Proton Emittance Growth Caused by Electron rf-Noise and the Beam-Beam Interaction in HERA

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Abstract

Due to nonvanishing dispersion in the rf-sections of the HERA electron ring, noise in the accelerating field excites coherent transverse oscillations. The effect of these excitations on the proton beams via the beam-beam force is studied in a simple analytical approach. From measurements of the noise spectrum of the PETRA rf-system, an estimate for the proton emittance growth rate is derived. It is shown that it can be larger than 1 hr^{-1} unless the tunes of the collider are not carefully chosen.

Introduction

In the HERA collider presently under construction at DESY, protons at an energy of up to about 1 TeV will collide with electron (or positron) bunches of 30 GeV. One of the effects which may limit the maximum achievable luminosity in storage rings is the beam-beam interaction ("space charge effect"). Usually this limit is expressed in terms of the maximum tolerable linear tune shift, ΔQ, caused by the space charge force. The tune shift limit is known to be about an order of magnitude smaller for protons than for electrons. This is reflected in the ΔQ-values for HERA (see Table 1). According to the generally accepted "typical" limits of $\Delta Q_{max} = 5 \cdot 10^{-2}$ for electrons and $\Delta Q_{max} = 5 \cdot 10^{-3}$ for protons, the HERA design appears to be rather conservative. However, in a two-ring machine like HERA, effects usually not present in a single-ring collider can become important. A striking example is the proton beam blow-up caused by nonlinear synchro-betatron resonances when the electron and proton beams cross at an angle. This was extensively studied by Piwinski[1], and it led to the decision for a head-on interaction geometry in HERA[2,3]. Other effects which may reduce the proton beam lifetime can be caused by transverse displacements of the colliding bunches with respect to each other (e.g. induced by ground motion waves[4]).

	e	p	
Energy	30	820	GeV
Number of Particles per bunch	$3.5 \cdot 10^{10}$	10^{11}	
β-function at I.P. $\beta^*_{x/z}$	2/0.7	10/1	m
Beam size at I.P. hor./vert.	0.20/0.030	0.24/0.07	m
Beam-beam tune Shift hor./vert.	0.017/0.019	0.0022/0.0017	
Luminosity	$2 \cdot 10^{31}$		$cm^{-2}s^{-1}$

Table 1: HERA interaction parameter

The effect I am considering in this paper is caused by coherent transverse oscillations of the electron bunches. The oscillations are excited by noise in the electron rf-cavities where the dispersion function cannot be made zero due to the specialities of the HERA electron ring lattice (see refs.[2,3]). A simple analytical description of the excitation mechanism is given in the following section. Also, the results of experimental studies at PETRA are given. In the subsequent section, the emittance growth of the proton beam due to the coherent electron oscillations is discussed on the basis of a strongly simplified model. The analytical description is compared to the results of a computer simulation study. Finally some conclusions are drawn.

Coherent Excitation of Electrons

In the description of the excitation of coherent oscillations driven by noise, I make the following simplifying assumptions:

- the localization of the cavities in the ring is ignored; the beam is assumed to behave like a simple harmonic damped oscillator in both the longitudinal and horizontal plane. The tune is assumed "far" away from synchro-betatron resonances and the (small) longitudinal-transverse coupling due to the dispersion in the cavities is ignored.
- the rf-noise in the cavities has a "white" frequency spectrum (the noise is uncorrelated at different times).
- any effects of the beam-beam interaction on the **electron** motion are ignored.

The equations of coherent synchrotron and betatron oscillations can then be written as:

$$\frac{d^2}{dt^2}\frac{\Delta p}{p} + 2\delta_s \frac{d}{dt}\frac{\Delta p}{p} + \omega_s^2 \frac{\Delta p}{p} = \frac{eU_{rf}}{E_o}\frac{d^2}{dt^2}\delta\varphi(t) \qquad (1)$$

$$\frac{d^2}{dt^2}x_e + 2\delta_x \frac{d}{dt}x_e + \omega_x^2 x_e = (< H > \beta^*_{x,e})^{1/2}\frac{d^2}{dt^2}\delta\varphi(t) \qquad (2)$$

where

ω_s:	synchrotron frequency	ω_x:	(fractional part of) betatron frequency
δ_s:	longitudinal damping constant	δ_x:	transv. damping constant
U_{rf}:	rf-voltage	E_o:	electron energy
$\delta\varphi(t)$:	rf-phase noise	$\beta^*_{x,e}$:	β-function at I.P.

