PULSE PROPAGATION IN FREE ELECTRON LASERS: THE OSCILLATOR MODE AND PHASE SENSITIVE GAIN MEASUREMENTS

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Abstract

We present numerical and analytical results for FEL in the small signal, low gain regime. Both the oscillator mode and the cw injected case are studied.

1 Introduction

Free electron lasers (FEL) operating with radio frequency accelerators run into the well known difficulty connected with the "Lethargy", which is a by-product of the combined gain and slippage mechanisms [1]. Owing to their different velocities the optical pulse slips over the electron bunch, consequently its front part experiences larger gain than the rear part.The laser pulse is reshaped and its centroid moves at a velocity lower than the velocity of light. The result is therefore a deceleration of the optical pulse. Two parameters play a central role in the FEL pulse propagation problem:

1) The coupling parameter $\mu_c = \Delta/\sigma_z$ which is a measure of the relative slippage between optical and electron bunches ($\Delta = N\lambda$ is the slippage length and σ_z the rms electron bunch length).

2) The cavity length shortening δL, necessary to compensate for the reduction in the group velocity of the optical pulse and ensure continuous synchronysm between optical and electron pulse after each round trip (see Fig. 1).

The gain of an FEL depends on the above parameters. In this paper we will discuss a model of the FEL pulse propagation which relates lethargy to the dispersive property of the interaction. We present an analysis of the gain dependence on the coupling and cavity mismatch parameters and give simple formulae which reproduce the numerical scaling. Finally we apply the theory to the analysis of an injected signal [2].

2 Optical Pulse Propagation in an FEL

The FEL pulse evolution equation valid for low gain ($g_0 < 1$) and small signal is:

$$2\tau_c \frac{\partial E}{\partial t}(z,t) + \left\{\gamma_T + i g_0 \Theta \left[\nu - 2\pi N\right]\right\} E(z,t)$$

$$+\Delta\Theta g_0 \frac{\partial E}{\partial z}(z,t) = -\frac{i g_0 (2\pi)^{3/2}}{\mu_c \Delta^2}\int_0^{\Delta} d\eta \cdot \eta e^{i\nu\eta/\Delta} E(z+\eta,t)\int_{z+\eta}^{z+\Delta} dz' f(z') \tag{2.1}$$

where $E(z,t)$ is the slowly varying part of the electric field. ν is the normalised frequency of the the field $\nu = 2\pi N(\omega_0 - \omega)/\omega_0$ (ω_0 is the resonance frequency) and N is the number of undulator periods. $f(z)$ is the longitudinal electron distribution and g_0 is the gain coefficient. The integral over η accounts for the effective gain. γ_T is the rate of loss of optical energy from the cavity. The delay parameter Θ is defined as $\Theta = -4\delta L/g_0\Delta$.

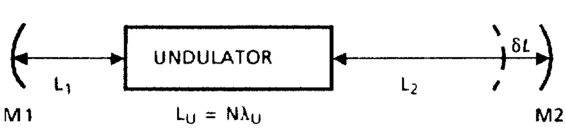

Fig. 1 Schematic of the FEL optical cavity. δL represents the detuning from the nominal length L_c.

3 Analytic Results

Approximate solutions of Eq. 2.1 can be found for long electron bunch, $\Delta \ll \sigma_z$ (i.e. for $\mu_c \ll 1$)

$$E(z,t) \propto e^{g(\tau)} e^{-(z-z_0(\tau))^2/2\sigma_E^2(\tau)}, \qquad \tau = \frac{g_0 t}{2\tau_c}, \tag{3.1}$$

where:

$$g(\tau) \simeq \left\{ -\gamma_T/g_0 + G_1(\nu)\left[1 - \frac{1}{2}\frac{\mu_c}{\sigma_E^2(0)} z_0^2(0)\right] - \frac{1}{2}\mu_c G_3(\nu) \right\}(\tau - \tau_0)$$

$$z_0(\tau) \simeq z_0(0) - \Delta[G_2(\nu) - \Theta](\tau - \tau_0) \tag{3.2}$$

$$\sigma_E(\tau) = \sigma_E(0)\left\{1 + \frac{1}{2}\mu_c\left[G_3(\nu) - G_1(\nu)\right]\right\}(\tau - \tau_0)$$

