CALCULATION OF PROTONIUM PRODUCTION RATE FROM COROTATING BEAMS OF $\bar{P}$ AND H_- IN LEAR

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Abstract

The rate of protonium formation with corotating beams of $\bar{p}$ and H_- in LEAR is calculated taking into account the density and velocity distributions of the two beams, the ion-optical properties of LEAR, and the formation cross sections recently published.

Introduction

The number of protonium atoms produced per sec in LEAR by the reaction

$$\bar{p} + H_- = (\bar{p}p) + 2e \qquad (1)$$

induced by corotating beams of H_- and $\bar{p}$ [1,2], is calculated in the present work. Details on calculations are available in Ref. 3.

The $\bar{p}$ ($\approx 10^9$) injected at 609 MeV/c are cooled, decelerated to 308 MeV/c, and bunched over a fraction of the circumference. Then, an H_- bunch with about the same intensity is injected onto the free part. The two beams can then be merged by adiabatic debunching. As the H_- beam decays due to various stripping mechanisms[1,2,4] the sequence of cooling, bunching and H_- re-injection is repeated frequently until a new $\bar{p}$ beam is taken.

Cooling of the corotating beams can be useful to improve the rate. Electron cooling may, however, lead to a fast destruction of the H_- beam due to stripping by the electrons[2]. If a sufficient lifetime can be obtained in these conditions the two beams will have the same averaged velocity, imposed by the electrons. With stochastic cooling or after debunching with the same rf, both beams will have the same revolution frequency. The calculations will be done for both cases.

It may be useful to reduce to zero the dispersion (orbit separation due to a momentum difference) in the region where the protonium is produced (straight section 1 of LEAR) using trimming-power supplies for some quadrupoles to adjust the optics[5]. To find out whether this is worthwile, calculations have been done with both normal and zero dispersion lattices.

The cross section $\sigma(v_r)$ for protonium production versus relative velocity v_r between $\bar{p}$ and H_- is taken from the recent publications[6,7].

A General Expression for the Production Rate

Assume that an H_- ion of speed v_- and a $\bar{p}$ of speed v cross each other (we measure velocities in a system moving with the $\bar{p}$ beam). We have

$$\vec{v}_- = (v_{\xi_-}, v_{y_-}, v_{s_-}), \quad \vec{v} = (v_\xi, v_y, v_s), \quad \vec{v}_r = \vec{v}_- - \vec{v} \ . \qquad (2)$$

Here $x = (\xi, y, s)$ is the position of the particle, and ξ includes both the betatron oscillation (x) and the displacement due to momentum deviation (D dP/P):

$$\begin{aligned} \xi &= x + D(\delta P/P) \\ \xi_- &= x_- + D(\delta P_-/P) + D(\Delta P/P) \end{aligned} \qquad (3)$$

Above and in the following we use a subscript "$_-$" for the H_- beam, no subscript for $\bar{p}$ beam. Speeds in the system moving with the $\bar{p}$ are denoted by lower case v, speeds in the Lab system by capital V. We use δP to denote deviations from the central values within the same beam, and ΔP to denote the difference between the central momentum of the H_- and the $\bar{p}$ beam. Betatron oscillation (x, x_-) is defined with respect to the central value of each beam.

For protonium formation at time t it is necessary that the ξ, y, s of the two particles is equal:

$$y_- = y \qquad (4)$$

$$\xi_- = \xi \quad \text{i.e. } x_- = x + D(\delta P/P_0) - D(\delta P_-/P_0) - D(\Delta P/P)$$

$$s_- = s \ .$$

To get the formation rate we have to sum over all $\bar{p}$-H_- encounters per sec and use the cross section for the corresponding relative speed. To this end, we need the density and speed distribution functions of the beam. As these functions have a simple form in the "betatron phase space" (x, v_x, y, v_y) rather than in the "real" space (ξ, v_ξ, y, v_y), we shall use the first ones (with Eqs. (3) to go back to real space).

