LEAR BEAM STABILITY IMPROVEMENTS USING FFT ANALYSIS

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Abstract

To measure the behaviour of particles at large amplitudes in LEAR, we have developed a bunch autosynchronized acquisition system together with precice FFT alogrithms to analyse the data. Time-independent perturbation theory has been used to find analytical expressions for the particle behaviour and has been applied to interpret the Fourier analysis response to a transverse kick of the beam. Non-linear amplitude dependent tune-shift due to sextupoles and resonance compensation were obtained with this system resulting in an improvement of the beam stability and ultra-slow extraction performance mainly at low momenta (100 MeV/c).

1. ACQUISITION SYSTEM

A pulse train which is synchronized with the bunch centre is electronically generated [1]. The synchronization is automatic and essentially independent of changes in revolution frequency and in length of the bunch. This pulse train is used to trigger the acquisition of changes in radial beam position after an automatic substraction of the residual closed orbit at the time of the measurement. The bunch length is comprised between 40 and 400 nanoseconds.

2. THE MATHEMATICAL TREATMENT

The use of Discrete Fourier Transform (DFT) presents a number of major handicaps if one wishes to make selective and absolute measurements of the characteristics other than the frequency (phase, amplitude, damping) of the betatron oscillations.

The mathematical treatment of the raw data uses a spectral analysis (FFT) combined with mathematical algorithms and iterative methods. This technique and the algorithms which it uses are derived from the analysis of errors introduced by the Fourier transform when applied to the measurements of betatron oscillations (frequency, phase, amplitude and damping factor).

2.1 Sources of the errors

The physical process is sampled at the revolution frequency ($T_s = 1/f_{rev}$). It is causal (triggered at an arbitrary time t = 0 corresponding to the turn n = 0) and has an indefinite duration ($n \to \infty$) This can be represented in time domain by:

$$y_\infty(nT_s) = \sum_{n=0}^{n=\infty} y_n(nT_s) \tag{1}$$

The spectrum of this signal is continuous and periodic ($= f_{rev}$) and correspond to the true spectrum in the limits of Shannon's theory (for frequency $< f_{rev}/2$). It could be produced by a hypothetical DFT with an infinite number of samples separated by T_s :

$$Y_\infty(f') = \overline{Y_\infty(f')}\, e^{\,j \overset{\frown}{Y_\infty(f')}} \tag{2}$$

where $Y_\infty(f')$ is the continuous and periodic true spectrum, $\overline{Y_\infty(f')}$ the true modulus, $\overset{\frown}{Y_\infty(f')}$ the true phase and with $0 \le f' =$ continuous frequency $< f_{rev}$ and $j = \sqrt{-1}$.

In practice the DFT is made with a limited number of samples N corresponding to the original process y_∞ seen during a time limited ($N.T_s$) window $w(t)$.

The resulting spectrum is discrete and periodic; each line is separated by $\Delta F = f_{rev}/N$. One can express the spectrum given by the DFT, by the convolution:

$$\overline{Y_N(F_k)}\, e^{\,j \overset{\frown}{Y_N(F_k)}} = \int_{f'=0}^{f'=f_{rev}} \overline{Y_\infty(f')} \times \overline{W(F_k - f')} \times e^{\,j\,[\overset{\frown}{Y_\infty(f')} + \overset{\frown}{W(F_k - f')}]}\, df' \tag{3}$$

with $F_k = k\Delta F$, k is an integer; for the principal period $0 \le k \le N-1$.

Each component $Y_N(F_k)$ of the spectrum given by the DFT of N samples should be considered as a continuous vector summation of all the true vector components $Y_\infty(f')$ of a period which are distributed on each line by a modulation phenomena with all components of the spectrum of the window.

The values of interest are the true modulus $\overline{Y_\infty(f')}$ and phase $\overset{\frown}{Y_\infty(f')}$. In this way we can see the errors introduced by the DFT. To show that each complex component $Y_N(F_k)$ of the DFT represents the vector resulting from the window distribution phenomena of the true spectrum on each line F_k, we write:

$$Y_N(F_k) = D_W[Y_\infty \to k] \tag{4}$$

2.2 Correction of the errors

Practically the true process can be written as the sum of a principal damped oscillation (frequency $= q_H \cdot f_{rev}$) and some perturbative terms: an other component of damped oscillation (frequency $= q_V \cdot f_{rev}$) and a supposed random noise. We have q_H, $q_V < 0.5$.

$$y_\infty(n) = \sum_{n=0}^{n=\infty} [h(n) + v(n) + b(n)] \quad \text{with}$$

$$h(n) = M_H \cdot e^{-n/\delta_H} \cos(2\pi q_H n + \phi_H) \tag{5}$$

$$v(n) = M_V \cdot e^{-n/\delta_V} \cos(2\pi q_V n + \phi_V)$$

$$b(n) = M_b \cdot r(n)$$

where: M_H and M_V are the initial amplitudes (for n = 0); δ_H, δ_V are the damping constants; M_b= noise amplitude, r(n)= random function such that $-1 \le r(n) \le 1$. We want to measure q_H, ϕ_H, δ_H, M_H and we assume that

$f_{rev}=1/T_s=1$. Generally q_H and q_V are non rational and they are comprised between two lines (k_H and k_H+1 for q_H and k_V and k_V+1 for q_V) of the DFT. The analysis has shown that it is interesting to consider two types of windows, rectangular $W_r(t)$ and sine $W_s(t)$.

