

The conceptual "three-lump" correction scheme is not very different from the more practical scheme of two lumped correctors per half cell.

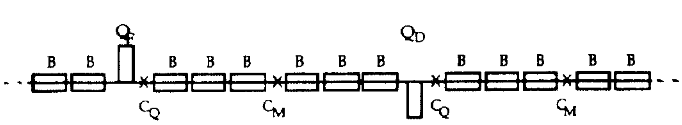

Fig. 2. The practical Neuffer scheme with two lumped correctors per half cell.

In this scheme (Fig. 2) a single corrector C_Q at each quadrupole replaces the two correctors C_1 and C_2 that straddle the quadrupole in Fig. 1.

The virtue of the conceptual three-lump scheme is that it allows the correction of the errors in all six dipoles of a half cell to be analyzed without the complication of intervening quadrupoles. Then it seems reasonable to combine the C_1 evaluated for the errors in one half cell with the adjacent C_2 (evaluated for the errors in the adjacent half cell), the total to become the C_Q of the two-lump scheme. This combination of correctors C_1 and C_2 into a single corrector C_Q has been found to produce an effective correction scheme.

3. Analysis

We shall develop a simple but general and powerful argument that can be applied to a variety of effects produced by multipole errors in the dipoles of a half cell.[2] Consider an effect E linear in the magnetic-field errors $\alpha(s)$ at points s and in the discrete set of correction fields $\beta(s)$ evaluated at the end (s_e) of the half cell. E can represent any of a variety of beam effects, such as tune shift, "smear" (to be discussed in section 4), or any other dynamical effect. The total effect E can be evaluated through an integral over the half cell of the form

$$E = \int_0^{s_e} [\alpha(s) + \beta(s)]\ T(s_e - s)\ ds \tag{1}$$

where $T(s_e - s)$ represents the effect at s_e of a unit error at s. We now expand the transform T(s) in a power series in s

$$T(s_e - s) = T_0\,[1 + A_1 (s_e - s) + A_2 (s_e - s)^2 + \cdots]$$

or its equivalent

$$= T_0\,[1 + B_1 s + B_2 s^2 + \cdots] \tag{2}$$

where T_0, A_n, and B_n are constants. Then the net effect E can be written in the form

$$E = T_0 \left\{ \int_0^{s_e} [\alpha(s) + \beta(s)]\ ds + B_1 \int_0^{s_e} [\alpha(s) + \beta(s)]\ s\ ds + B_2 \int_0^{s_e} [\alpha(s) + \beta(s)]\ s^2\ ds \cdots \right\} \tag{3}$$

If the phase advance over the half cell is small and if there are no strong focusing effects within the half cell, we can expect rapid convergence of the series and terminate with the term quadratic in s. Furthermore, we can set each of the three integrals equal to zero and thereby derive three conditions on the three corrector strengths $\beta(s)$ in terms of the six known error strengths $\alpha(s)$. Note that in this method the constants T_0, B_1, and B_2 need not be known. They will differ for the various effects represented by E (tune shifts, distortions, etc.).

Setting the three integrals to zero has a ready interpretation.[3] The first integral

$$\int_0^{s_e} [\alpha(s) + \beta(s)] = 0 \tag{4}$$

simply requires that there be no net angular deflection at the end of the half cell for a ray parallel the central orbit. The second integral

$$\int_0^{s_c} [\alpha(s) + \beta(s)]\ s\ ds = 0 \tag{5}$$

similarly requires that there be no net displacement at the end of the half cell. The third integral

$$\int_0^{s_c} [\alpha(s) + \beta(s)]\ s^2\ ds = 0 \tag{6}$$

is the first higher-order correction.

Applying Eqs. (4), (5), and (6) to an ideal six-magnet half cell of length L, we derive the following formulas for the three corrector strengths for any given multipole:

$$\beta_1 L_1 = \frac{L}{648}[-83\alpha_1 - 41\alpha_2 - 11\alpha_3 + 7\alpha_4 + 13\alpha_5 + 7\alpha_6] \tag{7}$$

$$\beta_M L_M = -\frac{L}{81}[4\alpha_1 + 10\alpha_2 + 13\alpha_3 + 13\alpha_4 + 10\alpha_5 + 4\alpha_6] \tag{8}$$

$$\beta_2 L_2 = \frac{L}{648}[7\alpha_1 + 13\alpha_2 + 7\alpha_3 - 11\alpha_4 - 41\alpha_5 - 83\alpha_6] \tag{9}$$

where α_j is the multipole error field of interest in the j^{th} dipole magnet. For the n^{th} normal multipole, for example, $\alpha = 10^{-4}\ b_n B_0$ is the average field strength of that multipole[4] at x = 1 cm in the magnet being considered. $\beta_1 L_1$ is the integral strength of corrector magnet 1, et cetera.