Let $\varrho(x, y, s, v_x, v_y, v_s)$ be the phase space density of the beam where $\varrho d^3x\, d^3v$ gives the fraction of beam with betatron oscillations in the range x to x+dx, v_x to $vx+dv_x$, y to y+dy, v_y to v_y+dv_y, and longitudinal position and speed deviations in the range s to s+ds, v_s to v_s+dv_s. Here we use the "short-hand" notation $d^3v = dv_x dv_y dv_s$, $d^3x = dx dy ds$.

A "test $\bar{p}$" with a cross section σ will, in a time dt, "scan" a volume $\sigma v_r dt$ for H_- with relative velocity v_r. As there are $N_-\varrho_- d^3v_-$ H_- in the unit volume, the interaction rate of the test antiproton is $dn/dt = N_- \int \varrho_-(x_-, y_-, s_-, v_{x_-}, v_{y_-}, v_{\sigma_-})\sigma(v_r)v_r d^3v_-$ with the constraint (Eq. (4)) on the positions. To include all interactions in the unit volume at (x,y,s) we sum over all $N\varrho d^3v$ $\bar{p}$ present. Finally, summing over the total volume we get

$$\boxed{\begin{aligned} dN/dt = NN_- \int & v_r\sigma(v_r)\varrho_-(x_-, y, s, v_{x_-} v_{y_-}, v_{s_-}) \\ & \varrho(x, y, s, v_x, v_y v_s) d^3x\, d^3v\, d^3v_- \end{aligned}} \qquad (5)$$

In the following, to evaluate Eq. (5), we will express v_r and $P_- - P$ in term of $\vec{v}$ and $\vec{v}_-$, and assume Gaussian phase space distributions.

Momentum and Velocity Difference Between the Beams

To perform the integrations in Eq. (5) we will use the expression of the central momentum difference between the two beams

$$\left.\frac{\Delta P}{P}\right|_T = \frac{\Delta m_0}{m_0} + \gamma^2 \frac{\Delta\beta}{\beta} = \frac{\Delta m_0}{m_0} + \gamma^2 \frac{\Delta V_s}{V_s} \ . \qquad (6)$$

Distribution in the Longitudinal Plane

For the coasting $\bar{p}$ beam we use a uniform spacial and a Gaussian speed distribution

$$\varrho_s = (1/2\pi R\sigma_{vs})(1/\sqrt{2\pi})e^{-(v_s^2/2\sigma_{vs}^2)} \ . \qquad (7)$$

The standard speed deviation σ_{vs} in the $\bar{p}$ beam system is related to the velocity spread in the Lab system. Using the Lorentz transformation we get

$$v_S = \frac{v_S + V}{1 + v_S V/c^2} \approx \left(1 - \frac{V^2}{c^2}\right)v_S + V = \frac{v_S}{\gamma^2} + V \quad (8)$$

i.e. for small deviations we have

$$\delta V_S = V_S - V = v_S/\gamma^2 \ . \quad (9)$$

Also in the Lab system (Eq. (6)) we have

$$\delta V_S/V = (1/\gamma^2)(\delta P/P) \ . \quad (10)$$

We can therefore express σ_{vs} in terms of the Lab (rms) momentum spread as

$$\sigma_{vs} = \beta c(\delta P/P)\big|_{rms} \ . \quad (11)$$

Next we turn to the H_- beam. The centre of the v_{s_-} distribution is displaced by Δv_s, which (Eq. (9)) is related to the Lab value ΔV_S as follows

$$\Delta v_s = \gamma^2 \, \Delta V_S \ . \quad (12)$$

We write then

$$\varrho_{s_-} = (1/2\pi R\sigma_{vs_-})(1/\sqrt{2\pi})e^{-(v_{s_-}-\Delta v_s)^2/2\sigma^2_{vs_-}} \ . \quad (13)$$

Here σ_{vs_-} is related to the H_- beam momentum width by a relation analogous to Eq. (11).

