Optics Effects & Options for TeV Colliders - II*

J.E. Spencer and B. Zotter

CERN, CH-1211 Geneva 23, Switzerland

Abstract In a previous report¹ a number of possibilities were considered with emphasis on the linac lattice. Here we concentrate on the final focus system. Even with the small emittances assumed, one still has significant nonlinearities at the interaction point which must be corrected locally and/or globally or otherwise nullified by upstream matching or by reduction or distortion of the phase space volume. Questions of round or flat beams and achromatic, spatially achromatic or dispersive optics are considered in this context.

1. Introduction

We begin by considering the transfer functions for the basic dipole, quadrupole and higher-order multipoles required for such systems. The transfer functions for individual elements or combined function systems can be calculated in a way that satisfies Maxwell's equations. Some useful applications for combined function magnets are given. Next, we consider their combination into the basic subsystems that have been used to design complete final focus systems. Different solutions are then compared and analyzed with respect to emittance and optics.

2. Optical Transfer Functions

The transfer function is suppose to transform a single particle or distribution of particles of type a with *any* initial coordinates to final coordinates for particle type **b**, as accurately as practicable *i.e.* $\{\vec{r}_i, \vec{p}_i, \vec{s}_i, t_i\}_a \rightarrow \{\vec{r}_f, \vec{p}_f, \vec{s}_f, t_f\}_b$. By convention, we call this mapping \mathcal{M}_{ab} and the set of variables to be transformed \vec{x}_i :

$$\vec{x}_{f}^{\ b} = \mathcal{M}_{ab} \cdot \vec{x}_{i}^{\ a} \ . \tag{1}$$

In the linear approximation, calculations simplify considerably since it may be possible to ignore individual multipoles and deal with lumped elements such as cells or superperiods even with considerable energy loss. The cost of this simplicity, while convenient for the design process, is a loss of information content e.g. one essentially determines only the paraxial interaction point and various tunes but in no way determines whether the system will work or not!

A. Zeroth and First Order

The zeroth order defines our choice of variables and reference systems. A reference orbit which is a straight line such as required for a linac or final focus system(FFS) is defined absolutely as soon as we place the first element. The reference trajectory in a bendfree, perfectly-aligned system is then a straight line which can be a physical trajectory even in the presence of the beam-beam force.

Bending effects complicate the situation in various ways but can be calculated and compared to observation so that the zeroth order agrees with the asymptotic trajectories outside the magnets[1]. For our purposes, bends are to be avoided because: 1) The central orbit radiates - complicating the optics and increasing emittance. 2) They dictate an 'unnatural' spin quantization axis and 3) Generate chromatic and geometric aberrations in all orders. 4) The strengths of these abberations compete with those of quads and sextupoles in typical lattices[2]. 5) They are also the sole source of pure chromatic aberrations like $(\mathbf{x}|\delta^n)$ which must be corrected. All of this leads us to use a different kind of optical element that combines dipole and sextupole fields[3] which allows reducing the bend field at the expense of the sextupole. This, in turn, implies a combined quad and octupole system[3] for the chromatic correction cells.

Referring to Fig. 1, we define the zeroth order as

$$\vec{x}_0 = \{\vec{r}, \vec{p}_0, \hat{n}\}_a \equiv \{\vec{z}, p_0 \hat{z}, \hat{z}\}_a$$
(2)

which is a helicity representation for the spin and an actual orbit for a real particle. In first order, relative to \vec{x}_0 , we use the canonical variables

$$\vec{x}_{1} = \{x, y, x', y', z, \delta, m_{z}, \phi\}_{a}$$
(3)

where the spin coordinates m_z, ϕ are equivalent to Chao's α, β . With symmetry of the fields about a midplane(y = 0 in Fig. 1) and perfect alignment, it follows from the transformation properties of the basic optical elements[2] that a system will be:

- 1. Achromatic to first order in any direction free of bends.
- 2. In second order it is 'ageometric' in these directions.
- 3. In higher order, it is generally neither of these.

Brown and coworkers[4] have shown how to make such systems achromatic to second order. However, this involves the imposition of bends in one form or another.

C. Second and Higher Order

Suppose we solve the differential equations of motion for each element of a system to the necessary accuracy and represent the results by a Taylor expansion. Some terms may not be independent – some of which are numerically equal under symplectic conditions. These can serve as a check on the calculations but such symmetries and conservation laws are not imposed. For instance, quantum fluctuations can be included in the design process or used only for subsequent tracking.



Figure 1: Layout(to scale) of a Final Focus System for CLIC.

^{*}Partially supported by U.S. Department of Energy contract DE-AC03-76SF00515.

¹Proc. 1986 Linear Accelerator Conference, Stanford, Calif., June, 1986.