The corrector strength formulas (7), (8), and (9) have some interesting features. The errors closest to each corrector are weighted the most, as is expected intuitively. However, for the end correctors C_1 and C_2, the contributions due to errors in the the three farthest magnets are of the "wrong" sign.

If the six errors α_j are identical (the case of systematic errors with no variation), the three corrector strengths are in the ratios of 1:4:1, the Simpson's Rule ratio, as pointed out by Neuffer. For purely random multipole errors in the dipoles, the average corrector strengths can be evaluated by summing the coefficients in Eqs. (7), (8), and (9) in quadrature. In this case the three average corrector strengths are in the ratios 1:2.02:1. These ratios point up the relative importance of the mid-cell corrector, for both systematic and random errors.

4. Application to the SSC

The linear aperture[5] of the SSC is dominated by the random multipole errors in the dipole magnets. The random errors are the principal source of "smear," the rms variation of the first-order betatron "invariant" amplitude a.[6] To test the effectiveness of the three-lump correction scheme for random errors, we calculate the smear[7] due to the random sextupole and octupole errors (which dominate the smear in the SSC), with and without the correction scheme, including the case of errors, or coarseness, in the applied corrections. These analytic smear estimates are spot checked by tracking results. A simplified SSC lattice consisting of 320 "90-degree" cells (adjusted to give fractional tunes of 0.285 and 0.265) was used.

For such a ring the square of the rms smear in the horizontal plane r_x^2 can be expressed to lowest order in multipole strength as a quadratic sum of terms as follows:

$$r_x^2 = (s_{xa2}^2\ \sigma_{a2}^2 + s_{xb2}^2\ \sigma_{b2}^2)\ A^2$$
$$+ (s_{xa3}^2\ \sigma_{a3}^2 + s_{xb3}^2\ \sigma_{b3}^2)\ A^4 \tag{10}$$

plus corresponding terms in successively higher powers in the betatron amplitude ($A^{2(n-1)}$) if higher order multipoles are included. σ_{a2}, σ_{b2}, σ_{a3}, and σ_{b3} are the rms variances in the a_2, b_2, a_3, and b_3 multipoles components in the dipole magnets. In this calculation an equal horizontal and vertical amplitude is assumed, and it is expressed as the maximum betatron amplitude in each plane at the point of maximum beta. The calculation of the coefficients s_{xa2}, etc., for any lattice is described in SSC-95.[7] There is, of course, a corresponding expression for the rms smear in the vertical plane r_y.

Since there are no cross terms in Eq. (10), it is possible to isolate the contributions to the smear from each component:

$$r_x(a_2) = s_{xa}\ \sigma_{a2}\ A \qquad \text{(skew sextupole)}$$
$$r_x(b_2) = s_{xb2}\ \sigma_{b2}\ A \qquad \text{(normal sextupole)}$$
$$r_x(a_3) = s_{xa3}\ \sigma_{a3}\ A^2 \qquad \text{(skew octupole)}$$
$$r_x(b_3) = s_{xb3}\ \sigma_{b3}\ A^2 \qquad \text{(normal octupole)} \tag{11}$$

In Eqs. (10) and (11), smear is expressed as a fraction, each σ is in "multipole units,"[4] and the betatron amplitude A in meters.

Evaluating Eqs. (11) at A = 0.01 m and all σ = 1 gives the smear per unit multipole strength near the desired SSC aperture. The results for three cases are shown in Table I: (a) with no correction applied, (b) with a coarse, or "binned" correction, and (c) with perfect three-lump correction. To represent coarse binning plus other errors, an rms error equal to 0.20 of the average corrector strength was assumed for each corrector.

Table I. Smear contributions at 10 mm betatron amplitude per unit multipole strength.

	Smear – x			Smear – y		
Multipole	No Corr	Coarse 3-lump	Perfect 3-lump	No Corr	Coarse 3-lump	Perfec 3-lum
a_2	0.0342	0.0066	0.0024	0.0327	0.0064	0.0024
b_2	0.0287	0.0057	0.0022	0.0347	0.0067	0.002:
a_3	0.0816	0.0214	0.0160	0.0801	0.0211	0.015ε
b_3	0.0477	0.0096	0.0041	0.0453	0.0090	0.003ϵ

These tables indicates that per unit-strength the random octupoles are more effective in producing smear than the corresponding sextupoles at betatron amplitudes greater than about 5 mm.

