LOSS PARAMETERS FOR VERY SHORT BUNCHES

J. J. Bisognano, S. A. Heifets, B. C. Yunn
Continuous Electron Beam Accelerator Facility,
12000 Jefferson Avenue, Newport News, VA 23606

Abstract

Semi-empirical formulas for the transverse and longitudinal loss factors generated by cavity and step discontinuities are given in the limit of short bunch length. Effects of tapering and the cross-talk between cavities are discussed. The formulas are compared with the numeric code TBCI.

The short bunch lengths common in recent designs require a thorough evaluation of the loss factors for the next generation of accelerators. In particular, the problem is important for CEBAF, where the bunches are only 0.25 mm long. However, the estimation of the loss factors with available numerical codes such as TBCI is not straightforward because of the limitation on the number of mesh points, which becomes very large for short bunches. The modal analysis, efficient for calculations of the narrow-band impedance, is not adequate in the high-frequency limit where the impedance is a smooth function of frequency. Analytic results are controversial: the Vainstein-Sessler optical resonator model[1] predicts that impedance decreases with frequency as $\omega^{-3/2}$, whereas recent results[2],[3],[4] give rather $\omega^{-1/2}$ dependence.

The main contribution to the loss factors of typical machines is given by elements of the system which can be approximately described as pill-box cavities with attached tubes, or as discontinuities of the beam-pipe cross-section due to an abrupt change of radius. We refer to these two basic elements as a cavity and a step. In this note we give handy analytic expressions for loss parameters for these two cases, compare them with results of numerical simulations with TBCI, and give the range of their applicability. At the end we discuss how tapering reduces the loss factor of a step.

For a single pill-box cavity with width g, radius b, and attached tubes with radius a, the impedance in high frequency limit is [3],[4]:

$$Z_l(\omega) = \frac{Z_0}{2\pi}\sqrt{\frac{g}{\pi a}}\frac{1}{\sqrt{ka}} \tag{1}$$

In this case the high-frequency tail is dominant and gives for the energy loss k_l

$$k_l(\sigma) = \frac{\Gamma(1/4)Z_0 c}{4\pi^2 a}\sqrt{\frac{g}{\pi\sigma}}, \qquad \frac{\Gamma(1/4)}{\pi} = 1.154 \tag{2}$$

where σ is rms bunch length. This formula was discussed also by K. Bane.[5]

The transverse impedance is built from a large number of modes, each of which satisfy the Panofsky–Wenzel theorem. If the longitudinal impedance of the transverse deflecting mode scales as $\omega^{-1/2}$, then the transverse impedance scales as $\omega^{-3/2}$. With this argument the transverse loss $k_\perp = < W_\perp^\delta/r >$ for a pill-box cavity with attached tubes may be written as

$$k_\perp = \frac{1}{a^3}\sqrt{\pi g\sigma} \tag{3}$$

We checked Eq. (2) with the code TBCI with parameters chosen to be close to CEBAF parameters for: the fundamental power couplers (a = 3.5 cm, g = 2.5 cm, cavity radius b = 5.5 cm), the higher order mode couplers (a = 3.75 cm, g = 3.75 cm, b = 5.5 cm), and the gate valves (a = 1.75 cm, g = 2 cm, b = 3.5 cm). For all three cases the dependence of the average loss vs. the rms bunch size in the range σ = 0.75 mm – 1.5 mm corresponds to Eq. (2) and numerical agreement is within 10% accuracy. Calculations with TBCI confirmed Eq. (3) with the same accuracy. (See Fig. 1.)

