# ACHROMATIC LATTICE COMPARISON FOR LIGHT SOURCES<sup>\*</sup> S. L. Kramer, E. A. Crosbie, and Y. Cho Argonne National Laboratory 9700 S. Cass Avenue

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## Introduction

The next generation of synchrotron light sources are being designed to support a large number of undulators and require long dispersion-free insertion regions. With less demand for radiation from the dipole magnets, the storage ring cost per undulator beam can be reduced by decreasing the number of dipole magnets and increasing the number of dispersion-free straight sections.

The two simplest achromatic lattices are the Chasman-Green<sup>(1)</sup> or double-bend achromatic (DBA) and the three-bend achromat (TBA). The DBA in its simplest form consists of a single horizontallyfocussing quadrupole between the two dipole magnets. Since this quadrupole strength is fixed by the achromatic condition, the natural emittance ( $\epsilon_n$ ) may vary as the beta functions in the insertion region (IR) are varied. The expanded Chasman-Green (also DBA) uses multiple quadrupoles in the dispersive section to provide emittance control independent of the beta functions in the IR. Although this provides flexibility in the ID beta functions, the horizontal phase advance is constrained to  $\phi \approx 180^\circ$  between approximately the centers of the dipole magnets. If small  $\varepsilon_n$  is required, the horizontal phase advance between the dipoles<sup>(2)</sup> will be near one and the lattice properties will be dominated by this systematic resonance. The TBA lattice places a third dipole between the DBA dipoles, eliminating the 180° horizontal phase advance constraint. However, the requirement of small  $\varepsilon_n$  limits the range of tune, since  $\mu_x \approx 1.29$  in the dipoles alone for  $\varepsilon_n$  near its minimum value. The minimum emittance is five times smaller for the TBA than for the DBA with the same number of periods and, therefore, its phase advance can be relaxed more than the DBA for the same natural emittance. Similar to the DBA lattice, a single quadrupole can be placed between each of the dipoles (2 per period) of the TBA, but a pair of doublets provides greater control of the beta functions in the IR without changing  $\varepsilon_n$ .

#### Linear Lattice Comparison

The requirements for the 7-GeV Advanced Photon Source lattice<sup>(3)</sup> are:  $E_0 = 7$  GeV,  $\varepsilon_x \approx 7$  nm,  $\beta_{x,y}(IR) \gtrsim 10$  m,  $\rho \approx 39$  m and at least 32 6-m-long dispersion-free straight sections. A DBA lattice with adequate flexibility was proposed and is shown in Fig. 1(a). In order to reduce the sensitivity to closedorbit errors and to increase the dynamic aperture, the phase advance was relaxed to  $\mu_x = 0.88$  per period and the number of periods increased to 40. This provided a natural emittance of 7.9 nm (3.5 times minimum) and adequate beta function tune range for the IR regions. The lattice requires 10 quadrupoles per period powered



**Figure 1.** Lattice Parameters  $\beta_x$ ,  $\beta_y$ , and  $n_x$  (dispersion function) for one-half period (symmetry point) of (a) the DBA lattice and (b) the TBA lattice compared in this study.

in 5 families. The vertically focussing quadrupoles in the dispersion region could have been included in the dipole magnet, but was ruled out because it reduces the tunability (e.g., emittance) of the lattice and makes construction and alignment of these long dipole magnets difficult.

Figure 1(b) shows the TBA lattice which meets the lattice requirements for the APS. For low emittance, the central dipole must have a horizontal waist at its center, requiring considerable horizontal focussing. As with the DBA lattice, vertical focussing is provided by separate quadrupoles rather than with gradients in the dipole magnets. The four quadrupoles in the dispersion section are powered as a triplet, with the central quadrupole split into two to provide a more suitable location for a chromaticitycorrecting sextupole. The natural emittance for this lattice is 7 nm (8.3 times the minimum).

Both lattices have the same non-integer tunes, but the higher tune per period of the TBA required a longer period length in order to maintain reasonable beta functions and quadrupole strengths. This resulted in approximately the same circumference as a 32-period TBA as for the 40 period DBA. The advantage of this larger number of insertion regions for light source users is obvious. Each lattice has similar flexibility for tune, emittance and beta function changes. Both lattices can have their IR region extended from 6 to 8 m by removing the first quadrupole magnet on either side of the 6-m IR and retuning the remaining doublet. Table 1 compares the parameters for these lattices.

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Lattice Parameter	DBA	тва
Circumference	1060 m	1088 m
Revolution Time, T	3.5358 µa	3.629 µs
Energy, E	7 GeV	7 GeV
No. of Insertion Regions	40	32
Length of Insertion Region	6.2 m	6.0 m
Dipole Length	3.06 m	2.6 m
Dipole Field	0.599 T	0.587 T
Bend Radius, p	38.9611 m	39.73 m
Maximum Quadrupole Strength	18.9 T/m	17.34 T/m
No. of Dipoles	80	96
No. of Quadrupoles	400	448
Tunes, Vy and Vy	35.22, 14.30	37.22, 15.30
Transition Gamma, Yr	64.91	52.5
Momentum Compaction Factor, a	2.374×10 <sup>-4</sup>	3.627×10 <sup>-4</sup>
Chromaticities, 5 <sub>x</sub> , 5 <sub>y</sub>	-63.7, -26.0	-54.429.8
Chrom. Corr. Sextupoles	$3.92, -4.27/m^2$	$2.99 - 3.88/m^2$
Number of Chromatic Sextupoles	120	192
Number of Harmonic Sextupoles	160	-
Maximum Dispersion	0.39 m	0.28 m
Maximum $\beta_x$ and $\beta_y$	22.3, 19.8 m	16.7, 19.2 m
Natural Emittancé, E	8×10 <sup>9</sup> ■	7.0×10 <sup>-9</sup> m
Transverse Damping Time, Ty = Ty	9.08 ms	9.5 m.s
Synchrotron Damping Time, Tg	4.54 ms	4.7 ms
Bending Magnet Critical Energy, c	19.5 keV	19.2 keV
Energy Loss per Turn, U	5.45 MeV	5.35 MeV
Radio Frequency, frf	352.96 MHz	352.7 MHz
Harmonic Number, h	1248	1280
Natural Energy Spread, o <sub>R</sub> /E	0.096%	0.095%
Horizontal Beam Size in Undulator		
(zero coupling) o <sub>x</sub>	0.32 mm	0.29 mm
Vertical Beam Size in Undulator		
(100% coupling) oy	0.20 mm	0.16 mm