$< H > = < D^2/\beta + 2\alpha DD' + \beta D'^2 >$ (average over cavity section)

From eqs. (1,2) the correlation functions for coherent motion can be derived:

$$< \frac{\Delta p}{p}(t+\tau)\frac{\Delta p}{p}(t) >= \left(\frac{\Delta p}{p}\right)^2_{rms} exp(-\delta_s\tau)cos(\omega_s\tau) \quad (3)$$

$$< x_e(t+\tau)x_e(t) >= (x_e)^2_{rms} exp(-\delta_x\tau)cos(\omega_{x,e}\tau) \quad (4)$$

with

$$\left(\frac{\Delta p}{p}\right)_{rms} = \frac{1}{2}\omega_s\frac{eU_{rf}}{E_o}\left(\frac{P_\varphi}{\delta_s}\right)^{1/2} \quad (5)$$

$$x_{e,rms} = \left(\frac{\Delta p}{p}\right)_{rms}\frac{\delta_s}{\delta_x}(< H > \beta^*_{x,e})^{1/2} \quad (6)$$

where P_φ is the frequency power spectrum of the noise of the rf-phase, which is assumed independent of frequency (at least over a range of frequencies corresponding to "typical" synchrotron and betatron tunes).

The rf-noise of the PETRA rf-system was measured with a spectrum analyser in the range of the synchrotron frequency ($\omega_s = 44.6$ KHz), yielding $P_\varphi = 10^{-(10\pm0.5)}\ Hz^{-1}$ [5]. Inserting the other parameters of PETRA at $E_o = 7$ GeV into eq. (5) gives a predicted coherent amplitude of $(\Delta p/p)_{rms} = (6\pm3)\cdot10^{-5}$ (about 0.1 standard deviation of momentum spread). The coherent excitation of the beam was measured with a longitudinal pick-up. From the signal (see Fig. 1), a coherent amplitude of $(\Delta p/p)_{rms} = (3\pm1)\cdot10^{-5}$ was derived, which is in agreement with the value computed from the noise spectrum within the limits of error.

Under the assumption that in the HERA electron ring the rf-noise level is the same as in PETRA, the rms-amplitude of coherent β-tron oscillations at the interaction point can be estimated. Scaling the measured longitudinal amplitude in PETRA according to the parameters of HERA at 30 GeV, the result is

$$x_{e,rms} = 2.2\cdot10^{-6}\ m \quad (7)$$

which corresponds to about one percent of the half-beam width.

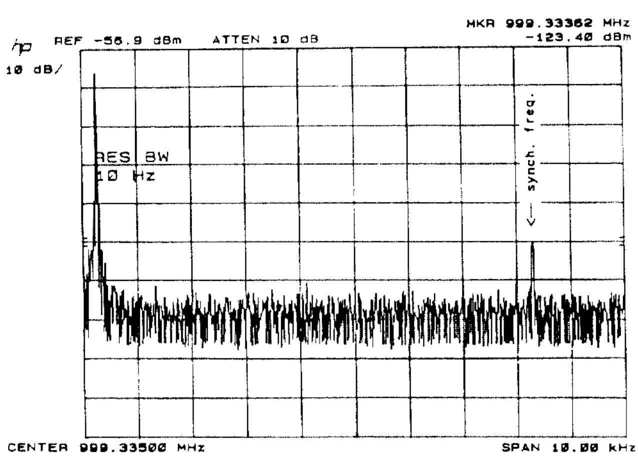

Figure 1: Frequency spectrum of the longitudinal pick-up signal in PETRA.

Proton Emittance Growth

In order to obtain an estimate for the proton emittance dilution caused by the randomly excited, coherently oscillating electron beam, we start from the continuous approximation for proton β-tron oscillations:

$$\frac{d^2}{dt^2}x_p + \omega_p x_p = 2\Delta\omega_p\omega_p f(x_p, z_p; x_e(t)) \quad (8)$$

where $\Delta\omega_p = 2\pi\Delta Q_{x,p}/T_o$ (T_o: rotation period) is the frequency shift caused by the linear part of the space charge force ($f_{lin} = x_p - x_e(t)$), and $f(x_p, z_p; x_e(t))$ describes the (nonlinear) dependence of the beam-beam interaction on the horizontal and vertical proton coordinate and the horizontal displacement $x_e(t)$ of the oscillating electron bunch.

As an extreme simplification, we now insert the **unperturbed linear** motion for x_p, z_p into f,

$$x_p \longrightarrow \mathring{x}_p = \hat{x}_p cos(\omega_{x,p}t + \varphi_{x,o}) \quad (9)$$

$$z_p \longrightarrow \mathring{z}_p = \hat{z}_p cos(\omega_{x,p}t - \varphi_{x,o}) \quad (10)$$

However, taking into account the dependence of the tunes on amplitude (see below),

$$\omega_{x,p} = \omega_{x,p}(\hat{x},\hat{z}) \ \text{ and } \ \omega_{z,p} = \omega_{x,p}(\hat{x},\hat{z}) \quad (11)$$

