EXHAUSTIVE SEARCH OF FIELD DEFECT COMPENSATION AND READING ERRORS IN A.G. MACHINES

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Summary

The methods usually considered for closed orbit correction in alternating gradient machines are probably not satisfactory for LEP for the vertical plane because the harmonics created may destroy the polarisation. Therefore it seems a priori interesting to correct locally the horizontal field defects. We then revised a method used a long time ago to detect alignment errors in the ISR [1] and tried to push it as far as possible towards a full orbit correction.

1. Fitting method with statistical analysis

In a circular machine, the closed orbit is nothing but a discontinuous betatron oscillation which closes itself over one turn. The discontinuities are kicks associated with dipole field errors. Thus, in a region where there are no dipole field errors or where they compensate between two measurement stations (BPM), the closed orbit position at the ith BPM is of the form :

$$y_i = a\sqrt{\beta_i}\,\sin\mu_i + b\sqrt{\beta_i}\,\cos\mu_i + D_i\delta \qquad (1)$$

where the letters with indices i are the well known optics functions [2]. The constants a and b and the relative energy error δ have to be determined by a least square fit of formula (1) on the BPM measurements.

It is then possible to do this computation starting at each BPM of the machine and using a certain number n of downstream BPM's. For each fit, the r.m.s. deviation with reference to the measured positions can be computed, which makes it possible to estimate the fit quality with the formula :

$$F = \sqrt{\frac{1}{n-c}\sum_{i=1}^{n}\left(\frac{y_{im}-y_i}{r}\right)^2} \qquad (2)$$

where n is the number of BPM used to make the fit, c the number of constants involved in the fit, r the r.m.s. reading error of the BPM, y_{im} the measured positions and y_i the calculated positions given by (1).

If the value of r is correct and if we assume that there is no field error in the machine, F should be of the order of 1, whatever the number n of BPM's used to make the fit. This leads naturally to computing the average value of F over all possible fits for a given n in order to obtain a statistically relevant number $\bar{F}$. The $\bar{F}$ should converge towards 1 when n is increased provided r is correct. This convergence criterion makes it possible to establish that there are no field errors in the machine : practically the discontinuities of the measured closed orbit are computed and their effect is subtracted from the measurements, then the convergence criterion is a sensitive mean of detecting whether all discontinuities have been really identified. Furthermore, if $\bar{F}$ converges towards a constant value when increasing n, r having been estimated, it is easy to adjust r so that the constant value is 1. This provides an estimation of the r.m.s. reading error r of the BPM's.

Up to now we assumed that formula (1) is exact and that the BPM's have a Gaussian distribution.

The validity of formula (1) can be tested with the measurement of the change of the closed orbit due to a known deflection. This has been done in ref. [3].

The hypothesis of a Gaussian distribution of the reading error of the BPM's - which is needed to state that formula (2) is a good estimator - implies that there cannot be too many BPM's with large reading errors. Practically this means that the fraction of BPM's with a reading error larger than R times the r.m.s. value is 1 - erf (R) which is evaluated in Table 1 :

R	1	2	3
fraction of BPM's	15.7 %	4.7 $^0/_{00}$	2.2 10^{-5}

Table 1

For instance, if the total number of BPM's is 20, the number of BPM with a reading error larger than 3 r.m.s. values should be of the order of 5.10^{-4}, i.e. the probability is small that there is any. In other words, if we find one we may consider that its reading is wrong. We will see in practice that the BPM's detected as wrong have reading errors usually larger than 3 r.m.s. reading errors.

The method practically used can be summarised with the following steps :

- make fits starting at each BPM with the maximum number n in order to find the largest kick,
- make fits starting at each BPM with the smallest number n in order to detect discontinuities or wrong BPM's,
- confirm these results by increasing the number n,
- compute the discontinuities, having removed the wrong BPM's,
- subtract the contribution of the discontinuities from the measurement,
- look at the evolution of $\bar{F}$ when increasing n,
- check the convergence of $\bar{F}$ which indicates that all detectable field defects have been found.

2. Test of the method with a closed orbit difference

The results obtained by evaluating fits, starting at each BPM and including n consecutive BPM, constitute a F-histogram, which is a set of N values of F, where N is the total number of monitors in the machine. The general rules empirically observed to distinguish between a wrong BPM reading and a kick can be summarized as follows : in the case of a fit with n consecutive monitors, a wrong reading induces n bad fits (all those which include the suspected monitor), whereas a kick signature would exhibit n-1 large F-values, appearing just before the kick. However, it must be stressed here that, when analysing a real measurement, mixture of signatures occurs very often.