Table 1Lattice Parameters of the DBA and TBALattices Compared in this Study

### Non-Linear Lattice Comparison

The large chromaticity of both lattices requires strong chromatic-correcting sextupoles. For the DBA, these sextupoles have been shown to drive  $v_x = 1$  and  $3v_x = 3$  (per period) resonances, which severely limit the dynamic aperture (4) and increase its sensitivity to closed-orbit errors. Consequently, two pair of harmonic sextupoles have been added to the non-dispersive section and tuned to suppress the tune shift with horizontal betatron amplitude resulting from the  $v_x = 1$  and  $3v_x = 3$  resonances. The resulting horizontal dynamic aperture<sup>(5)</sup> has increased about a factor of two and the vertical by 50%. The dynamic aperture for the harmonic corrected DBA lattice is shown in Fig. 2(a). The dynamic aperture for the TBA is shown in Fig. 2(b) and although somewhat smaller than the DBA lattice, it is more symmetric in the horizontal plane, indicating less nonlinear distortions of the phase space. The dynamic aperture for the TBA doesn't appear to be limited by one or two low-order resonant terms and therefore was not significantly increased by adding harmonic sextupoles.

The effect of tuning the harmonic sextupoles is shown in Fig. 3(a) for the DBA lattice. The actual tune of the harmonic sextupoles corresponds to 70% of the values required to balance the  $v_x = 1$  and  $3v_x = 3$ resonant terms. This compensates for the higher-order tune shifts that are opposite in sign from the loworder resonant terms, and are similar in magnitude to the TBA values shown in Fig. 3(b).

The effect of magnet field and alignment errors on the dynamic aperture have also been considered for these lattices. Those errors which induce a closed-orbit distortion have been specified by defining a tolerance level as:  $\Delta x$ ,  $\Delta y$ , the quadrupole alignment accuracy in meters;  $\frac{\Delta(Bk)}{Bk}$ , the



**Figure 2.** The dynamic aperture for (a) the harmonic corrected DBA lattice and (b) the TBA lattice. The solid line shows the dynamic aperture for the ideal lattice. The data points indicate the average and rms spread of the dynamic aperture of the lattice for 11 random machines with a  $10^{-4}$  closed-orbit tolerance level.

dipole field strength accuracy; and  $\Delta\phi$ , the dipole roll angle accuracy in radians. This tolerance level was then scaled as a whole to study the reduction of the dynamic aperture from the ideal lattice. This reduction of dynamic aperture with tolerance level is presented in Fig. 4. Although both lattices have significant reduction, only the DBA has sufficient aperture in which to inject at the state-of-the-art level of  $10^{-4}$ , as shown in Fig. 2. Both lattices were easily corrected with dipole correction magnets to a dynamic aperture and natural emittance which differed by less than 10% from the ideal lattice.

The effect on the dynamic aperture of magnet-to-magnet field strength errors for the quadrupole and sextupole magnets were also studied. The reduction of the dynamic aperture for the DBA lattice was about 15% for a  $10^{-3}$  quadrupole field accuracy, and a similar reduction was observed for a  $10^{-2}$  sextupole accuracy. The TBA lattice had a similar reduction of the dynamic aperture for a 3 x  $10^{-4}$  quadrupole field tolerance and a  $10^{-2}$  sextupole field tolerance.



**Figure 3.** The calculated tune shift versus the horizontal betatron amplitude obtained from tracking for (a) the harmonic corrected DBA and (b) the TBA lattice. The solid curve is for y = 0 and the dashed curve for  $y \approx 6$  mm.

#### Conclusion

The expanded DBA and TBA lattices compared here have similar linear properties for synchrotron light sources. The nonlinear properties of the DBA, with harmonic sextupole correction, provide a larger dynamic aperture and reduced sensitivity to closedorbit distortions compared to the TBA lattice. Harmonic correction of the TBA lattice was not found to yield significant improvement in the dynamic aperture. The higher tune shift per period for the TBA required a longer period length and therefore fewer undulator straight sections for a given circumference. Although the TBA lattice is capable of an emittance, 2.5 times less than the DBA, this was not found to be significant for this application. The 25% more IR's of the DBA and its reduced sensitivity to errors makes the DBA the lattice of choice for the 7-GeV Advanced Photon Source.

### References

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Figure 4. The reduction of the dynamic aperture for (a) the harmonic corrected DBA lattice and (b) the TBA lattice as a function of closed-orbit tolerance level. The dynamic aperture is expressed in normalized units for the coupling line  $N_x = N_y$ , where  $N_x = X/\sigma_x$  and  $N_y = Y/\sigma_y$  for  $\sigma_x$  and  $\sigma_y$  given in Table 1. The circle data points are the average and rms value for 11 random machines and the diamonds are for 4 random machines with closed-orbit corrections applied.

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