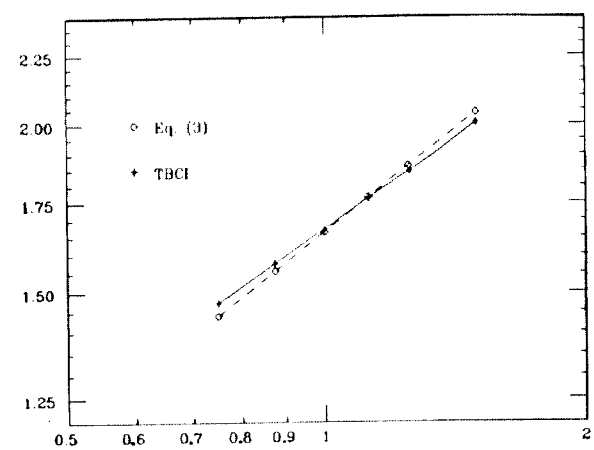

Figure 1

The transverse loss factor (V/pC/m) vs. σ (mm) for the fundamental power coupler, a=3.5, g=2.5, b=5.5 (cm).

For a wider range of parameters, in particular, for very long cavities, Eqs. (2) and (3) are not applicable. In this case we can expect a transition to formulas for impedances generated by a single discontinuity of the beam pipe radius. The energy loss per particle for a step was given by V. Balakin and A. Novokhatsky[6] and later studied in a semianalytical model.[7] If a particle enters a narrow pipe the impedance is negligibly small. Substantial impedance is generated only when a bunch traverses from the narrow side of the step to the wide side. The impedance, in this case, is approximately independent of frequency:

$$Z(\omega) = \frac{Z_0}{\pi}\ \ln\ \frac{b}{a},$$

where Z_0 = 377 ohm is the impedance of free space, b and a are the radii of the wide and narrow pipes, respectively. Notice that this frequency independent impedance corresponds to a δ-function wake $W^\delta(s)$.

The behavior for the average losses in this case is

$$k_l = \frac{2}{\sigma\sqrt{\pi}} \ln b/a, \tag{4}$$

$$k_\perp = \frac{2}{a^2\sqrt{\pi}} \ln \frac{b}{a} \ln \frac{b}{\sigma}. \tag{5}$$

These formulas depend on both radii, in contrast to Eqs. (2) and (3). The transition from the regime of a cavity to the regime of a step depends on whether or not the signal from the wall of a cavity can reach a bunch while it is within the cavity. For $a \approx b$, this transition is defined by the ratio of the width of the cavity g to the parameter $(b-a)^2/2\sigma$. If g is bigger than this parameter, the situation is similar to that for a step, otherwise Eqs. (2) and (3) are applicable. We checked this statement numerically with TBCI. The dependence of the losses on the pill-box cavity radius b, given in Fig. 2, clearly indicates the transition from one regime to another at

$$p = 2\sigma g/(b-a)^2 \approx 1 \tag{6}$$

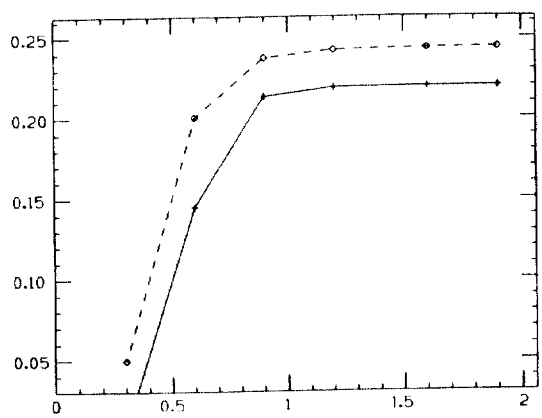

Figure 2

The longitudinal (s) loss factor (V/pC) and the transverse (d) loss factor (V/pC/m) vs. b (m) σ=0.06, a=0.25, g=6.0, L_{tot}=7.0

The more general formula for the transition parameter p, which coincides with Eq. (6) for $g \gg a \approx b$, was given by Wilson.[8] Numerically, the b-dependence is in agreement with Eqs. (2-5). Figure 3 shows dependence of the losses on rms bunch size.

