NUMERICAL COMPUTATION OF THE LONGITUDINAL COUPLING IMPEDANCE*

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Summary

A program[1] derived from the SUPERFISH cavity code is described, which computes the longitudinal coupling impedance for an azimuthally symmetric obstacle in a cylindrical beam pipe as a function of the frequency. The shape of the cavity is arbitrary, allowing one to examine the effect of rounding the corners of the obstacle on the impedance. Other effects studied are the parameters of the broad resonance as a function of the obstacle shape and the high frequency dependence of the impedance.

Introduction

In a recent paper[1] we showed that the longitudinal coupling impedance of a cavity with a beam pipe can be written as the sum of that for the beam pipe alone and a contribution from the cavity, which can be written as an integral of the fields over the surface of the cavity. A method for obtaining the fields and coupling impedance using an adaptation of the program SUPERFISH[2] was outlined.

We consider a beam pipe of cross sectional radius a and infinite length in both directions on which an azimuthally symmetric cavity-like obstacle is located. The longitudinal coupling impedance is defined as[3]

$$Z_L(\omega) = - \int \vec{J}^* \cdot \vec{E} \, dv / |I_o|^2 \qquad (1)$$

where $\vec{E}$ is the electric field in the cavity/beam pipe combination due to driving current given by

$$J_z(r,z,t) = \begin{cases} (I_o/\pi r_o^2)\, e^{i\omega z/v} & , \ r < r_o \\ 0 & , \ r > r_o \end{cases} \qquad (2)$$

The factor $\exp(-i\omega t)$ has been omitted from all fields, currents and charges.

We write the fields in the cavity/beam pipe combination (denoted by subscript 2) as the sum of the fields for the beam pipe alone (denoted by subscript 1) and $\vec{e},\vec{h}$, the field increments due to the cavity. The fields $\vec{e},\vec{h}$ satisfy the homogeneous Maxwell equations

$$\nabla \times \vec{e} = i\omega\mu\vec{h} \quad , \quad \nabla \times \vec{h} = - i\omega\epsilon\vec{e} \ , \qquad (3)$$

as well as the wall boundary condition

$$\vec{n}_2 \times \vec{e} = - \vec{n}_2 \times \vec{E}_1 \ \text{on surface} \ S_2, \qquad (4)$$

where $\vec{n}_2$ is a unit vector normal to the cavity/beam pipe wall surface S_2.

Equations (3) and (4) represent an equivalent SUPERFISH problem, with specified frequency and boundary conditions. In addition to the boundary conditions of eq. (4), we need to solve the problem of the boundary conditions at the pipe-cavity interface. The problem is solved by using very long pipes filled with a slightly conductive medium (simulated with a very small imaginary part in ϵ). Physically this is equivalent to terminating the pipes with "matching loads". The numerical problem of handling the very long pipes is manageable because the perfect periodicity of the mesh in the pipe allows to solve the problem by cyclic reduction. At each step of the process the field on "odd" numbered columns is eliminated giving equations for only the even columns. This gives a logarithmic solution time. As an inessential detail the pipe is in fact formally considered periodic, but in the limit of a very long pipe the communication between left and right sides disappears. What is left are modified equations for the last columns inside the cavity, involving complex matrices that correctly describe the outgoing wave conditions. Since the matrices inside the cavity are real, we perform the solution of the cavity problem by Gaussian elimination leaving the first and last column to the end. In this way all steps but the last are performed using only real matrices.

Longitudinal Impedance

By using Maxwell's equations we showed[1] that the increment in coupling impedance due to the cavity could be written as

$$\Delta Z_L = \frac{1}{|I_o|^2} \int_{S_2 \neq S_1} dS \ \vec{n}_2 \times \vec{E}_1^* \cdot \vec{H}_2 \qquad (5)$$

where the surface integral is evaluated only over that part of the cavity which differs from the beam pipe. Further analysis shows that the contribution to Eq. (5) can be split into two parts by using

$$\vec{H}_2 = \vec{H}_1 + \vec{h} \qquad (6)$$

and that the contribution from $\vec{H}_1$ is imaginary, and inversely proportional to γ^2. In the relativistic limit, one then can write ΔZ_L as a line integral

$$\Delta Z_L \cong - \frac{Z_o}{I_o} \int_{s_2 \neq s_1} r dr \, e^{-i\frac{\omega z}{c}} h_\phi(r,z), \qquad (7)$$

where we have used

$$E_{1r} = \frac{Z_o I_o}{2\pi r} e^{i\frac{\omega z}{c}} \qquad (8)$$

and where $Z_o = \sqrt{\mu_o/\epsilon_o}$ is the impedance of free space.

