

The Design Of Low Emittance Electron Storage Rings

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ABSTRACT: We have considered "high tune" electron storage rings as a possible source of low emittance beams. The parameters of such rings are studied in the limit where the emittance is determined by intrabeam scattering. Rings with either superconducting or conventional magnets are considered. The object is to maximize the ratio of electrons/bunch to invariant emittance while maintaining a certain fixed intensity.

We have also calculated the dynamic aperture for one ring of this type.

Introduction

The normalized horizontal emittance, $\epsilon^* \equiv \gamma \epsilon_x$, of an electron beam in a storage ring is determined by a competition between quantum fluctuations of the synchrotron radiation and classical radiation damping. ϵ^* is proportional to the cube of the ratio of the beam energy E_0 to the horizontal betatron tune Q_x . Usually the energy is fixed by other constraints, so the tune must be high to achieve low emittance.

Low emittance is very desirable for at least two applications: injection into a high energy linear collider, where the storage ring functions as a damping ring, and for creating a "point" source of x-rays in some synchrotron radiation applications. In the case of a linear collider, assuming that the emittance is preserved during acceleration of the electrons, there is a direct connection between the focussing strength required and the emittance divided by the number of electrons in a single pulse. The relationship is:

$$\beta_{FF} = \left(\frac{Nr_0}{\epsilon^*} \right) \left(\frac{\sigma_L}{D} \right) \quad (1)$$

In the formula above, β_{FF} is the focal length of the final focus optics, N , the number of electrons per pulse, σ_L the pulse length and D the disruption parameter. r_0 is the classical electron radius, $r_0 = 2.8 \times 10^{-15}$ meters. The second term in Eq. 1 must be minimized because it determines the ratio $\hat{L}$ of the luminosity to single beam power:

$$\hat{L} = \frac{1}{4\pi e^2} \left(\frac{D}{\sigma_L} \right) \sim 3.5 \times 10^{30} \left(\frac{D}{\sigma_L} \right) \text{ cm}^3 \text{ sec}^{-1} \text{ megawatt}^{-1} \quad (2)$$

e is the electronic charge. The only degree of freedom, once $\hat{L}$ is determined, is the "invariant brightness" $B^* \equiv \frac{Nr_0}{\epsilon^*}$. We have:

$$\beta_{FF} \propto \frac{B^*}{\hat{L}} \quad (3)$$

Thus it is important to maximize B^* for this type of application to reduce demands on the requirements for the high energy final focus optics*.

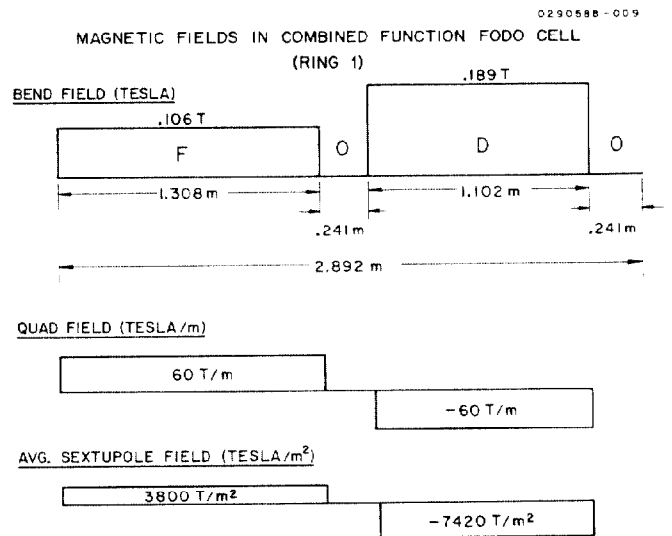
We might also wish to use rings of this type to make an intense point source of x-rays, perhaps for high resolution x-ray lithography or x-ray microscopy. In this case, ϵ^* is to be reduced to a value comparable to the wavelength λ of the x-rays of interest multiplied by $\gamma = \frac{E_0}{m_e}$, where m_e is the electron rest mass. The resolution limit from diffraction will predominate if:

$$\epsilon^* < \lambda \gamma \quad (4)$$

Recent advances in superconducting magnets have made it possible to design quadrupoles with much higher gradients than are normally used in electron storage rings. We present here a comparison of such superconducting rings with rings having only conventional magnets.