The modulus of the two lines k_H and $k_H + 1$ of a DFT made on N samples of the true signal (5), comes from the vector composition of the true spectrum of (5) distributed on the lines k_H and $k_H + 1$:

$$\overline{Y_N(k_H)} = \left(\overline{D_W[H_\infty \to k_H]} + \overline{D_W[V_\infty \to k_H]} + \overline{D_W[B_\infty \to k_H]}\right)$$

$$\overline{Y_N(k_H + 1)} = \left(\overline{D_W[H_\infty \to (k_H + 1)]} + \overline{D_W[V_\infty \to (k_H + 1)]} + \overline{D_W[B_\infty \to (k_H + 1)]}\right) \quad (6)$$

2.2.1 Frequency measurement with an analytical interpolation method

If the damping factor N/δ_H is zero, we have shown that [2-4]:

with the rectangular window:

$$q_H = \frac{1}{N}\left[k_H + \frac{\overline{D_W[H_\infty \to (k_H+1)]}}{\overline{D_W[H_\infty \to k_H]} + \overline{D_W[H_\infty \to (k_H+1)]}}\right] \quad (7)$$

with the sine window:

$$q_H = \frac{1}{N}\left[k_H + \frac{2\cdot\overline{D_W[H_\infty \to (k_H+1)]}}{\overline{D_W[H_\infty \to k_H]} + \overline{D_W[H_\infty \to (k_H+1)]}} - \frac{1}{2}\right] \quad (8)$$

We have also shown that the perturbative distributions (noise and v(n) oscillation) are decreasing if N is increasing and if we use the sine window. With a good choise of N it is possible to neglect these distributions and we have in this case:

$$\overline{Y_N(k_H)} \approx \overline{D_W[H_\infty \to k_H]} \quad \text{and}$$
$$\overline{Y_N(k_H+1)} \approx \overline{D_W[H_\infty \to (k_H+1)]} \quad (9)$$

This interpolation formula assumes that $N/\delta_H = 0$. If N/δ_H is non zero, the minimum error (equal to zero) introduced by the interpolation, occurs when the true frequency q_H correspond exactly to the middle of the interval k_H, $k_H + 1$. Hence, when the damping factor is not negligible ($N/\delta_H > 1$) we combine the analytic interpolation method with an iterative convergent algorithm which displaces frequency (by an adequate modulation) to be measured to the middle of the interval between two consecutive lines of the DFT. In this method, the evaluation of the value of δ_H is made with a "moving FFT". The residual error is caused by the distribution of the noise and other parasitic signals which have been neglected.

The analytical interpolation method gives a possible decrease of frequency error by a factor of 10 to 1000. It is important because the accuracy of the other measurements depends directly on the accuracy of the frequency measurement.

2.2.2 Phase measurement

The method uses the fact that if the frequency is known we can, by an adequate modulation, displace the spectral component such that it coincides with one of the line of the DFT. In this case, by neglecting the distibution of the other components, we can obtain with the DFT a spectrum which ressembles the true spectrum of the displaced component.

2.2.3 Damping measurement

For the damping measurement it is necessary to increase the sensibility by using the rectangular window and choose N such that we have $N/\delta_H \geq 2$. We displace the spectral component q_H on one of the lines of the DFT and we measure the frequency spread.

2.2.4 Modulus measurements

$\overline{Y_N(k_H)}$ and $\overline{Y_N(k_H+1)}$ being the modulus of the two lines given by the DFT when the true frequency was displaced to the middle of the interval between two lines (frequency measurement) we obtain the modulus by an analytical interpolation where the damping factor is included.

3. BEAM MEASUREMENTS

The unperturbed movement of a particle in a storage ring can be written as [5]

$$z = \sqrt{2\beta_z(s)J_z}\,\cos\,[\mu_z(s) + \phi_0] \quad (10)$$

where z stands for a particle's position in the horizontal or vertical plane. β (s) and μ (s) are called betatron functions and phase advance at location s along the trajectory, and are given by the ion optical properties of the storage ring. J is an invariant of the motion given by the initial conditions of a particle.

3.1 Tune and phase advances

The measurements are of particular interest for the knowledge of the machine working point (Q_H, Q_V) and also to correct beam trajectory misteering during the injection process.
If we use two horizontal (or vertical) pick-ups at different places we can measure the phase advance between these points, compare it with theoritical values and eventually find focusing errors.

