MAXIMUM BEAM CURRENTS OF LIGHT IONS IN RFQS

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The topic of principal limitations for beam intensities in RFQs will be considered. What are the highest currents possible that can be accelerated, when radiofreqency f, electrode voltage V, injection voltage U_0 and minimum aperture R_1 are chosen arbitrarily up to certain limits?

1. Limitations

The first and substantial limit is set by sparking and in this context we take Kilpatrick's criterion[1] in the approximation[2]

$$\frac{V^3}{g^2}\left(1 - e^{-\frac{9.33\cdot 10^{-11} V\lambda}{g^2}}\right) e^{-\frac{1.7\cdot 10^{-7} g}{V}} = 1.8\cdot 10^{18} \quad (1)$$

with λ rf wave length, g minimum gap distance, both in meter, voltage V in Volt. According to present opinion[3] sparking is mainly determined by total voltage and gap distance. For any given g, V, λ situation equ. (1) exhibits a voltage, which is said to be 1 Kilpatrick. With regard to RFQ geometry we assume circular rods according to fig. 1 and determine gap distances between points A and B by

$$g = (R_1 + R_2)(\sqrt{2} - 1) \quad (2)$$

Variations of electrode distance may occur at other z, resulting differences in Kilpatrick limits are considered small, however. As a second assumption we exclude beam bunches with oplate geometry. In this way we consider RFQs, where the initial cell length $\beta_0\lambda/2$ of the adiabatic buncher is equal or larger than the minimum aperture

$$R_1 \le \frac{\beta_0\lambda}{2} = \frac{v_0}{2f} \quad (3)$$

This does not mean a severe restriction, as it is impossible to design a proper RFQ contrarily. For the initial velocity v_0 we have $v_0 = \sqrt{2eU_0/m}$. A possible third limitation that zero current phase advances should not surpass 90° does not play a role in this considerations.

2. Current Formulae

The following expressions are used[4,5] at given U_0, V, f, R_1, φ_s, φ_{so}

$$I_{max}^{trans} = \frac{\varepsilon_0 m\omega^3 \bar{a}_t^2 \bar{a}_l^2 \sigma_{ot}^2 (1 - \frac{3}{8}q^2)}{e\pi^2(3\bar{a}_l - \bar{a}_t)}$$

$$I_{max}^{long} = \frac{\varepsilon_0 m\omega^3 \bar{a}_t \bar{a}_l^2 \sigma_{ol}^2}{2e\pi^2} \quad (4)$$

ε_0 dielectric constant, $\bar{a}_t$ resp. $\bar{a}_l$ stand for mean transverse resp. longitudinal envelope radii, synchronous phase φ_s corresponds to velocity v, φ_{so} to v_o

$$\bar{a}_t = \frac{R_1}{1 + \frac{q}{2}}$$

$$\bar{a}_l = \frac{2v}{\omega}\sqrt{1 - \varphi_s \mathrm{ctg}\varphi_s} \quad (5)$$

Zero current phase advances can in good approximation be written

$$\frac{\sigma_{ot}^2}{\pi^2} = \frac{a + \frac{q^2}{2}}{1 - \frac{3}{8}q^2}$$

$$\frac{\sigma_{ol}^2}{\pi^2} = -2a \quad (6)$$

with parameters a and q

$$a = \frac{eVA\sin\varphi_s}{2mv^2}$$

$$q = \frac{2eCV}{m\omega^2 R_1} \quad (7)$$

Velocity v at the end, when the bunch length is kept constant along the buncher, with φ_s is related to input velocity v_o with φ_{so} according to

$$v = v_o\sqrt{\frac{1 - \varphi_{so}\mathrm{ctg}\varphi_{so}}{1 - \varphi_s\mathrm{ctg}\varphi_s}} \quad (8)$$

