COLLECTIVE COUPLING STATES IN A QUASINEUTRAL ION BEAM

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The possibility of the existence of spherically symmetric neutrals (electrostatic potential drops faster than 1/R) of collective coupling states formed by a large number of electrons and ions is discussed. It is shown that such states may exist in a quasineutral ion beam and in the frame of ions may be attached to the neutral plasma forming of chaotic moving electrons and ions at rest.

Methods of transport of high-current ion beams (HIB) [1-4] have to meet the requirements of neutralization of large self fields and conservation of HIB quality. The main paradox of the problem of neutralization is the following: initially, there are two cold flows of electrons and ions, finally, there is one uniform mixed electron-ion flow of larger entropy. Thereafter, the growth of entropy is the consequence of neutralization processes and a high degree of charge and current neutralization may be attained only if there is *an effective channel of relaxation of the electron distribution function.* A mechanism of instability of the virtual cathode train was cosidered in our report [5] where we investigated mainly time-dependent evolution of electron flow and did not consider the behaviour of ions.

Below we shall consider an intermediate stage of less entropy than the neutralized stationary state. This stage permits fast thermalization of the electron-ion system, i.e., the growth of HIB emittance may be considered in the following.

The main concept of this stage is the collective coupling state (CCS). At first sight a quasineutral ion beam is similar to a moving plasma medium. But as distinct from the plasma even the concept of partial thermodynamic equilibrium is not applicable to its description, at least up to the stage of full mixing of ions and electrons. Therefore, the concept of temperature is absent also. The correlation phenomena appearing in a quasineutral HIB arise from the CCS energy advantage. The moving regions of potential significantly differing from the average (zero) potential of HIB is a manifestation of CCS. Such regions consist of quite a number of electrons and ions and the cross section of the CCS interaction may be large enough to lead to fast thermalization of the system.

Here we try to construct the simplest spherically-symmetric CCS in the ion beam frame. We shall find neutral CCS (the electrostatic potential drops faster than 1/R) transferred to the neutral plasma consists of chaotic moving electrons and ions at rest in the limiting case of large radius.

The Lagrangian of a nonrelativistic particle in spherical coordinates r, ϑ, φ is given by $\mathcal{L} = (1/2m)(p_r^2 + p_\vartheta^2 + p_\varphi^2) - e\varphi + (e/mc)(p_r A_r + p_\vartheta A_\vartheta + p_\varphi A_\varphi)$, where $p_r = m\dot{r}$, $p_\vartheta = mr\dot{\vartheta}$, $p_\varphi = mr\dot{\varphi}\sin\vartheta$ are mechanical momenta. Since we are interested in spherically symmetric CCS, the conditions for the components of the vector-potential are $A_r = A_\vartheta = A_\varphi = 0$. There are known three constants of motion of a particle in the field of central forces: H - hamiltonian, N - full momentum and L - the projection of the full momentum on the polar axis. The transformation from the mechanical momenta to H, N, L is given by

$$D = \frac{m \cdot N}{R^3 \cdot \sin\vartheta} \cdot \frac{1}{\sqrt{2m(H \pm e\varphi) - N^2/R^2}} \cdot \frac{1}{\sqrt{N^2/R^2 - L^2/R^2 \sin^2\vartheta}}$$

where the plus and minus signs represent ions and electrons, respectively.

The distribution functions of the particles should not depend on L in the case of spherically symmetric states and we define these as follows

$$f_e = f_e^o \cdot \delta(H - H_e) \cdot F_e(N), \quad f_i = f_i^o \cdot \delta(H - H_i) \cdot F_i(N) \qquad (1)$$

To simplify calculations and evaluate restriction to the behaviour of the particle distributions, we take the N-distribution in the following form:

$$F_e(N) = N^\alpha, \quad F_i(N) = N^\beta. \qquad (2)$$

The constants H_e and H_i are defined from the conditions: the full energies $H_i = 0$ (ions at rest) and $H_e = W$ provided $\Phi = 0$. Using eq. 1,2, one obtains the charge densities of electrons and ions:

$$\rho_e = \pi m_e f_e^o \frac{1}{R^2\sqrt{m_e W}} \int_0^{N_m} \frac{N\cdot F_e(N)\cdot dN}{\sqrt{2(1-\Phi) - N^2/m_e W R^2}},$$

$$\rho_i = \pi m_i f_i^o \frac{1}{R^2\sqrt{m_e W}} \int_0^{N_m} \frac{N\cdot F_i(N)\cdot dN}{\sqrt{2(m_e/m_i)\Phi - N^2/m_e W R^2}}, \tag{3}$$

where $\Phi = e\varphi/W$ and the integration is performed in the region of nonnegative values of the radical functions. Substituting eq. 2 in eq. 3 and defining the quantities E and I by