The right hand side of eq. (8) then becomes a **given function** of time which as the next step is expanded in powers of $x_e(t)$:

$$f(\mathring{x}_p(t), z_p(t); \mathring{x}_e(t) = x_e(t)\frac{\partial f}{\partial x_e}(\mathring{x}_p(t), \mathring{z}_p(t); 0) + O(x_e^2) \quad (12)$$

Keeping only the term linear in the (small) coordinate x_e, the mean square growth of the proton β-tron amplitude can be calculated from the solution of eq. (8):

$$< x_p^2(t) - x_p^2(o) >=$$

$$4\Delta\omega_p^2\int_o^t d\tau\int_o^t d\tau' sin\omega_{x,p}(t-\tau)sin\omega_{x,p}(t-\tau')$$

$$< x_e(\tau)x_e(\tau') > < \frac{\partial f}{\partial x_e}(\tau)\frac{\partial f}{\partial x_e}(\tau') >_{\varphi_{x,o},\varphi_{z,o}} \quad (13)$$

Expanding $\partial f/\partial x_e$ in a Fourier series,

$$\frac{\partial f}{\partial x_e}(\tau) = \sum_{k,l} a_{kl} exp(i(k\omega_{x,p} + l\omega_{z,p})) \quad (14)$$

taking the average over the initial phases $\varphi_{x,o}$ and $\varphi_{z,o}$, and inserting the correlation function (eq. (4)) yields:

$$< x_p^2(t) - x_p^2(o) > = 2\Delta\omega_p^2 x_{e,rms}^2\delta_x$$

$$\left(\sum_{j=-1,1}\sum_{k,l}\frac{|a_{kl}|^2}{\delta_x^2 + ((k+j)\omega_{p,x} + l\omega_{p,z} - \omega_{x,e})^2}\right)t \quad (15)$$

and thus a diffusive growth of proton emittance. The diffusion rate is peaked not only for $\omega_{x,e} = \pm\omega_{x,p}$, but also at higher order resonances due to the nonlinearity of the beam-beam force.

The Fourier-coefficients a_{kl} up to k + l = 12 are calculated by numerical integration for different values of proton amplitude (the tune shift is determined by the coefficient a_{oo}). The diffusion rates are then calculated as a function of electron tune for fixed tunes $Q_{p,x/z} = .37/.36$. Around the resonances $\omega_{x,e} = 3\omega_x$ and $\omega_{x,e} = \omega_{x,p} + 2\omega_{z,p}$, the results are compared to the growth rates obtained from computer tracking simulations for one interaction point with $\Delta Q_{x/z} = .005$ (see Fig. 2). Taking into account the typical error of the simulation due to statistical uncertainities, the agreement is rather good. (Note: The parameters used in the simulation correspond to an overestimate of the actual effect in HERA by about two orders of magnitude.)

Conclusions

With the analytical model presented in the preceding section the estimated luminosity lifetime in HERA due to electron rf-noise is definitely larger than 10 hr, if the distance to the linear resonance ($Q_{x,p} = Q_{x,e}$) is kept larger than about 0.1. The agreement of the simple ansatz with the computer simulations for the **nonlinear** effects is surprisingly good. The model calculation implies that the luminosity lifetime will not be undesirably reduced if the distance to the strongest nonlinear resonances is kept larger than about 0.03.

A more refined version of the analytical model (including localization of interaction points and rf-sections), which was done earlier[6], gives a better description of the dependence of growth rate on tunes in detail, but does not change the results qualitatively.

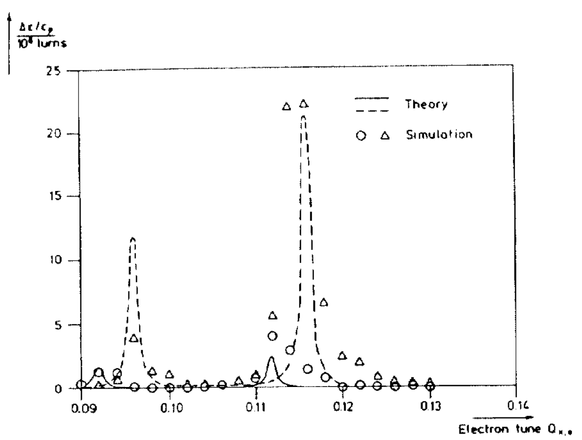

Figure 2: Proton emittance diffusion rate $(\Delta\varepsilon/\varepsilon_p)/(10^6$ rotations) from computer simulations for an initial amplitude of 0.5 $\sigma_{x,e}$ (circles) and 1 $\sigma_{x,e}$ (triangles), and the predicted rates from the analytic model (solid and dashed curves). The parameters are: $\Delta Q = 0.005, Q_{x/z,p} = .37/.36, x_{e,rms} = 0.05\ \sigma_{x,e}, \delta_x = 0.0034$ per rotation (one rotation in HERA = 21 μs).

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