We assume $\varphi_{so} = 75°$, $\varphi_s = 30°$, $v = v_o \cdot 2.64$ throughout in this paper. A and C apply to the well known two term RFQ potential

$$A = \frac{m^2 - 1}{I_o(\frac{\omega}{v} mR_1) + m^2 I_o(\frac{\omega}{v} R_1)}$$

$$C = I - AI_o(\frac{\omega}{v} R_1) \quad (9)$$

(I_o modified Bessel function of zero order)
State of affairs is illustrated in fig. 2. Point P, where $I^{opt} = I^{long}_{max} = I^{trans}_{max}$, corresponds to optimum RFQ parameters

$$q_{opt} = \sqrt{P^2 + O} - P$$

$$a_{opt} = \frac{\frac{\alpha^2}{2} q_{opt} - \chi}{K} \quad (7b)$$

with abbreviations

$$P = \frac{\alpha^2 - \chi}{\frac{2R_1K}{3\bar{a}_1} + \alpha^2}, \quad O = \frac{4\chi}{\frac{2R_1K}{3\bar{a}_1} + \alpha^2}, \quad K = \frac{2I_o(\alpha)}{\sin\varphi_s} \qquad (10)$$

$$\chi = \frac{eV}{mv^2} \quad \left(= \frac{1}{2 \cdot 2.64^2} \cdot \frac{V}{U_o} \approx 0.072 \frac{V}{U_o} \right) \qquad (11)$$

$$\alpha = \frac{\omega}{v} R_1 \quad \left(= 1.1919 \text{ for maximum } R_1 = \frac{v_o}{2f} \right) \qquad (12)$$

Evidently from adds given above relevant degrees of freedom reduce to proper choice of V/U_o resp. χ (11) and α (12). Since it is our aim to estimate maximum beam currents, only I_o^{opt} is of interest and it should be emphasized that they scale with α and χ. On this base fig. 3 shows calculations of I_o^{opt}, σ_{ot} and m with α = 1.1919, amu = 1 protons, U_o = 100 kV. Denoting these I_o^{opt} (χ,α) currents of other ions (amu > 1) and different injection voltage scale according to

$$I^{opt}(\alpha, \chi) = \sqrt{\left(\frac{U_o[kV]}{100}\right)^3 \frac{1}{amu}} \cdot I_o^{opt}(\alpha,\chi) \qquad (4a)$$

Since apertures turn out large with α = 1.1919 for low frequencies (12) and in order to increase σ_{ot} (6) by a larger q_{opt} (7b) fig. 4 displays situations, when apertures are reduced by a factor of 2. We state, however, that this causes a significant sacrifice of current I_o^{opt}. Variation of injection voltage or ion mass changes velocities, so according to (12) R_1 (and R_2) has to match α's. This of course leads to different gaps (2), electrode voltages (11) and Kilpatrick factors (1). Figs. 5 and 6 illustrate calculations, where assumed modulation 2.4 corresponds to χ = 0.918 for α = 1.1919 and χ = 0.344 for α = 0.5959 as seen from figs. 3 and 4. Table I comprehends RFQ examples for highest possible beam currents and their feasibility of practical realization, disregarding present ion source limitations. Summarizing efforts, we can say: Maximum beam current is obtained with maximum transverse phase advance and maximum modulation. Scaling laws (11) and (12) favour low frequencies and not too high injection voltages. Excamples exhibit protons up to 4 amps and deuterons up to 2 amps. Because of demands on RF powers, beam powers, electrode voltages, low frequencies etc. comments should be considered speculatively.