A first example is given in Table 2, with EPA measured data corresponding to the effect of a horizontal single kick [3]. In addition to the signature of the kick (i.e. 2 consecutive large F-values at BPM's M061 and M063), we recognise the signature of a wrong BPM (2 large F-values surrounding

	COD	All BPM's	M091 removed	M091 M055 M023 M003 removed
M003	3.10000	2.01643	2.01643	
M005	-3.50000	1.36623	1.36623	1.36623
M011	-1.40000	1.21211	1.21211	0.13649
M013	3.90000	0.46765	0.46766	0.28098
M023	2.10000	2.26337	2.26337	
M033	-2.80000	0.46207	0.46207	0.46207
M041	-10.30000	0.81934	0.81934	0.81934
M045	3.10000	0.33812	0.33812	0.33812
M047	2.70000	0.10920	0.10920	0.10920
M049	1.20000	0.42830	0.42830	0.59300
M053	-2.00000	0.54642	0.54642	0.79186
M055	-3.90000	2.20889	2.20889	
M061	9.40000	15.83638	15.83638	15.83638
M063	4.30000	13.62012	13.62012	13.62012
M073	5.00000	3.41517	1.35955	1.35955
M083	3.30000	1.20712	1.07425	1.07425
M091	-4.20000	2.52141		
M095	-3.90000	0.11476	0.11476	0.36186
M097	2.90000	0.35593	0.35593	0.44117

Table 2
F-values obtained for a closed orbit distortion due to a single kick in the horizontal plane. The two right columns are associated with suppressed BPM measurements (no F-value for the suppressed BPM). The fits are made with 3 BPM's.

a smaller one at BPM's M073, M083 and M091). If the F-histogram is remade without BPM 91, the latter signature disappears (third column in Table 2). We can then notice that 3 F-values of the order of 2 can be found in the F-histogram. If the latter is remade without the BPM's where these values appear, all large F-values disappear except those associated with the kick (fourth column in Table 2). In this example we see how to detect simply and unambiguously wrong BPM readings.

A somewhat more tricky situation is shown in Table 3. A single kick has been applied in the vertical plane [3]. The F-histogram is still made with 3 BPM's per fit and we notice only one single large F-value at BPM M073. If the F-histogram is made without M083, just downstream M073, a kick signature appears (see third column in Table 3). If the F-histogram is made without M063, just upstream M073, it is similar to that shown in the second column. This shows that the reading of M083 is probably wrong. If the kick effect is removed, no signature remains. However, the convergence test on F is only good if M083 is not included.

	Measured COD (mm)	F(3) All BPM's	F(3) M083 suppressed	F(3) Kick removed	COD effect of kick removed (mm)
M003	2.40000	0.28650	0.28650	0.28650	-0.05923
M005	1.60000	0.32953	0.32953	0.32953	0.17175
M011	-0.50000	0.28431	0.28431	0.28431	-0.03926
M013	-6.60000	0.36751	0.36751	0.36751	-0.00060
M023	0.70000	0.25210	0.25210	0.25210	0.34043
M033	6.90000	0.33513	0.33513	0.33513	0.29772
M041	0.30000	0.46958	0.46958	0.46958	-0.03406
M045	-1.70000	0.56010	0.56010	0.56010	-0.25272
M047	-2.00000	0.39135	0.39135	0.39135	0.28533
M049	1.60000	0.16841	0.16841	0.16841	-0.06687
M053	5.10000	1.14056	1.14056	1.14056	-0.98653
M055	-0.10000	0.06371	0.06371	0.06371	0.00155
M061	-4.20000	0.16456	0.16456	0.16456	-0.20869
M063	-2.40000	0.71867	13.01215	0.71867	-0.29184
M073	6.50000	13.02627	13.24809	0.54150	0.45249
M083	2.40000	0.86100		0.73470	-0.20077
M091	4.00000	0.50901	0.50901	0.50901	0.04112
M095	0.50000	0.76662	0.76662	0.76661	0.44679
M097	-6.20000	0.24749	0.24749	0.24749	-0.10496

Table 3
Closed orbit distortion and F-values associated with a single vertical kick in EPA 3 BPM's used for the fits.

For both cases presented here, the result of the convergence test is that r should be 0.35 mm. As this is computed with a difference measurement, we conclude that the uncertainty on a single measurement is 0.25 mm. This includes noise, reproducibility and calibration error if we assume no error on the betatron functions.

