FOCUSING PROPERTIES OF A PLASMA WAVE

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The focusing properties of electron plasma waves with a two dimensional structure are investigated numerically. Trajectories are determined for a large set of initial conditions: phase velocity of the wave, energy and relative phase of the particles to be accelerated. The relative phase is found to be of paramount importance.

Electrons can be accelerated in high amplitude electron plasma waves created either by beating of 2 laser waves [1] or in the wake of a relativistic bunch of charged particles [2]. It was pointed out in the first report of the R.A.L. study group on Beat-Wave Laser Accelerators [3], that whenever the plasma wave has a transverse structure, radial forces occur which could either focus or defocus the accelerated or decelerated particles.

1. Equipotentials and electric field lines in a plasma wave.

The longitudinal electric field E_z in a beat-wave driven by 2 lasers with frequencies ω_1 and ω_2 such that ω_1-ω_2=ω_p (plasma frequency), has an intensity dependant amplitude through the interaction parameter Λ according to

$$E_z = \frac{m_0 c \omega_p}{e}\left(\frac{16\Lambda}{3}\right)^{1/3} \qquad \text{with} \qquad \Lambda = \left(\frac{e}{m_0 c}\right)^2 \frac{E_1}{\omega_1}\frac{E_2}{\omega_2} \propto I\lambda^2 .$$

Since the laser beam intensity I is radially dependant, so is E_z. Let in the reference frame moving with the phase velocity v_F=ω_p/k

$$E_z = f(r)\cos kz = -\frac{\partial\phi}{\partial z} .$$

The electric field is irrotational. In cylindrical geometry

$$\frac{\partial E_z}{\partial r} = \frac{\partial f}{\partial r}\cos kz = \frac{\partial E_r}{\partial z} .$$

This is readily integrated yielding

$$E_r = \frac{1}{k}\frac{\partial f}{\partial r}\sin kz = -\frac{\partial\phi}{\partial r} \qquad \text{and} \qquad \phi = -\frac{f(r)}{k}\sin kz .$$

The corresponding differential equation for the field lines in a meridian plane is then

$$\frac{dr}{dz} = \frac{1}{kf}\frac{\partial f}{\partial r}\frac{\sin kz}{\cos kz} .$$

As an example, choose a gaussian f(r)

$$f(r) = E_0\, e^{-\frac{r^2}{R_0^2}}, \qquad \frac{\partial f}{\partial r} = -\frac{2r}{R_0^2} f, \qquad \frac{dr}{dz} = -\frac{2r}{kR_0^2}\frac{\sin kz}{\cos kz} .$$

Singularities in the r,z plane are located on the z axis

$$r = 0, \qquad kz = \frac{\pi}{2} + n\pi .$$

Setting

$$z = \frac{\pi}{2} + \alpha, \qquad \cos kz = -\sin k\alpha \approx -k\alpha, \qquad \frac{dr}{dz} = \frac{dr}{d\alpha} \approx \frac{2r}{k^2R_0^2}\frac{1}{\alpha}$$

one gets the solution near the singularity at kz=π/2

$$r \propto (\alpha)^{\frac{2}{k^2R_0^2}} .$$

The relevant two dimensional maps of equipotential and of electric field lines are displayed on figure 1.

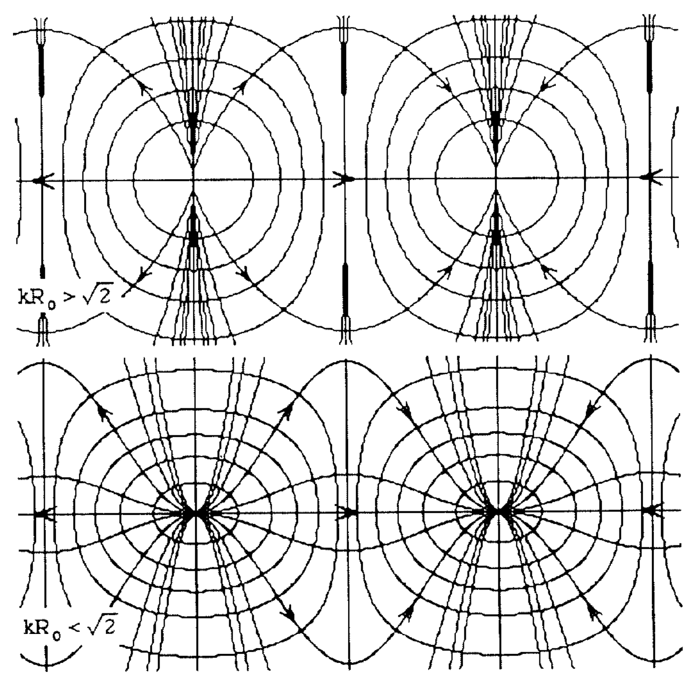

Figure 1. Equipotentials and E field lines for a gaussian profile.