Table I

α	amu	f [MHz]	U_o [kV]	V [kV]	R_1 [cm]	I^{opt} [A]	KPT	Comments
1.1919	1	27	25	320	4.06	0.544	0.76	feasible
"	"	54	25	320	2.03	0.544	1.26	feasible
"	"	108	25	320	1.015	0.544	2.02	high KPT
"	"	13.5	100	1280	16.24	4.35	0.95	high V and R_1
"	"	27	100	1280	8.12	4.35	1.6	doubtful KPT
0.5956	"	13.5	100	480	8.12	1.22	0.65	feasible
"	"	27	100	480	4.06	1.22	1.10	feasible
"	"	54	100	480	2.03	1.22	1.83	doubtful KPT
"	"	13.5	200	933	11.48	3.44	0.95	large R_1
"	"	27	200	933	5.74	3.44	1.61	doubtful KPT
1.1919	2	13.5	100	1200	11.48	2.87	1.24	high V and R_1
0.5959	2	27	100	467	2.87	0.86	1.5	feasible
0.5959	2	13.5	200	933	8.12	2.43	1.29	feasible

References

1. W.D. Kilpatrick, UCRL-2321, Univ. of California, Berkeley 1952
2. K. Mittag, KFK 2555, Kernforschungszentrum Karlsruhe 1978
3. A. Gerhard et al., this conference
4. P. Junior, Part. Acc. 13 (1983) p. 231
5. P. Junior et al., Fusion Symposium GSI 1988

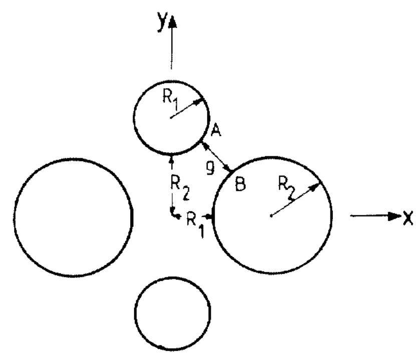
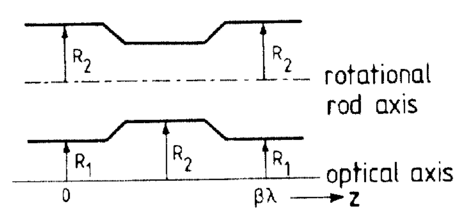


Fig. 1 Four rod geometry with modulation m = R_2/R_1
a) x-y intersection at z = 0



b) x-z intersection at y = 0

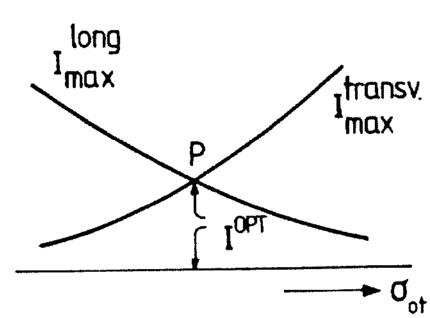


Fig. 2

Current limits from equ. (4) versus zero current transverse phase advance

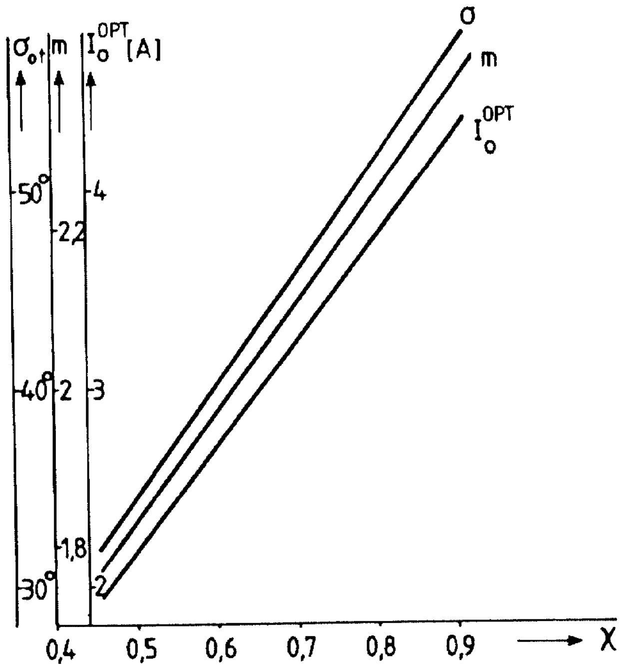


Fig. 3 Current I^{opt}, modulation m and phase advance σ_{ot} versus normalized voltage for normalized aperture α = 1.1919, protons, injection voltage 100 kV