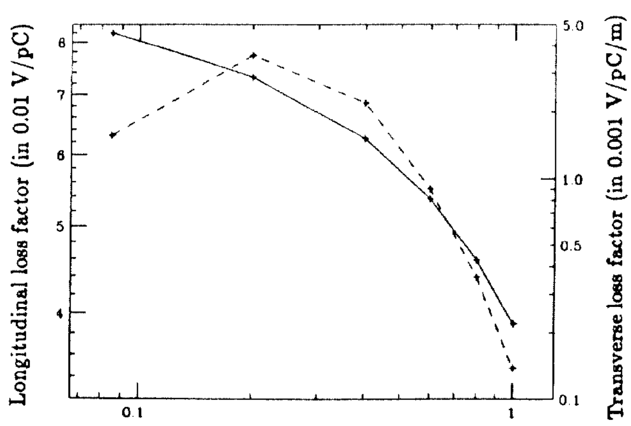

Figure 3

Transition from the regime of the cavity to the regime of a step. The longitudinal (s) and the transverse (d) loss factors vs. σ (m). a=1.25, b=2.50, g=8.8, L_{tot}=15.5

If a beam pipe is tapered, the losses become smaller. There are no analytic results available on the effect of tapering. We studied the effect numerically with TBCI for a long pill box cavity with attached tubes, with parameters $a = 0.01$ m, $b = 0.03$ m, and $g = 1.0$ m. The transition from one radius to another was described by the function

$$r(z) = 0.5\left\{b + a - (b-a)\tanh\left(\frac{|z| - g/2}{\delta}\right)\right\}$$

with δ in the range from 0.0 to 0.10 m. The result for the longitudinal and transverse wakes, averaged with a Gaussian bunch, are shown by the solid line in Fig. 4 for σ = 0.6 cm and for σ = 0.25 cm. The δ-dependence is steeper for larger σ. The maximum value for the longitudinal wake, which affects the energy spread, is about two times larger than the average value and it depends on δ similarly to the loss factor. The same is true for the maximum value of the transverse impedance. Figure 4 shows that tapering can decrease loss factors by several times.

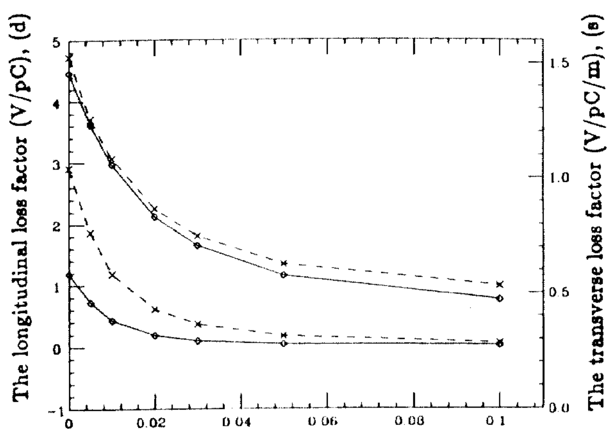

Figure 4

The longitudinal (sld) and transverse (dsh) loss factors vs. the width of a step (m), a=0.01, b=0.03 (m). Upper curves correspond to σ=0.0025, lower curves to σ=0.0060

The contradiction between the optical resonator model and the results for a single cavity might be understandable if the result of the optical resonator model is valid for a periodic structure only. Today this problem has not been rigorously solved. The related problem, of how the impedance scales with the number of adjacent cavities, has not been answered either. We compared the losses for a 6-convolution bellows with a pitch 2.1 mm, a = 1.74 cm, and b = 2.26 cm, with the total loss factor of six independent single pill-box cavities with attached tubes. Each cavity simulates a single convolution. The losses of the bellows, given by TBCI for σ = 0.25 mm, are k_l = 3.56 V/pC and $k_\perp$ = 8.28 V/pC/m. Equations (2) and (3) give correspondingly 5.81 V/pC and 13.05 V/pC/m. Hence, longitudinal and transverse losses for a bellows with 6 convolutions are about 40% less than that for 6 independent cavities.

Conclusion

The main results of the paper are given in Eqs. (2-5). The loss parameters for a cavity and a step given in these formulas are in good agreement with numerical results given by TBCI. These expressions provide simple but reasonable estimates of the loss factors of short bunches.

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