$z_0(0)$ and $\sigma_E(0)$ are the initial position and the r.m.s. half-width of the initial optical pulse assumed to be a Gaussian and finally the functions $G_{1,2,3}(\nu)$ are defined by

$$G_\alpha(\nu) = -2\pi(-i)^{\alpha-1}\frac{\partial^\alpha}{\partial\nu^\alpha}\left(1 + i\frac{\partial}{\partial\nu}\right)\frac{\sin\nu/2}{\nu/2}e^{i\nu/2} \tag{3.3}$$

$G_1(\nu)$ being the complex gain function.Equation (3.1) states that the optical packet centroid is shifted after each round trip, by a quantity $z_0(\tau)$ depending on both $G_2(\nu)$ and Θ, and is a direct manifestation of the FEL lethargy. In fact, depending on whether

$$\mathrm{Re}\, G_2(\nu) - \Theta < 0 \text{ or } > 0 \tag{3.4}$$

the optical pulse is ahead or behind the electron bunch. The synchronism condition is satisfied by

$$\Theta = -\frac{4\delta L}{g_0\Delta} = \mathrm{Re}G_2(\nu) \tag{3.5}$$

Referring to Fig. 1 we write therefore

$$\frac{2L_c}{c} = \frac{L_1}{c} + \frac{L_u}{v} + \frac{L_2 + \delta L}{c} + \frac{L_c + \delta L}{c} \tag{3.6}$$

where v is the optical packet velocity in the interaction region and according to (3.5) and (3.6) is

$$v = \frac{c}{1 + \dfrac{g_0\Delta}{2L_u}\mathrm{Re}G_2(\nu)} \tag{3.7}$$

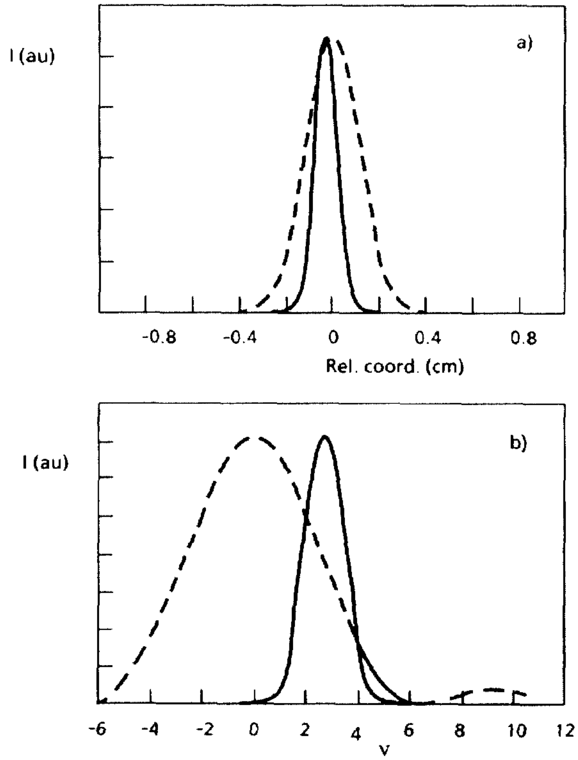

Fig. 2 Stationary distribution of the optical pulse:(a) spatial profile (solid line) and electron bunch shape(dotted line) (b) spectral profile (solid line) and spectrum of spontaneous emission (dotted line).

The relation (3.7) which represents the average velocity of the optical pulse in the undulator suggests the introduction of the following refractive index

$$n - 1 = \frac{g_0 \Delta}{2L_u} G_2(v) \tag{3.8}$$

According to the above definition we can introduce a dielectric constant for the FEL, whose real and imaginary parts satisfie the Kramers-Kronig relations. In Figure 2 we show the stationary distribution of an optical pulse starting from an initially constant field for $\mu_c = 0.7$.