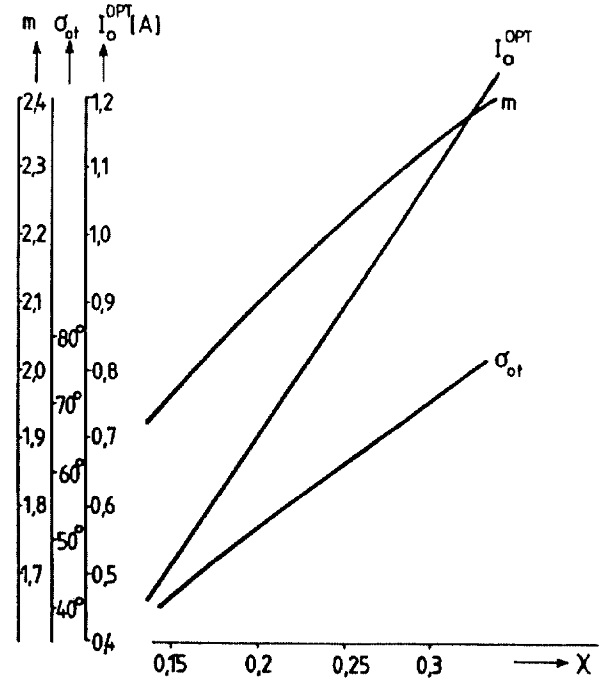


Fig. 4 Same as in fig. 3, but normalized aperture α = 0.5959

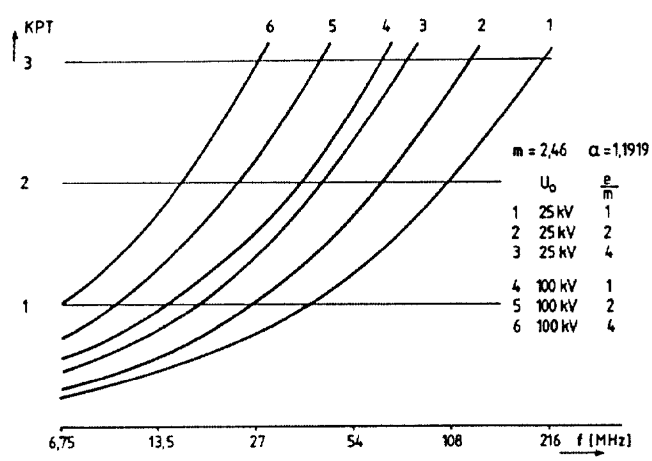


Fig. 5 Kilpatrick factors versus frequency for modulation m = 2.46 and normalized aperture α = 1.1919

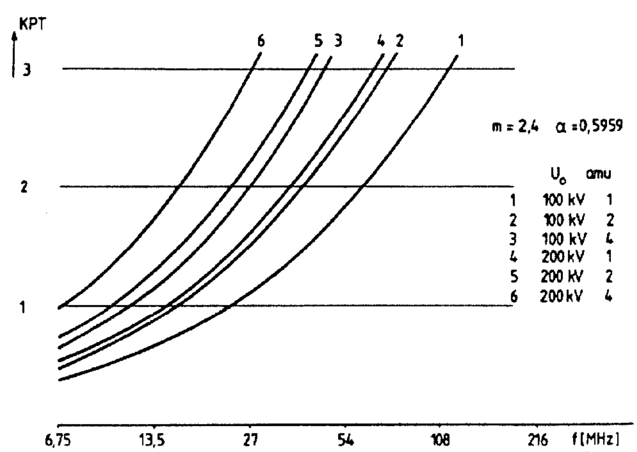


Fig. 6 Kilpatrick factors versus frequency for modulation m = 2.4 and normalized aperture α = 0.5959