Next we consider the smear produced by the random multipoles expected in the SSC dipoles.[8]

Table II. Expected rms variations of multipole errors in SSC dipoles in "multipole units" ($10^{-4}\ B_0$ at 1 cm).

n =	1	2	3	4	5	6	7	8
σ_{an} =	0.70	0.61	0.69	0.14	0.16	0.034	0.030	0.0064
σ_{bn} =	0.70	2.0	0.35	0.59	0.059	0.075	0.016	0.021

The smear contributions due to random decapole and higher terms have not been included because tracking studies indicate that the smear at betatron amplitudes less than 10 mm in the SSC is dominated by the sextupole and octupole errors. The contribution to the rms smear from all the higher multipoles in Table II are shown by tracking studies to be less than 0.01 at 10 mm betatron amplitude. In Table III are the smear contributions at 10 mm betatron amplitude for σ_{a2} = 0.61, σ_{b2} = 2.0, σ_{a3} = 0.69, and σ_{b3} = 0.35 units for four cases:

(a) no correction
(b) coarse correction for only b_2
(c) coarse correction for only b_2 and a_3
(d) perfect 3-lump correction of a_2, b_2, a_3, and b_3

Table III. Horizontal rms smear contributions at 10 mm betatron amplitude with expected sextupole and octupole errors

		Coarse 3-lump Correction		Perfect
	No Corr.	b_2 only	b_2 and a_3	3-lump
a_2	0.021	0.021	0.021	0.0015
b_2	0.057	0.011	0.011	0.0043
a_3	0.056	0.056	0.015	0.0110
b_3	0.017	0.017	0.017	0.0014
Total Smear	0.085	0.063	0.033	0.012

We see that the smear caused by the expected random multipole errors are dominated at a betatron amplitude of 10 mm by equal contributions from the b_2 and a_3 components. Thus, while there is an appreciable improvement in smear from correcting only the normal sextupole, the gain by correcting the normal sextupole plus the skew octupole is much more impressive.

This analytic procedure for calculating smear has been well supported by corresponding Monte Carlo tracking results.[7] In this study of the three-lump correction scheme, similar comparisons have been made. For example, in a single-cell system with only a random skew octupole error in the dipoles, the smear reduction by a perfect three-lump correction scheme was about 4.4 by the analytic method and 4.9 ± 0.8 by tracking using 21 different random seeds.

An additional contribution to the smear is produced by the systematic normal sextupole due to persistent currents if the correction is by lumped correctors. For a systematic normal sextupole of minus 6 "units" at injection corrected by the two-lump scheme in the Simpson"s Rule ratio, the resultant smear is calculated to be 0.0148 at a betatron amplitude of 10 mm. Adding this systematic contribution to the smear in quadrature to that due to the random multipoles given in Table III, the total smear for the two cases of coarse three-lump correction become 0.065 (only b_2 corrected) and 0.036 (b_2 and a_3 corrected). Thus the contribution of the persistent-current sextupole to the smear at injection is negligible with this correction scheme.

Conclusion

This study has shown that the "three-lump" correction scheme, proposed by Neuffer for correcting systematic multipole errors, is effective also for correcting random multipole errors. The random normal sextupole and skew octupole errors expected in the SSC dipoles are the principal contributors to the smear in the SSC. If only the normal sextupole is corrected by lumped correctors with coarse binning, the smear is reduced from 8.5 percent to 6.3 percent at a betatron amplitude of 10 mm. If in addition the skew octupole is similarly corrected, the smear is reduced to 3.3 percent.

References

[1] D. Neuffer, "Lumped Correction of Systematic Multipoles in Large Synchrotrons," SSC-132 (June 1987).

[2] E. Forest, "Correction by the Quasi-Local Scheme," SSC-N-366 (Neuffer) (July 1987).

[3] Suggested by Alex Chao.

[4] The normal multipole coefficient b_n is defined as the strength of the 2(n+1) multipole at x = 1 cm in units of $10^{-4}B_0$, where B_0 is the dipole field strength.

[5] *Conceptual Design of the Superconducting Super Collider*, SSC-SR-2020, edited by J. D. Jackson (March 1986) p. 129.

[6] In the horizontal plane $a_x^2 = [x^2 + (\alpha_x x + \beta_x x')^2]/\beta_x$.

[7] E. Forest "Analytical Computation of the Smear," SSC-95 (October 1986).

[8] Fisk, *et al.*, "Magnetic Errors in the SSC," SSC-7 (April 1985).