Similar results are obtained with a parabolic distribution

$$f(r) = 1 - \frac{r^2}{R_0^2} .$$

The wakefield induced by a relativistic bunch of particles is basically two dimensional. Furthermore, in a cylindrical plasma column with a finite transverse extent, the boundary conditions for guided modes [4] apply [5].

2. Electron trajectory calculations.

In the reference frame moving with the phase velocity of the wave, the electron equation of motion is

$$m_0\frac{d(\gamma v)}{dt} = m_0\left(v\frac{d\gamma}{dt}+\gamma\frac{dv}{dt}\right) = -eE, \qquad \text{with} \qquad \gamma = \frac{1}{\sqrt{1-\dfrac{u^2+v^2}{c^2}}}\,.$$

One then gets for the components of the velocity, the Lorentz factor and the radial and axial coordinates

$$\left\{\begin{aligned}
\frac{du}{dt} &= -\frac{e}{m_0\gamma}\left[\left(1-\frac{u^2}{c^2}\right)E_r - \frac{uv}{c^2}E_z\right]\\
\frac{dv}{dt} &= \frac{e}{m_0\gamma}\left[\frac{uv}{c^2}E_r - \left(1-\frac{v^2}{c^2}\right)E_z\right]\\
\frac{d\gamma}{dt} &= -\frac{e}{m_0c^2}\left(uE_r + vE_z\right)\\
\frac{dr}{dt} &= u, \qquad \frac{dz}{dt} = v\,.
\end{aligned}\right.$$

This first order differential system is solved by a centered finite difference scheme. The initial conditions are the radial positions of particles with the same initial phase and the same velocity. The Lorentz factor γ_0 is entered in the laboratory frame together with the Lorentz factor γ_F of the phase velocity and

$$\gamma = \gamma_0\gamma_F\left(1-\frac{v_0v_F}{c^2}\right) \approx \frac{\gamma_0}{2\gamma_F} \qquad \text{if} \qquad \gamma_0 >> \gamma_F$$

is used.

3. Examples of results.

They are given on figures 2 to 4. The abscissa is the relative phase angle ξ=kz. The upper part of each figure is a phase space plot with ordinate γ. Particles are accelerated for $0<\xi<\pi$, decelerated for $\pi<\xi<2\pi$, modulo 2π. The electric field amplitude on axis has a comparatively low value so that overall energy gains or losses are less than 30%. The lower part represents symetrical particle trajectories in a meridian plane.

In figure 2, the wave structure has a small radius to wavelength ratio. The initial radial position R_b of the outermost particle under study is of order R_0. Focusing depends on the initial position: trajectories starting from different radii cross the axis at various locations and times even when the initial phase has its optimum value π. When the initial phase is of order $\pi/2$, only the particle initially close to the axis are focused. In all cases these poor focusing conditions are associated with a large energy spread.

In figures 3 and 4, R_b is of order $R_0/10$ or less. The focusing is good with no visible energy spread. Any initial phase leads to focusing over a length which is shorter for $\xi(0)=\pi$. This minimum is almost independant of γ_0. It compares with the predicted acceleration length for a beat wave accelerator [1], which in the laboratory frame is

$$l_A = 2\,\gamma_F^2\,\frac{c}{\omega_p}.$$

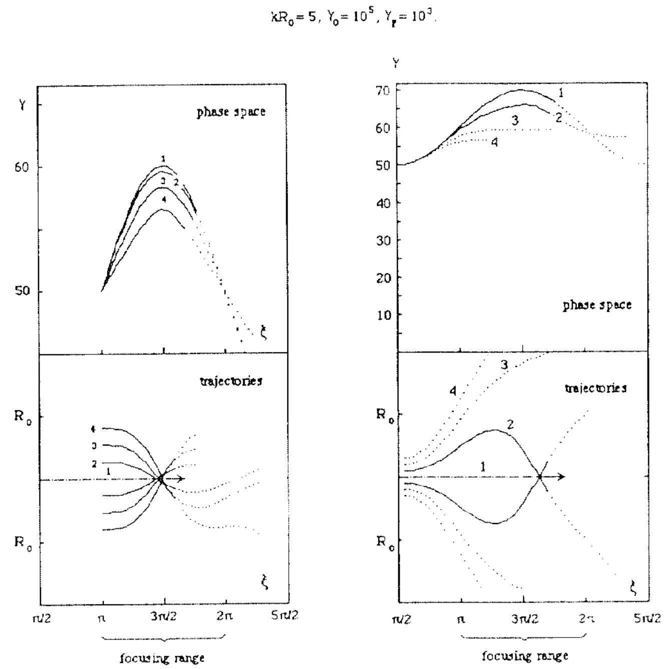

Figure 2. Phase space plot and trajectories in the moving frame for 4 particles labelled 1 (on axis) to 4. Large energy spread and poor focusing.