3.2 Perturbations

An error of the electromagnetic guiding- and focusing fields in a storage ring gives a perturbation to the movement of particles. The possible effects are:

i. Tune shifts as a function of the amplitude of oscillation. To measure these, kicks of increasing force are applied to the whole beam and the tune change are measured [6]. Figure 1 shows the change of tune versus the square of the applied horizontal kick for different compensations of the systematic sextupolar resonance $Q_H + 2Q_V = 8$ close to the working point.

ii. Excitation of resonances along certain lines $nQ_H + mQ_V$ in the tune diagram. The perturbations can act in one plane (nQ_H = integer or nQ_V = integer) or gener-

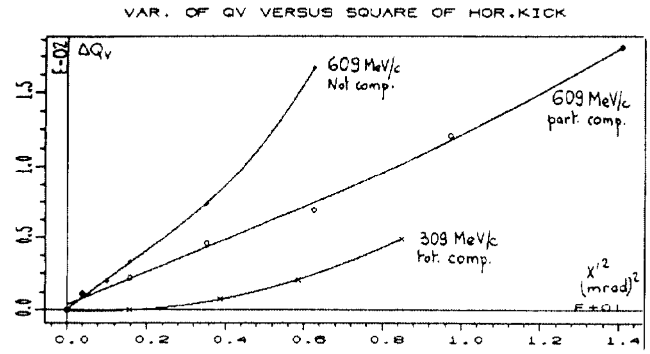

Figure 1: Amplitude dependent tune shift

ate coupling between the two transverse planes. Using a Hamiltonian formulation and canonical transformations it is possible to find the perturbed motion of a particle [7,8]. Consequently the additionnal frequencies appearing in the spectra of transverse oscillations can be related to particular resonance lines. The amplitudes and the phases of the perturbation can be extracted from the spectra, see Figure 2. The beam was kicked in the H plane. From the oscillation in the H plane (a) we can find the tune Q_H of this plane and the phase of oscillation. Due to coupling two peaks appear in the Fourier spectrum of V oscillation (b). One corresponding to V tune Q_V , the other to H tune that indicates a skew quadrupole perturbation i.e. extraction of the closest resonance line $Q_H + Q_V = 5$.

From these spectra the phase of a correction was found. After only three iterations this resonance was compensated. Using this information, correction elements can be devised and powered to compensate the perturbation.

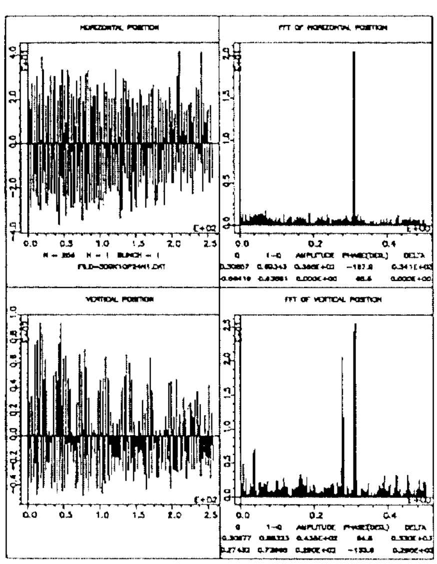

Figure 2: Measurement of linear coupling

3.3 Phase-space

In the special case where a resonance line is used to extract the beam it is of particular interest to measure the behaviour of particles. To reconstruct the normalized phase space at the observed point, a possible way exists to find the derivative of the position of the particle ($z' = \frac{dz}{ds}$). This derivative is computed using Fourier transform and Lanczos factors [9], see Figure 3. For extraction at LEAR we use the resonance $3Q_H = 7$ excited by normal sextupolar field. Fig A shows the recorded oscillations at a working point close to resonance line. Fig B shows the spectrum where 2 peaks appear, one for the tune Q_H and one at $2Q_H$ indicating a resonance of type $3Q_H$. Fig C shows the computed derivative of x and Fig D the normalised phase space (dots). It shows a rounded regular triangle. The stability limit given by the regular triangle is drawn in plain line. The result in phase space is then compared to the theoretical set-up [10].

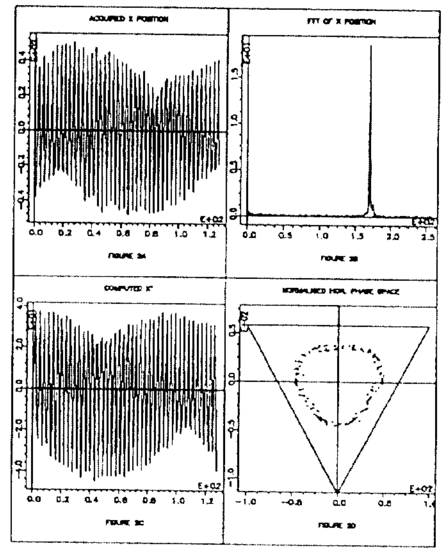

Figure 3: Reconstruction of phase space

3.4 Simulation

These Fourier methods are also used to analyse the motion of particle from simulation. By using accelerator "modelling" programs we can simulate the behaviour of a particle (or a pseudo-beam of particles) and analyse the oscillations. It permits us to predict the necessary corrections elements that will be installed in the machine.

Acknowledgement

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