Let R,S and T represent the first, second and third order contributions to the total transfer function, in terms of the initial coordinate variables, so the transform from $i \rightarrow o$ is [1,2]:

$$\vec{x}_o = R_o^i \cdot \vec{x}_i + S_o^{ij} \cdot \vec{x}_i \cdot \vec{x}_j + T_o^{ijk} \cdot \vec{x}_i \cdot \vec{x}_j \cdot \vec{x}_k + \cdots$$
(4)

where dots imply summations over repeated indices i,j,k. The transform to a final location $o \rightarrow f$ gives a similar result:

$$\vec{x}_f = R_f^o \cdot \vec{x}_o + S_f^{op} \cdot \vec{x}_o \cdot \vec{x}_p + \cdots$$
(5)

Substitution of Eq. 4 into 5 then leads to

$$\vec{x}_f = (R_f^i + (S_f^{ij} + (T_f^{ijk} + (\cdots) \cdot \vec{x}_k) \cdot \vec{x}_j) \cdot \vec{x}_i) \cdot \cdots$$
 (6)

where

$$R_f^i = R_f^o \cdot R_o^i$$

$$S_f^{ij} = R_f^o \cdot S_o^{ij} + S_f^{op} \cdot R_o^i \cdot R_p^j$$
(7)

 $T^{ijk}_f = R^o_f \cdot T^{ijk}_o + S^{op}_f \cdot (S^{ij}_o \cdot R^k_p + R^i_o \cdot S^{jk}_p) + T^{opq}_f \cdot R^i_o \cdot R^j_p \cdot R^k_q$

$$F_f^{ijkl} = R_f^o \cdot F_o^{ijkl} + S_f^{op} \cdot (R_o^i \cdot T_p^{jkl} + S_o^{ij} \cdot S_p^{kl}) + T_o^{ijk} \cdot R_p^l) + \cdots$$

Introducing only third order terms initially, gives terms up to sixth order. This would seem to justify the use of the full achromat principle i.e. correction of all 2^{nd} order terms. However, the second order chromatic correction is global whereas third order geometric corrections can be done more locally, consistent with their sources[2]. This is also the basis of using even and odd, combined-function magnets which concentrate the common sources of different aberrations. Lastly, reducing bends, effectively makes a system achromatic in higher orders.

D. Higher Orders

The phrase 'linear' in acronyms like SLC, CLIC or TLC is rather remarkable since it has been known for some time that these systems are quite nonlinear[5] and that this presents very real problems. A good illustration of high-order effects arose here. While it is well known how to get first-order, stigmatic focusing(i.e. a common crossover for x and y), this is not so easy in higher-order even with the low emittances assumed. Fig. 2 shows the effect schematically for a CLIC solution that was uncorrected in third order. This case had $(y!y'^3) > 0$ which effectively foreshortens the vertical crossover. When the corresponding x term is negative, the points of minimum confusion occur at opposite sides of the paraxial crossover(z=0 at the IP).



Figure 2: Third-Order Geometric Effect in a Second-Order Achromat.

3. Emittance Effects

Emittance becomes a problem only when one wants to make it small or to keep it that way. The overall parameters for the CLIC FFS have been outlined by Schnell[7]. We have obtained some solutions with characteristics approaching these. A major problem is maintaining the emittance which is delivered to the FFS. In this respect, it seems useful to try to integrate the FFS with the Linac since it appears that it may well have similar problems. Regardless, one needs a scaling relation for emittance or preferably spot size at the FFS in terms of magnetic rigidity, bend radius, magnifications and the like.

4. The Final Focus System(FFS)

There are a number of approaches one can adopt such as not localizing specific sources but dealing with the problem globally e.g. minimizing the rms spot size at the final focus. Flat-beam FFS's are an example[6] which concentrate on the direction with fewer chromatic terms – the non-bend plane. Another, tried here, is to concentrate only on spatial abberations.

A. Telescopic Transformer

It requires a minimum of four quads to get a first-order telescope with angular magnifications $M_{x'}, M_{y'}$. In second-order one must deal with two chromatic aberrations in each variable. It appears impossible to make all of these zero with symmetries such as multiple cells, equal magnifications and the like. Further, when one cascades n identical telescopes to compound the magnifications e.g. $(M_{x'})^n$, it follows that:

- Terms like (x|x'δ), (x|x'³) or (y|y'x'²) etc., which are predominantly positive definite, grow as their respective magnifications (M^j_{x'}M^k_{y'})ⁿ⁻¹ where j,k are the orders of x', y'.
- 2. Thus, the last cell(and lens) dominate the aberrations! This has implications for correction schemes, magnet strengths and the number and symmetry of cells.
- 3. The magnifications alternate in sign depending on whether n is even or odd as do *all* the various aberrations.
- 4. Clearly, the higher order terms grow most rapidly.
- 5. However, higher-order geometric terms such as from fringing fields are usually small compared to the chromatics.
- Through fourth order, there is only one aberration in each direction which limits the luminosity: (x|x'δ) and (y|y'δ). This was verified with TURTLE[9].

Since we are interested in luminosity, we relax the telescope by constraining only spatial aberrations i.e. $(x|\cdots)$ and $(y|\cdots)$. Using two telescopes at 1TeV with $M_x = 6$ and $M_y = 18$, as used in the configuration shown in Fig. 3, you obtain first-order spot sizes $(\sigma_y^*=15nm, \sigma_x^*=125nm)$ with:

- Input emittance $\epsilon_x = \epsilon_y = 10^{-6}$ m and $\delta_{max} = \pm 0.01\%$.
- Likewise, for an input $\epsilon_x = \epsilon_y = 10^{-7}$ m and $\delta_{max} = \pm 0.1\%$.