Gaussian Distributions in Transverse Planes

We take the phase planes uncorrelated such that

$$\varrho = \varrho_x(x,v_x)\varrho_y(y,v_y)\varrho_s(s,v_s) \quad (14)$$

and we assume bivariate Gaussian distributions

$$\varrho_y(y,v_y) = (\sqrt{\beta_y\gamma_y}/2\pi\sigma_y\sigma_{vy})e^{Arg} \quad (15)$$

with

$$Arg = (-\beta_y\gamma_y/2)(y^2/\sigma_y^2 + 2yv_y\alpha_y/\sigma_y\sigma_{vy}\sqrt{\beta_y\gamma_y} + v_y^2/\sigma_{vy}^2) \quad (16)$$

where $\alpha_y, \beta_y, \gamma_y$ are the Twiss parameters[8] of the ring lattice. In the Lab system the standard deviations

$$\sigma_{yLab} = (1/f_\varepsilon)\sqrt{\varepsilon_y\beta_y}, \quad \sigma_{vyLab} = (\beta c/f_\varepsilon)\sqrt{\varepsilon_y\gamma_y} \quad (17)$$

are given by the beam emittance ε_y (defined here as "area/π").

A Lorentz transformation of the transverse components

$$Y = y, \quad V_y = (1/\gamma)\, v_y/(1 + Vv_s/c^2) \approx v_y/\gamma \quad (18)$$

gives the standard deviations in the $\bar{p}$ beam system.

$$\sigma_y = (1/f_\varepsilon)\sqrt{\varepsilon_y\beta_y}, \quad \sigma_{vy} = (\beta\gamma c/f_\varepsilon)\sqrt{\varepsilon_y\gamma_y} \ . \quad (19)$$

Analogous expressions hold for the H_- distribution.

In the horizontal plane a similar bivariate Gaussian distribution in x, v_x is assumed. As D is $\neq 0$ we have to fold the betatron and the momentum distribution to get the (ξ, v_ξ) density. Using Eqs. (10) and (19) we get

$$\begin{aligned} \xi_{Lab} \equiv \xi &= x + D\gamma^2(\delta V_s/\beta c) = x + D(v_s/\beta c) \\ \xi_- &= x_- + D(\Delta m_0/m_0) + D(v_{s_-}/\beta c) \end{aligned} \quad (20)$$

$$\begin{aligned} v_\xi &= v_x + \gamma D' v_s \\ v_{\xi_-} &= v_{x_-} + \gamma D' v_{s_-} + \beta\gamma c D'(\Delta m_0/m_0) \ . \end{aligned} \quad (21)$$

Evaluation of the Integrals in the Transverse Planes

Substituting the Gaussians (7), (13), (15), and expressions similar to (15) for ϱ_{y_-}, ϱ_ξ and ϱ_{ξ_-} into our basic Eq. (5) we can evaluate analytically five of the nine integrations.

Performing the change of variable $v_{y_-} = v_y + v_{yr}$, the integrals over y and v_y can be carried out. Denoting

$$I_y(v_{ry}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \varrho_y(y,v_y)\varrho_{y_-}(y,v_y+v_{ry})dydv_y \quad (22)$$

we get

$$I_y = (1/4\pi\langle\varepsilon_y\rangle\gamma)e^{-(\beta_y/4\langle\varepsilon_y\rangle)(v_{ry}/\beta c\gamma)^2} \quad (23)$$

with

$$\langle\varepsilon_y\rangle = (\sigma_y\sigma_{vy} + \sigma_{y_-}\sigma_{vy_-})/(2\beta c\gamma\sqrt{\beta_y\gamma_y}) \ . \quad (24)$$

In the horizontal plane two integrations can be done despite the complication given by Eqs. (20) and (21). In analogy with Eq. (22) we define I_x and we get

$$I_x = (1/4\pi\langle\varepsilon_x\rangle\gamma)e^{-(1/4\langle\varepsilon_x\rangle)(\gamma_x U_r^2 + 2\alpha_x U_r X_r + \beta_x X_r^2)} \quad (25)$$

with

$$\langle\varepsilon_x\rangle = (\sigma_x\sigma_{vx} + \sigma_{x_-}\sigma_{v_-})/(2\beta\gamma c\sqrt{\beta_x\gamma_x})$$

in complete analogy with Eq. (23), and

$$\begin{aligned} U_r &= v_{\xi r} - \gamma D' v_{sr} - \beta\gamma c D'\Delta m_0/m_0 \\ X_r &= -D(v_{sr}/\beta c + \Delta m_0/m_0) \end{aligned} \quad (26)$$