Applications

The program we have written has been tested for the case of a cylindrical pillbox with the semi-analytical results at Henke[4], giving good agreement

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(see ref. 8). As a further test we tried to reproduce the $(k)^{-1/2}$ behavior predicted for the large frequency limit (see refs. 9 and 10). The results are shown in Fig. 1 and 2 for the cavity considered in ref. 9. The impedance in the $k \to \infty$ limit for a pillbox cavity is given by the expression

$$Z = \frac{Z_o}{2\pi} \cdot \sqrt{\frac{g}{\pi a}}\, \frac{1-i}{\sqrt{ka}} \, , \tag{9}$$

for a cavity of length g. a is the pipe radius. Eq. 9 is independent from the radius of the pillbox. In our case g = b−a = .5a, where b is the pillbox radius. Eq. 9 predicts a value for the coefficient of $(ka)^{-1/2}$ of 23.92. A least square fit to a ~ $ka^{-1/2}$ law is shown as a dashed line of fig. 1 and 2. The coefficients of the real and imaginary part are 24.03 and −26.45. The reasonable agreement shows that we can reproduce the analytic result in a region where many modes propagate. Also note that our detailed plots differ from the ones in ref. 9. We believe our results are correct in view of the approximations used in the numerical calculations in that paper. The main feature of our program is the ability to compute the impedance for a cylindrically symmetric cavity of otherwise arbitrary shape. As an example fig. 3 shows the section of a free electron laser injector linac cavity considered at Los Alamos. Figures 4 and 5 show the real and imaginary part of the longitudinal coupling impedance as computed by our program. Below the cut-off frequency of 9500 MH_z (the pipe diameter is 1.2 cm), the real part of the impedance vanishes, while the imaginary part shows simple poles at the resonant frequencies of the cavity (the lowest is at 1300 MH_z). Above cut-off the picture is very complex.

As a test of the effect of rounding the corners of an obstacle, we have computed the impedance for the two obstacles of fig. 6. Figure 7 shows the real part of the impedance for the pillbox with square corners, fig. 8 for the rounded obstacle. The conclusion seems to be that rounding corners does not improve the longitudinal coupling impedance in any significant way.

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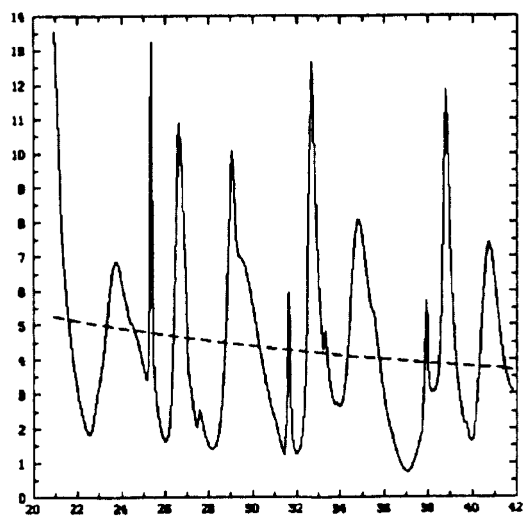

Fig. 1. Real part of the impedance (in Ohms) for a pillbox cavity of length g = .5a and radius b = 1.5a. Horizontal scale is ka, which is dimensionless. The dashed curve is the fit to the $(ka)^{-1/2}$ behavior for large k, as discussed in the text.

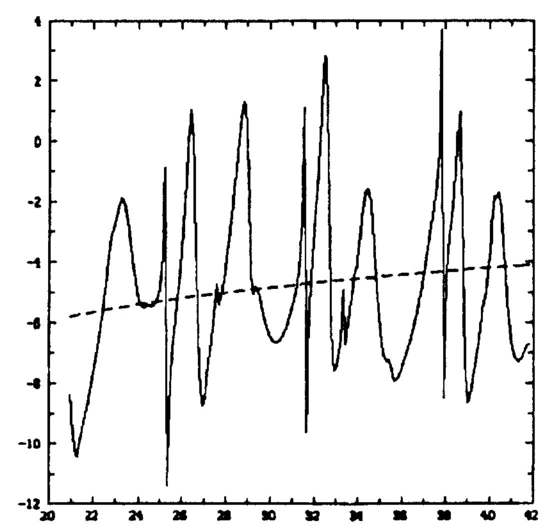

Fig. 2. Imaginary part of the impedance for the cavity of Fig. 1.

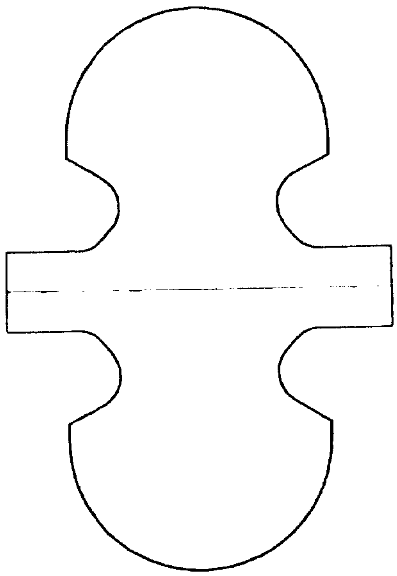

Fig. 3. Section of the FEL linac cavity. The radius of the pipe is a = 1.2 cm.