In addition to this, since the kicks were artificially introduced in the machine, it is possible to compare the experimental values to the computed ones. In the vertical plane, the estimation is in excess by 0.07 mrad (β_V = 13.3 m) and by 0.12 mrad (β_H = 13.8 m) in the horizontal plane. Once the contribution of the kick to the closed orbit distortion has been subtracted, the residual signal behaves like a Gaussian noise. We therefore conclude that, for the machine under consideration, we cannot hope to localise precisely kicks of about one tenth of miliradian at places where the betas are of the order of 13 m.

3. Analysis of an absolute closed orbit measurement

We have seen above that, even for an extremely simple case, the fit analysis can be somewhat complicated. We can imagine that for a general case where reading errors and kicks effects can be mixed, the fit analysis is far from being straightforward. The simple rules given at the end of section 1 must be completed by other tests such as suppressing systematically BPM's in a suspicious area and look at the evolution of the histograms.

The method has been applied to the correction of the closed orbit of EPA in the vertical plane. A very large kick was detected after the monitor M095, but since M097 was found to be wrong, we were forced to extend our search down to M003. In this range, the best candidate for compensating the kick happened to be the quadrupole QFL96 (it is worth noting that in the case where M003 would also be considered as wrong, then the answer of the method could not detect a single candidate since there would then be more than π vertical phase advance between M095 and the next monitor M005). Although the proposed compensation appeared to be very efficient, it was not possible - despite careful checks - to find any defect on the suspected quadrupole. This problem is still unresolved and more detailed information can be found in a companion paper [5].

In the course of the orbit correction process, the predicted closed orbit distortion and the measured one could be compared. Once the wrong BPM's were removed, the r.m.s. difference between the two is 0.25 mm, which is about the same as estimated above. This comparison is interesting as it is an additional confirmation that the optics functions computed with the MAD program [4] using the EPA optics model in [3] are accurate enough and do not introduce any error at our level of accuracy.

During the exhaustive search of the defects, it was found that the convergence test on $\bar{F}$ is an extremely sensitive tool. For instance, a kick of - 0.08 mrad at a place where β_V is 3.5 m is enough to change the convergence behaviour : when this kick is compensated, the $\bar{F}$ starts to increase and then decreases when n is increased; if it is included, the $\bar{F}$-values tend towards a constant for n > 9. Furthermore, if the contribution of the - 0.28 mrad kick (at a β_V of 3.5 m) in Table 4 is not compensated, it prevents the $\bar{F}$ convergence. The closed orbit positions after subtraction of the kicks effects is given in Table 4. If we merely compare the positions with and without the smallest kick, we hardly see any difference.

It is clear that, once all the defects contributions have been subtracted, the wrong BPM's appear at a single glance to the list of closed orbit positions. In Table 4, the BPM's 33, 63 and 97 emerge. If they are removed from the calculation, the r.m.s. amplitude of the distortion is 0.28 mm, whereas the convergence test on $\bar{F}$ leads to a value of r of

	F (3 BPM's) all kicks subtracted	COD (mm) all kicks subtracted	COD (mm) all kicks but last one subtracted
M003	1.15402	0.40393	0.61517
M005	0.74975	0.28596	0.32882
M011	1.49761	0.45759	0.35506
M013	2.63769	0.05899	-0.19413
M023	0.02701	0.11987	0.26854
M033	0.19714	1.06948	1.33276
M041	0.57674	0.02487	-0.04655
M045	1.58094	-0.29993	-0.37015
M047	0.73007	-0.02122	0.00427
M049	1.41489	-0.43410	-0.31307
M053	0.54203	-0.34602	-0.11026
M055	1.32106	-0.29260	-0.32884
M061	1.44907	-0.23858	-0.41857
M063	6.50665	2.95029	2.71829
M073	0.29213	-0.04136	-0.23767
M083	1.49835	-0.19480	0.02277
M091	3.49091	0.12257	0.25372
M095	2.09934	-0.40357	-0.43421
M097	1.64779	-3.00346	-3.24442

Table 4

Vertical closed orbit distorsions computed after subtraction of defects. The kicks subtracted for the middle column are :

0.56 mrad on QFLH96 -0.28 mrad on QFNH32
0.18 mrad on QFLH54 -0.08 mrad on QFNH62
0.33 mrad on QFLH56

0.29 mm. This concordance is quite remarkable and contributes to justify the convergence conjecture. For what concerns the wrong BPM's, two of them have an offset of 10 times the r.m.s. error reading, and one 3 times. The distribution histograms of the BPM's considered as good is shown on Fig. 1. They are far from being Gaussian : the standard deviation of the Gaussian fits of the histograms are respectively 0.42 mm and 0.37 mm for the left and right histogram.