4 Stationary Solutions (Super Modes)

The optical pulse reaches a stationary configuration after a number of round trips. These solutions can be found analytically for $\mu_c \ll 1$ [6] because Eq. (2.1) reduces to a Schrodinger type equation. Therefore S.M.s can be written in terms of harmonic oscillator eigenfunctions and eigenvalues

$$\Phi_n(Z) = \frac{1}{(n!2^n\sqrt{\pi\sigma_E})^{1/2}} \exp\left\{\omega_2^2 \frac{\sigma_E^2}{2}\right\}\cdot$$

$$\cdot\ H_n\left(\frac{Z}{\sigma_E}\right) \exp\left\{-\frac{(Z-Z_0)^2}{2\sigma_E^2}\right\}, \quad Z = z/\sigma_z \tag{4.1}$$

$$\lambda_n = (G_1 - \gamma_T/g_0) - (n + 1/2)\mu_c\sqrt{G_1 G_3} - \frac{1}{2G_3}(G_2 - \Theta)^2 + 0(\mu_c^2)$$

where

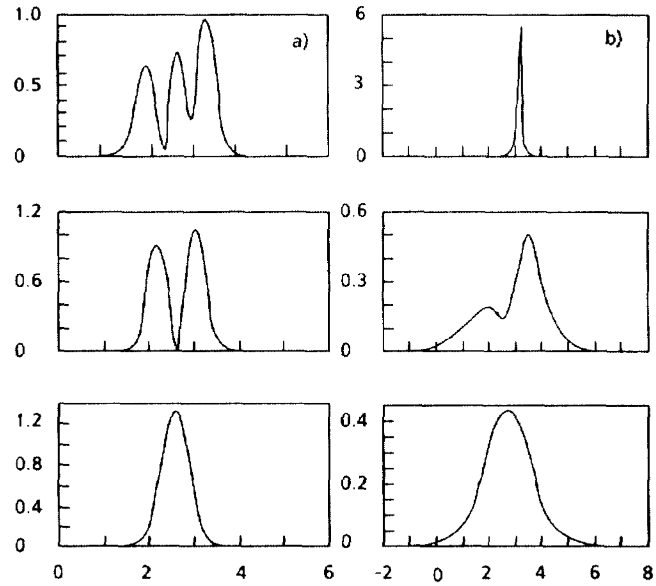

Fig. 3 Spectral shape of the first three SMs for μ_c=0.1 (a), and μ_c = 1 (b)

$$\omega_2 = -\frac{G_2 - \Theta}{\mu_c G_3}, \quad \sigma_E^2 = \mu_c\sqrt{G_3/G_1}, \quad Z_0 = -\frac{(G_3 - \Theta)}{\sqrt{G_1 G_3}} \tag{4.2}$$

Evaluating the G functions at v = 2.6 we get

$$\mathrm{Re}\lambda_n \simeq 0.85\,[1 - 0.6(n+1/2)\,\mu_c - 2(0.46 - \Theta)^2\,], \mathrm{Im}\lambda_n \simeq 0 \tag{4.3}$$

Some examples of S.M. spectral shapes are shown in Fig 3 for μ_c =0.1 and 1. It has been possible to derive a scaling law for the gain

$$\mathrm{Re}\lambda_n = -0.85\,\frac{\Theta}{\Theta_s}\left\{\ell n\left[\frac{\Theta}{\Theta_s}\left(1 + \frac{\mu_c}{3}\right)^{2n+1}\right] - 1\right\} \qquad \Theta_s \simeq 0.46 \tag{4.4}$$

The above parametization is valid for μ_c up to 3.

An interesting point is that the S.M.'s in general form a bi-orthogoanl basis [7].

5 Optical Pulse Evolution in the Presence of an External Field

To include a continuously injected external optical field Eq.(2.1) must be modified to include a source term:

$$E(n+1) = S\{E(n) + HE(n)\}R_1R_2e^{i\Phi} + E_{inj} \tag{5.1}$$

The operator S shifts the field by δL after it has experienced gain and HE(n) is the integral. The index n refers to the electric field after n passes and the coordinate z is implied. Φ is the round trip phase change. Solving Eq.(5.1) numerically with the initial conditions

$$E(0) = \frac{1 - R_1}{1 - R_1R_2e^{i\Phi}} \tag{5.2}$$

we can establish the changes that result from both interference effects arising from the dispersive nature

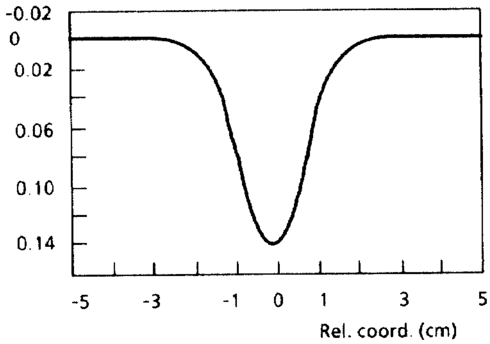

Fig. 4 Phase of the e.m. field as a function of position across the micropulse after 10 round trips for δL = 0. and v= 2.6.