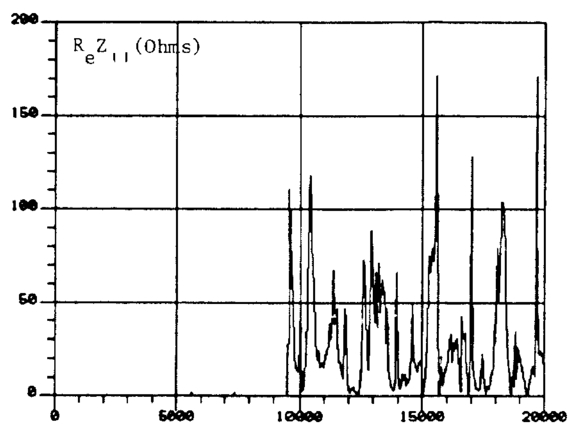

Fig. 4. Real part of the longitudinal coupling impedance for the cavity of Fig. 3. Horizontal axis is frequency in MH_z.

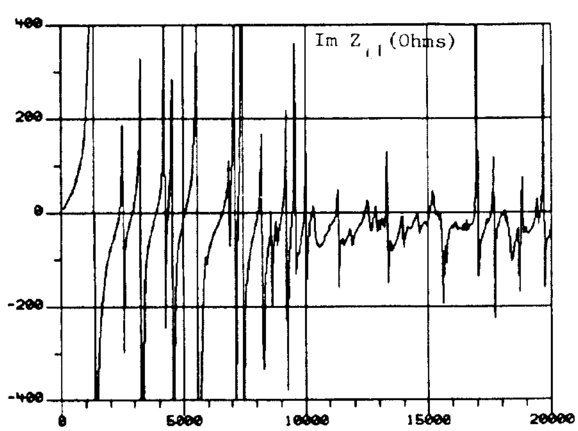

Fig. 5. Imaginary part of the impedance for the cavity of Fig. 3.

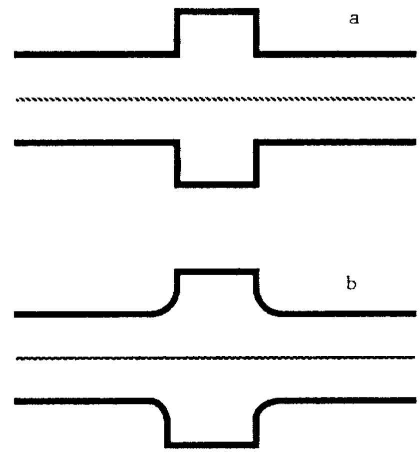

Fig. 6. The top obstacle has length g = 2a, the bottom has the same length and smooth inner corners with radius r = 0.5a.

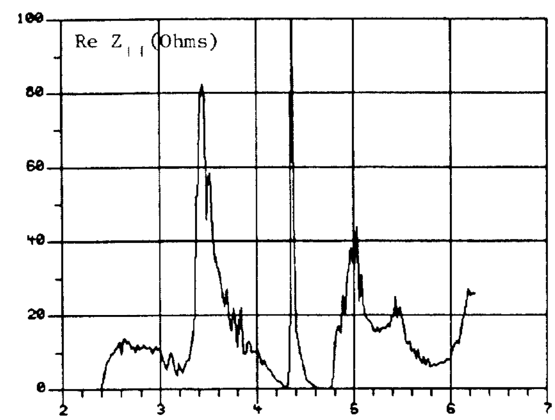

Fig. 7. Real part of the longitudinal coupling impedance for the pillbox of Fig. 6a.

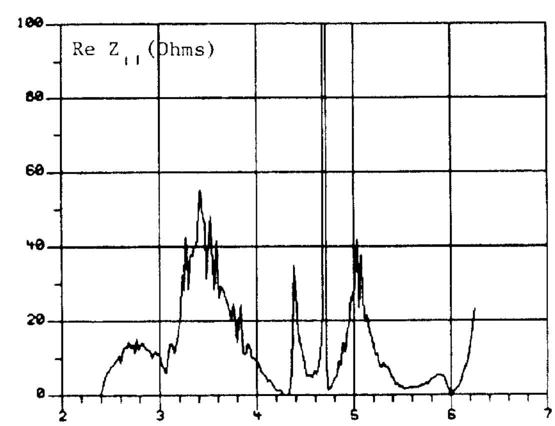

Fig. 8. Real part of the longitudinal coupling impedance for the rounded obstacle of Fig. 6b.