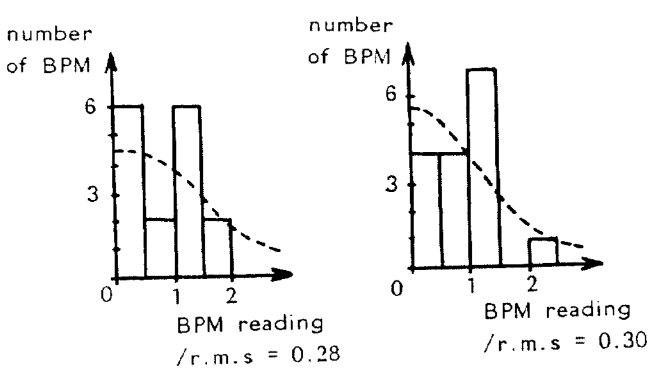

Fig. 1 - Distribution histogram of good BPM's readings for the two closed orbit positions given in Table 3.

These values are larger than the computed r.m.s. errors which are respectively 0.28 mm and 0.30 mm. Nevertheless, even when using the standard deviations of the Gaussian fits, the reading of BPM's 63 and 97 are still substantially larger than 3 standard deviations, and the reading error of BPM 33 is at the limit of 3 times.

Knowing both the r.m.s. reading error and the r.m.s. error due to noise and calibration (0.25 mm from Sect. 2), and remembering that the errors related to the optical model were found to be negligible, it is then possible to calculate the r.m.s. alignment error of the BPM's. Assuming a quadratic addition and accounting for an uncertainty of 0.01 mm on the errors mentioned above, one obtains r.m.s. alignment errors of 0.14 ± 0.04 mm and 0.2 ± 0.04 mm respectively. These estimations compare fairly well with the value of 0.1 mm given by the survey people for the vertical alignment tolerance.

Three exhaustive searches have been done with three vertical closed orbit measurements performed in May 1987 (from which Table 4 was obtained), November 1987 and March 1988. The values of r associated with these three searches are comprised between 0.32 mm and 0.34 mm. In the first two measurements, the same defects (around QFL96 and QFL56) emerged from the calculations, but the additional small kicks (angles less than 0.3 mrad) were found at different locations, without any clear relationship between them. In the last measurement of March 1988, the strongest kick (QFL96) had been fully compensated with a quadrupole displacement. The other kicks were found at locations still differing from the previous ones, probably related to the quality of the orbit measurement.

The differences found in the locations of small kicks (which are responsible for an oscillation emittance of the order of $3 \cdot 10^{-7}$ m), indicate the limitations of the search method for the system of monitors under consideration (with an r.m.s. absolute reading error of about 0.3 mm).

After correction with a quadrupole displacement of the strongest kick found by the method, the resulting closed orbit distortions were found to be already satisfactory [5]. For this reason no attempt was made to further compensate for the remaining kicks. However, when considering the predicted closed orbit, resulting from the compensation of all the computed kicks, we feel that it might be worth to try this scheme once and check experimentally the level of correction which could be indeed achieved in the machine.

4. Conclusion

The fitting method is a powerful tool to predict both the location and the correction of field defects, provided the latter are large enough and the neighbouring beam position monitors are properly functionning. A major ingredient of this approach is its ability to check the correctness of the BPM readings, which is fundamental for a precise location of the compensation. A convergence criterion is used to ensure that all detectable defects have been removed. The application and the limitations of the method have been discussed for the case of the EPA machine. An interesting by-product of this so-called exhaustive analysis is the determination of the various contributions to the monitor reading errors. It opens also the possibility of checking the optics model of the machine in the particular case of the analysis of the effect of a known kick.

References

1. G. Guignard, The closed orbit measurements as a diagnostic tool for localisation and correction of the misalignments in the ISR. XIth Intern. Conf. on High Energy Accelerators, CERN, Geneva, July 7-11, 1980. Also in CERN ISR-BOM/80-21.
2. E.D. Courant, H.S. Snyder, Theory of the alternating gradient synchrotron, Ann. of Phys. Vol.3, (1958)1.
3. M. Bell, J.P. Delahaye, H. Kugler, The model of the bending magnets used in the 600 MeV e^+e^- accumulator of LEP, IEE P.A.C. Washington D.C. (March 16-19 1987).
4. F.C. Iselin, J. Niederer, The MAD program (Version 6), User's Reference Manual, CERN/LEP-TH/87-33.
5. H. Kugler, S. Battisti, D. Brandt, J.P. Potier, A. Verdier, Closed orbit distortion and their correction in the 600 MeV Electron-Positron Accumulator at LEP. This Conference.