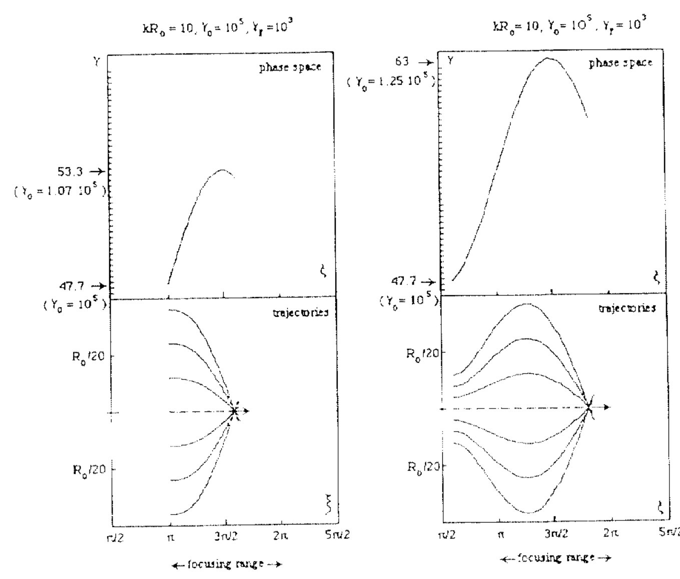

Figure 3. Phase space plot and trajectories for particle close to the axis. No energy spread and good focusing.

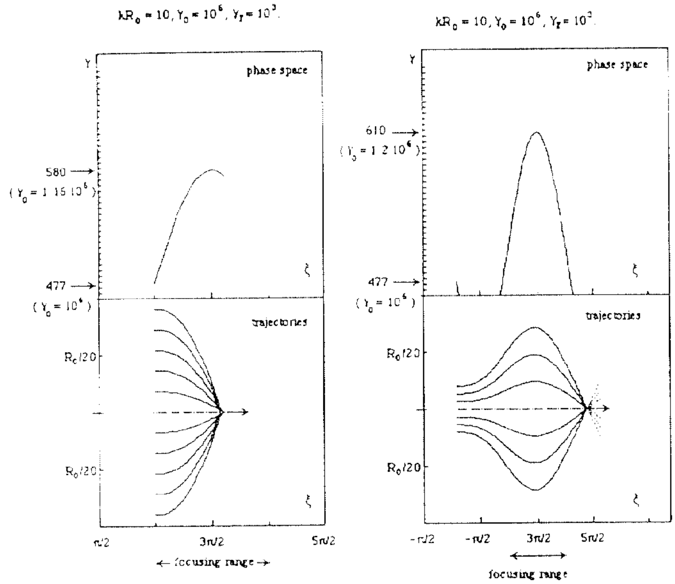

Figure 4. Phase space plot and trajectories for particle close to the axis with a larger initial velocity than in figure 3.

The computations reported in the figures were all made with $\gamma_F=10^3$. Now, $\gamma_F=\omega_L/\omega_p$ where ω_L is the mean laser frequency which induces the beat wave. Consistent sets of parameters are given below (table 1). They refer to the usual high power lasers Nd and CO_2.

Table 1

Laser	Nd	CO_2
λ_L, ω_L	1.06µm, 2 $10^{15}s^{-1}$	10.6µm, 2 $10^{14}s^{-1}$
γ_F	10^3	10^2
ω_p, electron density	2 $10^{12}s^{-1}$, $10^{15}cm^{-3}$	2 $10^{12}s^{-1}$, $10^{15}cm^{-3}$
minimum l_A	300m	3m

4. Conclusion.

Plasma waves with a transverse structure, efficiently focus small radius bunches with a longitudinal extent smaller than 1/10th of the wavelength. The initial phase has to be carefully adjusted in order to minimize the focusing length. Only individual particle trajectories in a static electric field were considered. Further studies should include space charge effects in dense bunches, and magnetic fields.

References.

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