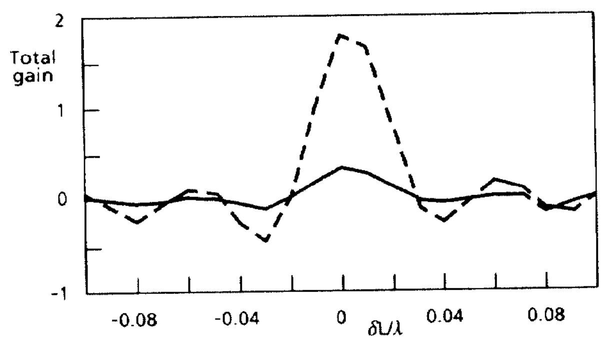

Fig. 5 The gain as a function of cavity length after 10 round trips and for v=2.6; centroid gain (dotted line), integrated gain (solid line).

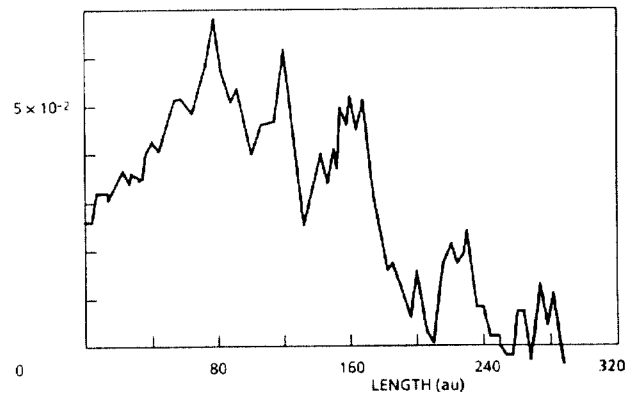

Fig.6 Gain as a function of δL, as observed in the UK experiment.

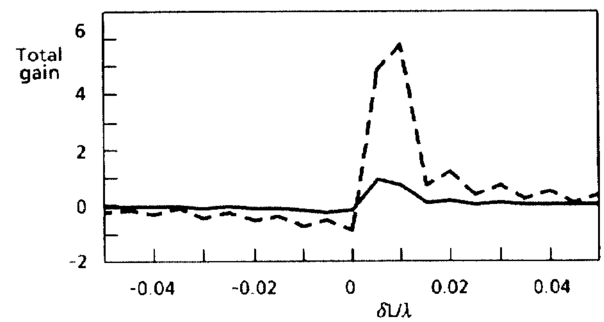

Fig. 7 Total gain as a function of δL after 50 round trips, for v=0; centroid gain (dotted line), integrated gain (solid line)

of the gain and amplitude changes arising from the "absorptive" part of the gain.

The total gain is enhanced by the high Q cavity and the peak of the gain as a function of cavity length is shifted towards longer cavity lengths. The phase shifts linearly with the number of round-trip passes. Numerical solutions to Eq.(5.1) with parameters corresponding to the UK FEL [2] were found for a variety of δL and v. Fig. 4shows the phase as a function of position across the pulse for injection on resonance with the cavity longitudinal mode (ie. δL= 0) after 10 round-trips.

It is convenient to divide the cavity length scans into two categories: a)micro-scans and b) macro-scans. Micro-scans are defined as δL variations less than one wavelength whereas the macro-scans are over many wavelengths.

Injecting at a wavelength corresponding to v=2.6 and scanning over 1μm the gain grows exponentially whereas the gain outside this range decreases to zero as a result of the fluctuating gain per pass. This is a result of the away from cavity resonance wavelength experiencing large phase changes per round-trip causing equal loss and gain on average. The peak of the gain curve (as a function of δL) is shifted away from resonance as shown in Figure 5. Experimentally [2]this shift of the peak has been observed in the UK experiment and is shown in Figure 6.

The dispersive nature of the gain medium can be observed by injecting at v=0. Only phase changes occur and any intensity gain inside the cavity is due to interference effects see (Fig. 7).

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