Integration in the Longitudinal Plane

In analogy with Eq. (22) we define I_s and, putting $v_{s_-} = v_s + v_{rs}$, we get for the longitudinal plane

$$I_s(v_{sr}) = \{1/[(2\pi R)^2 2\langle\sigma\rangle\sqrt{\pi}]\}e^{-[(v_{sr}-\Delta v_s)/2\langle\sigma\rangle]^2} \quad (27)$$

where

$$\langle\sigma\rangle^2 = (\sigma_{vs}^2 + \sigma_{vs_-}^2)/2 \ .$$

Numerical Integration Over the Relative Velocity and the Longitudinal Coordinate

Using the integrals I_y, I_x and I_s we can write

$$dN/dt = NN_- \iiiint ds dv_{\xi r} dv_{yr} dv_{sr} \sigma(|v_r|)|v_r| I_x(s, v_{\xi r}, v_{sr}) \, I_y(s, v_{yr}) I_s(v_{sr}) \quad (28)$$

From the integration over the $\vec{v}_s$ components at s, we get the differential rate $dn(s)/dt = dN/(dtds)$ at s. To get all protonium formed we integrate numerically dn/dt over the LEAR straight section 1.

A computer code has been developed to perform these integrations, taking the lattice properties of LEAR and measured or calculated cross sections into account. Some representative results will be discussed.

Protonium Losses on the Walls

Protonium emerging from SS1 with a too big transverse speed will be lost on the exit window that has an effective radial width of 2W = 95 mm and an effective height of 2h = 50 mm.

From the momentum conservation it turns out that the transverse velocity of protonium is nearly equal to that of the $\bar{p}$ that created it. If that assumption is used it appears that the losses are negligible for ε_y and ε_x less than 10π mm.mrad, $2(\delta P/P)rms \leqslant 0.001$.

For large emittances or large $\delta P/P$ more detailed loss calculations are necessary.

Results on the Formation Rate

Formation rates for various representative values of beams emittances and $\delta P/P$ are shown in Tables 1 and 2 for $N = 10^9\ \bar{p}$ and $N_- = 10^9\ H_-$. The formation cross sections used are that published by Bracci[6] and Cohen[7].

In Table 1 the emittances and $\delta P/P$ of the two beams are supposed equal.

Table 1

Protonium rate for various beam and lattice conditions. Emittances here are defined to contain 95% of the beam. Momentum spread is such that 95.5% of beam is in the range $P_0 \pm \delta P$ $[\delta P = 2(\delta P)rms]$.

	$\bar{p}$ & H_-		Atoms/sec (10^4)			
D	$\epsilon_x = \epsilon_y$	$2\delta P/P$	Equal Frequency		Equal Velocity	
m	π mm.mrad	$^0/_{00}$	σ_{Bracci}	σ_{Cohen}	σ_{Bracci}	σ_{Cohen}
3.59	0.1	0.1	0.0000	0.0000	0.0000	0.0000
3.59	0.2	0.2	0.0001	0.0077	0.0000	0.0034
3.59	0.25	0.25	0.0009	0.0317	0.0004	0.0169
3.59	0.4	0.4	0.0320	0.2940	0.0221	0.2067
3.59	0.5	0.5	0.0928	0.6066	0.0716	0.4669
3.59	1.0	1.0	0.4793	1.7634	0.4456	1.6134
3.59	1.5	1.5	0.5834	1.6775	0.5654	1.6109
3.59	2.0	2.0	0.5443	1.3325	0.5353	1.3032
3.59	5.0	5.0	0.2138	0.3556	0.2120	0.3546
3.59	10.0	10.0	0.0627	0.0929	0.0627	0.0928
0.0	0.1	0.1	724.31	26.966	189.099	7.3147
0.0	0.2	0.2	340.54	47.4774	201.632	17.5466
0.0	0.5	0.5	124.94	50.730	124.087	36.8787
0.0	1.0	1.0	46.022	31.050	49.058	31.3180
0.0	2.0	2.0	11.297	10.628	11.602	10.8713
0.0	5.0	5.0	1.1267	1.3441	1.133	1.3508
0.0	10.0	10.0	0.1659	0.2181	0.166	0.2184

In Table 2 relatively large emittances and momentum spread of the H_- beam are assumed as they were obtained in LEAR in the past with a low brightness H_- source and without cooling of the H_- beam.

