Analytical evaluation of synchrotron radiation integrals for isomagnetic lattices with rectangular dipole magnets.

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Abstract

An analytical evaluation is given of several expressions of averaged lattice functions in bending magnets in a synchrotron ring. In particular it turns out that a simple expression results for the fourth synchrotron radiation integral in an isomagnetic lattice with rectangular dipole magnets, which is independent of the chosen quadrupole structure and of the machine length. As a result damping rates of the synchrotron and betatron oscillations, and energy spread of the beam are independent of the chose type of lattice.

Introduction

The synchrotron radiation integrals I_1 to I_5 (1) are used for evaluating the physical properties of the beam in a synchrotron or storage ring. The integrals are non-zero in bending magnets, and are given in terms of combinations of lattice functions (Twiss-functions, dispersion function, etcetera), only at the segment boundaries: Helm et.al. (1).
In this paper an evaluation is given of the fourth synchrotron radiation integral I_4 in particular, following the the prescriptions of Helm et.al., for rings with rectangular homogenous dipole magnets. A very simple expression results, independent of the lattice functions. This example has considerable practical applications, since rectangular ("straight") dipole magnets are easy to construct, and are often used. Moreover it turns out the analytic expression is independent of the sign of curvature (direction of rotation). Hence approximate expressions of ΔI_4 for wigglers and undulators can be found by application of the given formula. The radial and longitudinal damping partition numbers (2) are given by $J_x = 1-D$, $J_\varepsilon = 2+D$, with $D = I_4/I_2$. It is well-known that for separated function isomagnetic lattices $D \ll 1$ and that D vanishes for homogenous bending magnets with parallel end faces (3). Our evaluation of I_4 in fact shows that for this case of parallel end faces $-I_4 \ll 1$ but $\neq 0$ for dipoles with small bending angle. In synchrotron lattice codes, eg. DIMAD (4), the numerical evaluation of damping partition numbers is done via Helm's formulas, and also always shows that $I4 < 0$ for straight homogenous bending magnets. In this paper we will give a derivation for the expression for I_4. The result was first found with the code REDUCE (5), which was used for checking the relations given by Helm, e.g. for finding the expression for the averaged H-function.

Definition of synchrotron integrals (1)

With the usual Twiss functions α, β, γ, and with the dispersion function η the synchrotron radiation integrals are in fact summations of relevant quantities in bending magnets:

$$I_1 = \Sigma_i \frac{\ell_i}{\rho_i} \langle\eta\rangle_i; \quad I_2 = \Sigma_i \frac{\ell_i}{\rho_i^2}; \quad I_3 = \Sigma_i \frac{\ell_i}{|\rho_i|^3};$$

$$I_4 = \Sigma_i \left[\frac{\ell_i}{\rho_i^3} \langle\eta\rangle_i - 2\ell_i \left\langle \frac{n\eta}{\rho^3} \right\rangle_i\right]; \quad I_5 = \Sigma_i \frac{\ell_i}{|\rho_i|^3} \langle H \rangle_i$$

where ρ and n_i are radius and field index in magnet i whose length is ℓ_i and where $H = \frac{1}{\beta}[\eta^2 + (\beta\eta' - \frac{1}{2}\beta'\eta)^2]$, with $\beta' = d\beta/ds$, $\eta' = d\eta/ds$.

Evaluation of I_4

Here we give the derivation of the formula for ΔI_4, the contribution to I_4 of each magnet, for rectangular homogenous bending magnets. The bending angle is 2 ϕ, the angle of entrance and exit is ϕ, see fig. 1. In this derivation we closely follow the prescriptions of Helm et.al. The general expression for ΔI_4 is:

$$\Delta I_4 = \frac{\ell}{\rho^3} \langle \eta \rangle - 2\ell \left\langle \frac{n\eta}{\rho^3} \right\rangle$$

where ℓ and ρ are the length and bending radius of the magnet.

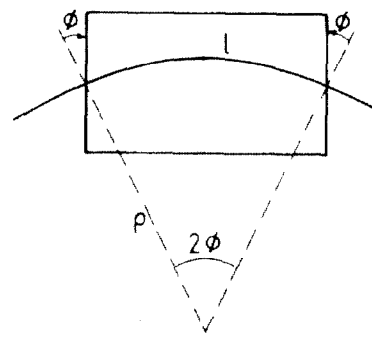

Fig. 1 Rectangular dipole magnet

In a homogenous sector magnet the dispersion function is given by:

$$\eta(s) = \eta_o \cos(s/\rho) + \eta_o' \sin(s/\rho) + \rho[1-\cos(s/\rho)],$$

where η_o and $\eta_o' = (d\eta/ds)_o$ are the values of the dispersion function and its derivative at the entrance of the dipole. Hence the averaged value, and the value at the end of the dipole are:

$$\langle \eta \rangle = \langle \eta(\eta_o, \eta_o') \rangle = \frac{1}{2\varphi}[\eta_o S + \eta_o' \rho(1-C) + \rho(2\varphi - S)],$$
and $\eta(L) = \eta_o C + \eta_o' \rho s + \rho(1-C)$,

where $S = \sin 2\varphi$, $C = \cos 2\varphi$ (double φ). For a rectangular dipole magnet a magnetic wedge of angle ϕ is added to the sector magnet at either end. This changes η_o' to η_1' (but not the value η_o):

$$\eta_1' = \eta_o' + (\eta_o/\rho)T,$$

where $T = \tan\varphi$ (single φ). Moreover the addition of wedge magnets gives contributions to $\langle n\eta/\rho^3 \rangle$ (1), which are absent in the case of the a sector magnet:

$$\delta \langle n\eta/\rho^3 \rangle_{entrance} = \frac{\eta_o T}{2\ell\rho^2}$$

$$\delta \langle n\eta/\rho^3 \rangle_{exit} = \frac{\eta_2 T}{2\ell\rho^2}$$

where $\eta_2 = \eta(\ell)$.

The expression for ΔI_4 now becomes:

$$\Delta I_4 = \frac{1}{\rho^2} [\eta_o S + \eta_1' \rho(1-C) + \rho(2\varphi-S) - (\eta_o+\eta_2)T],$$

with $\eta_2 = \eta_o C + \eta_1' \rho S + \rho(1-C)$.

This expression is independent of the values η_o and η_1': coefficient of $\rho\eta_1'$: $1 - C - TS = 0$,
coefficient of η_o : $S - T(1+C) = 0$.

Remaining terms: $\Delta I_4 = \frac{1}{\rho} [2\varphi-S-T(1-C)]$
$= \frac{1}{\rho} (2\varphi-2T)$, since $S + TC = T$.

Hence:

$$\boxed{\Delta I_4 = \frac{2}{\rho} (\varphi-\tan\varphi)}.$$

with φ half the bending angle of the magnet.
The formula given above shows that damping rates and r.m.s. energy spread of the beam are independent of the focussing structure of the machine, for the energy spread even independent of machine length.

Comment: Although it was seen that the coefficient of η_1' equals zero, without using the expression relating η_1' and η_o', which could lead to think that one could use any entrance angle, it is quite clear that the independence of ΔI_4 of η_o' and η_1' only occurs for the very special case of equal entrance and exit angles, i.e. for a straight magnet. This statement was also verified with the help of REDUCE.

The relations for damping rates of betatron and synchrotron oscillations, and for the relative r.m.s. energy spread are:

$$\tau_i^{-1} = J_i \frac{U_o}{2E_o T} = J_i I_2 C_\gamma E_o^3/(2T_o)$$

$$\tau_\varepsilon^2 = C_q \gamma^2 \frac{I_3}{I_2} J_\varepsilon^{-1}$$

with U_o the energy loss per turn, γ_o the relativistic factor, T_o the revolution time, the numerical factors for electrons: $C_\gamma = 8.85\ 10^{-5}$ m GeV^{-3},
$C_q = 3.84\ 10^{-13}$m,
and with damping partition numbers $J_x = 1-D$, $J_y = 1$, $J_\varepsilon = 2 + D_2$, $D = I_4/I_2$. For an isomagnetic lattice with $I_2 = 2\pi\rho^{-1}$ and $I_3 = 2\pi\rho^{-2}$ and having N straight dipole magnets the quantity D is given by:

$$D = 1 - \frac{N}{\pi} \tan \frac{\pi}{N}$$

Changing the polarity of a particular dipole implies the transformation $\rho \to -\rho$ and $\varphi \to -\varphi$. Inspection of the derivation of the formula for ΔI_4 shows that the coefficients of η and η' are zero anyway and that the remaining terms are invariant under this transformation. Hence the formula applies also to reversed bending magnets.

In particular it applies to each pole piece of a dipole undulator, and the total contribution to I_4 of an undulator of N pole pieces is given as

$$\Delta I_{4,\ \mathrm{und.}} = (N/\rho)(\varphi - \tan\varphi/2)$$

where φ is the bending angle of each pole, and ρ the bending radius. Moreover the formula applies to a dipole wiggler constructed out of straight dipoles, satisfying the condition of equal exit and entrance angles.

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