The results were obtained for a 151m straight FFS having $l^* = 2.5m$ and final quad gradient G=7.6kG/mm and length l=2m. They show the dominant aberrations are the two, second-order chromatics mentioned above since we reduced the input $\beta_{x,y}^i = 40m$ by 10 to compensate the δ variation. Such a system



Figure 3: Amplitude Functions[m] for CLIC with B=667 G.

is compatible with pure quadrupole compensation for the detector solenoid and allows the FFS to use pure permanent magnets with 5mm apertures. Because it has no bends, there is negligible perturbation to the incoming emittance and polarization.

B. Full Final Focus Systems

The telescopes used above were combined with two chromatic correction cells, each having 76m of 667 G dipoles, to give the configuration shown in Fig. 3 with l=471m. With $\beta_{x,y}^i=40$ m, we obtain first-order spot sizes($\sigma_y^*=15$ nm, $\sigma_x^*=125$ nm) with:

- Emittance $\epsilon_x = \epsilon_y = 10^{-6}$ m and $\delta_{max} = \pm 0.25\%$.
- This gives a factor of 25 improvement in acceptance for a factor of 3 increase in length.

Halving β_x^i and decreasing emittance to $2\epsilon_x = \epsilon_y = 10^{-6}$ m gives $\sigma_y = 16$ nm, $\sigma_x = 66$ nm. This configuration was not optimal in the sense of balancing the leading order chromatic and geometric aberrations. Further, it had unacceptable emittance blowup from synchrotron radiation[9].

We then reduced the dipoles to 100G to eliminate emittance blowup. This requires combined function magnets but without these or proper octupole corrections, this gave first-order spot sizes with:

• Emittance $\epsilon_x = 10^{-6}$ m, $\epsilon_y = 10^{-7}$ m and $\delta_{max} = \pm 0.25\%$.

Reducing either ϵ_x (or ϵ_y) by this amount is equivalent to making uncorrected aberrations like $(y|y'^3)$ (or $(x|x'^3)$) dominant. The result is shown in Fig. 4 with ϵ_y reduced because $M_{y'} \gg M_{x'}$.

5. Conclusions and Final Comments

While conclusions depend some on the specific solution, the full 6-D emittance and its shape at the IP, we find that the:

- 1. Dominant chromatic aberrations in order of importance for x and then y are[5]: $(x|x'\delta^2), (x|\delta^3), (x|yy'\delta), (x|xx'\delta) \cdots$ $(y|y'\delta^2), (y|xy'\delta), (y|y\delta^2), (y|x'y\delta), \cdots$
- 2. Dominant geometric aberrations in order of importance are^[5]: $(x|x'y'^2), (x|y'^4), (x|x'^3) \cdots (y|x'^2y'), (y|y'^3) \cdots$.



Figure 4: Distribution versus Position from the Paraxial Focus.

Whether terms like $(\mathbf{x}|\mathbf{x}'\delta^2)$ or $(\mathbf{x}|\mathbf{x}'y'^2)$ are larger depends on the emphasis between dipoles and sextupoles. Typically, one balances such terms and declares success when they get the required spot sizes for the required $\epsilon_{x,y}$ and δ .

Our approach is to make the leading order chromatics in x,y negligible by significantly reducing bends since sextupoles drive none of them[2]. One then corrects the geometrics locally with combined-function quad/octupoles in the first two weak quads of the telescope, taking advantage of the magnification factors discussed. Final focus systems at 1 TeV with lengths of \approx 500m and negligible emittance blowup with conventional technology appear practicable in this way.

Acknowledgements

We would like to thank Monica Gygi Hanney for help with Topdraw and La T_EX , John Jowett for use of his bag of T_EX tricks as well as Fulvia Pilat and Ghislain Roy for checking some of the calculations. Lastly, one of us would like to thank a number of people for their kind help in arranging his visit to CERN.

References

- [1] H.A.Enge, Rev. Sci. Instr., Vol. 35(1964)278.
- [2] J.E.Spencer, Proc. BNL Workshop on Accel. Orbit and Tracking, BNL-31761(1982)126-162.
- [3] H. Ferguson, J.E. Spencer and Klaus Halbach, A General Ion-Optical Correction Element, NIM, Vol.134(1976)409.
- [4] K.L.Brown, IEEE Trans. Nucl. Sci., Vol. NS-26(1979)3490.
- [5] K.L.Brown and J.E.Spencer, Non-Linear Optics for the FFS..., IEEE Trans. Nucl. Sci., Vol. NS-28(1981)2568.
- [6] A. Chao, J. Hagel, F. Ruggiero and B. Zotter, Proc. New Dev. in Part. Accel. Tech., CERN 87-11(1987)577-584.
- [7] W. Schnell, CERN CLIC Note-56, Dec. 1987.
- [8] D.C.Carey, K.L.Brown and F.C.Iselin, TURTLE, SLAC 246, Mar. 1982.
- [9] F. Ruggiero and B. Zotter, See these Proceedings.