Table 2

Protonium rate for an H_- beam with $2\delta P/P = 5\ ^0/_{00}$, $\epsilon_x = 40\pi$ mm.mrad, $\epsilon_y = 20\pi$ mm.mrad, and $\bar{p}$ beam conditions obtainable in LEAR. Definition of emittances and $\delta P/P$ as in Table 1.

	$\bar{p}$		Atoms/sec (10^4)	
D	$\epsilon_x = \epsilon_y$	$2\delta P/P$	Equal Frequency	
m	π mm.mrad	$^0/_{00}$	σ_{Bracci}	σ_{Cohen}
3.59	1.0	0.5	0.1341	0.1866
3.59	5.0	1.0	0.1072	0.1499
3.59	10.0	2.0	0.0808	0.1138
0.0	1.0	0.5	0.2180	0.2887
0.0	5.0	1.0	0.1677	0.2253
0.0	10.0	2.0	0.1194	0.1657

Equal Frequency

Zero-dispersion lattice. As the beams are perfectly overlapped the efficiency increases when ϵ and $\delta P/P$ decrease. The difference of speed insures that the range where the Bracci cross section is zero is never reached at low emittances values. Its longer tail at high velocity privilegiates slightly the Cohen's cross section at high ϵ and $\delta P/P$.

Normal lattice. The main effect is the higher rate obtained using the Cohen's cross section. This is again explained by the tail at large v_r: due to dispersion and mass difference a $\bar{p}$ and an H_- need a finite velocity difference to meet at the same horizontal position.

Equal Velocity

Zero-dispersion lattice. The efficiency grows at low ϵ and $\delta P/P$. As there is no difference of speed a maximum appears in correspondence with the maximum of the cross sections. At large ϵ and $\delta P/P$ the same considerations as for equal frequency apply.

Normal lattice. The same considerations apply as for equal frequency case. If electron cooling will allow to go down with ϵ and $\delta P/P$, one can reach the zone where the lattice with zero-dispersion is more efficient than the normal one.

Concluding remarks

For the relatively large emittances of the H_- beam as used in 1986[3] the protonium detectable per sec at the BHN10 window will range between 1000 and 3000. The exact value will depend on the efficiency of $\bar{p}$ cooling (maximum gain is a factor of 2) and on the presence of the trimming power supplies to have D = 0 (maximum gain 40%). The uncertainty due to different cross sections for protonium production is 30%.

Electron Cooling, if applicable without H_- stripping, will allow to reach the zone of maximum efficiency. The gain is then especially pronounced for the D = 0 lattice. Clearly, the improvement is due to the small H_- beam size. If a bright enough H_- source is available, these conditions could also be reached by beam collimation. However, in both cases, intrabeam stripping[4] can be very strong, leading to a fast decay of the H_- beam.

References

[1] U. Gastaldi, Fundamental Symmetries, Eds. P. Bloch, P. Paulopoulos, R. Klapisch, Plenum N.Y. 1987, p.307.

[2] D. Möhl, 3rd LEAR Workshop (1985), Ed. Frontières, Gif/Yvette, p.65.

[3] R. Giannini and D. Möhl, PS/AR/Note 88-13 (1988).

[4] M. Chanel et al., Phys. Lett., 192B, 475 (19).

[5] R. Giannini, CERN PS/LEA/87-19 (1987).

[6] L. Bracci et al., Phys. Lett., 85B, 280 (1979).

[7] J.S. Cohen, LA-UR-87-1179, Submitted to Phys. Rev. A.

[8] E.D. Courant and H.S. Snyder, Ann. Phys. 3, 11 (1958).