$$E = f_e^o\cdot\frac{4\pi^2 e m_e}{W}\cdot(2m_e W)^{\frac{1+\alpha}{2}}\cdot 2^{\alpha}\cdot B\left[\frac{\alpha+1}{2},\frac{\alpha+1}{2}\right],$$

$$I = f_i^o\cdot\frac{4\pi^2 e m_i}{W}\cdot(2m_e W m_i/m_e)^{\frac{1+\beta}{2}}\cdot 2^{\beta}\cdot B\left[\frac{\beta+1}{2},\frac{\beta+1}{2}\right]$$

eq. 3 can be written as

$$\rho_e = \frac{W}{4\pi e}\cdot E\cdot R^{\alpha}\cdot(1-\Phi)^{\frac{1+\alpha}{2}}, \quad \rho_i = \frac{W}{4\pi e}\cdot I\cdot R^{\beta}\cdot\Phi^{\frac{1+\beta}{2}}, \tag{4}$$

where B is Euler integral of 1-st kind. Defining the nondimensional radius X and parameter G by

$$X = R\cdot I^{\frac{1}{\beta+2}}, \quad G = E\cdot I^{-\frac{\alpha+2}{\beta+2}}, \tag{5}$$

the Poisson equation for self-consistent potential can be written as

$$\frac{1}{X^2}\cdot\frac{d}{dX}\left(X^2\cdot\frac{d\Phi}{dX}\right) = -G\cdot X^{\alpha}\cdot(1-\Phi)^{\frac{1+\alpha}{2}} + X^{\beta}\cdot\Phi^{\frac{1+\beta}{2}}. \tag{6}$$

This equation can be compared with the Thomas-Fermi equation, obtained under somewhat different conditions: distributed ion charge instead of point charge and non Fermi-statistics of energy distribution.

The boundary conditions are defined as

1. $\Phi(\infty) \to 0$, $\Phi'(\infty) \to 0$, i.e., $\rho_e(\infty) \to \rho_i(\infty) \to$ const.
2. net potential of CCS has to drop faster than 1/X when $X \to \infty$ i.e. we shall seek neutral CCS.
3. charge densities have to be integrated and the electrostatic potential has to be limited at the center of CCS.

In the following we restrict our consideration to the case of uniform electron distribution on N ($\alpha = 0$). The condition $\rho_e(\infty) \to$ const thus satisfied. Assuming that $\alpha = 0$, eq. 6 becomes

$$\frac{1}{X^2}\cdot\frac{d}{dX}\left(X^2\cdot\frac{d\Phi}{dX}\right) = -G\sqrt{1-\Phi} + X^{\beta}\cdot\Phi^{\frac{1+\beta}{2}}. \tag{7}$$

With the given boundary conditions the neutral CCS can exist in the following intervals of the exponent β of the ion distribution function:

$\infty > \beta > 1$ Here Φ drops asymptotically faster than $1/X$ and slower than $1/X^2$;

$-1 > \beta > -2$ Here Φ drops asymptotically faster than $1/X^4$.

The asymptotic solution of eq. 7 satisfying the given boundary conditions has the following form

$$\Phi = G^{\frac{2}{1+\beta}}\cdot t^{\frac{2\beta}{1+\beta}} - \frac{G^{\frac{4}{1+\beta}}\cdot t^{\frac{4\beta}{1+\beta}}}{1+\beta} - \frac{G^{\frac{6}{1+\beta}}\cdot t^{\frac{6\beta}{1+\beta}}\cdot(\beta-2)}{2(1+\beta)^2} + \frac{G^{\frac{3-\beta}{1+\beta}}\cdot t^{\frac{6\beta}{1+\beta}+\frac{2}{1+\beta}}}{(1+\beta)^3} + \ldots \tag{8}$$

where $t = 1/X$.

Eq. 6 was solved numerically to satisfy the asymptotic (8). There is only one value of the parameter G for each values $\Phi(0)$ and β satisfying the boundary conditions.

It will be noted that another equilibrium state can be described by eq. 6. This state is not transferred to neutral plasma. Let us change the boundary conditions as follows

$\Phi(\infty) \to 1$, $\Phi(\infty) \to 0$, Φ drops faster than $1/X$ when $X \to \infty$

It is seen that such states can exist only if $\beta < 0$ but the electron and ion densities diverge as X^{β} when $X \to 0$. As distinct from the first considered case, here ions move and electrons are at rest when $X \to \infty$. Neutral CCS can exist only if $\beta < -1/2$. While the densities diverge in the center of CCS the potential is limited and electric field is zero in the center if $-1 < \beta < -1/2$.

References

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