Parameters Of Rings At The Intrabeam Scattering Limit

In order to achieve a compact lattice, we have assumed a combined-function FODO lattice. We have used the design philosophy of Hofmann and Zotter¹. The lattice presented here was constrained to have equal F and D gradients for simplicity. The horizontal phase advance is taken to be 135 degrees per cell. This choice minimizes the horizontal emittance for a given number of unit cells. To further simplify this initial study, we assume here a purely circular machine with no straight sections. Figure 1 shows the dimensions of the magnets in a unit cell for the largest radius ring we have considered. The relative dimensions remain the same for all of the rings.



* Round gaussian beams are assumed. Higher luminosities per megawatt for a given amount of disruption may be obtained by using eccentric elliptical beams, but a similar relation to Eq. 3 will still hold.

As input parameters we have chosen the radius R , the quadrupole gradient G and the invariant emittance ϵ^* . The particular lattice we use follows the scaling law:

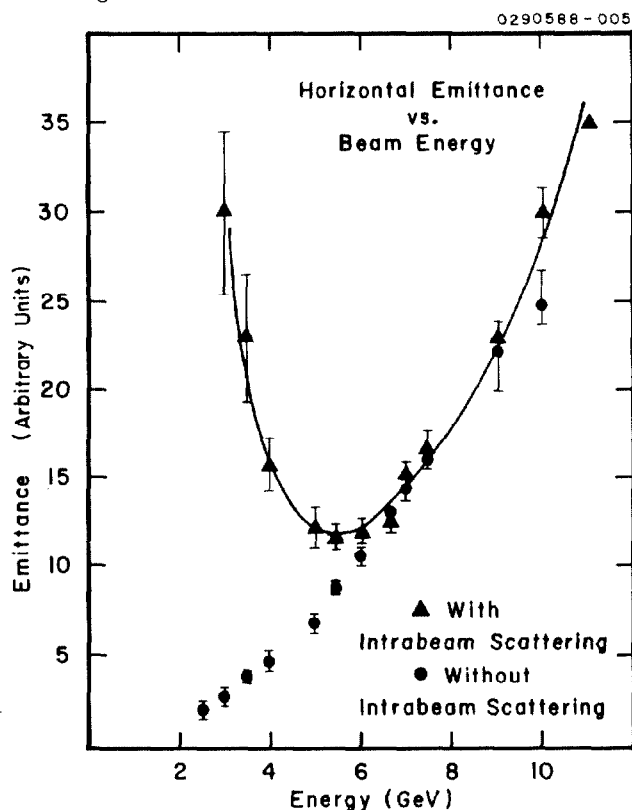
$$\gamma^3 = 3.519 \times 10^{10} G R^2 [\epsilon^*]^{\frac{2}{3}} \quad (5)$$

Once $\gamma = \frac{E_0}{m_e c^2}$ is known, we can calculate the electron energy. The number of unit cells, the tunes and the dimensions of the unit cell can all be derived from the relation:

$$N_{cell} = .4891 \frac{E_0}{[\epsilon^*]^{\frac{1}{3}}} \quad (6)$$

In these equations the units are: G [Tesla/meter], R [meters], ϵ^* [meter-radians], E_0 [GeV]. We chose three different rings for comparison. Rings 1 and 2 are made of conventional magnets, with $G = 60$ Tesla/meter, while Ring 3 is a superconducting ring with an optimistic choice for $G = 300$ Tesla/meter. The radius of Ring 1 is 300 meters, Rings 2 and 3 both have radii of 100 meters.

If we require that the horizontal emittance be determined by the combination of quantum fluctuations, intrabeam scattering^{2,3} and damping, we will obtain a relation between the number of electrons per pulse N_e and the emittance. If we increase N_e from a low value until the emittance has increased by a factor of $\frac{5}{3}$ over its value for $N_e = 0$, we can show that at this point $\epsilon^*(\gamma)$ is a minimum when the energy is varied⁴. We consider this to be the optimum $N_e \equiv N_e^*$ for this choice of G, R, ϵ^* . Varying the energy will produce a larger ϵ^* , hence a lower B^* . Increasing $N_e > N_e^*$ will produce only a marginal increase in B^* , due to the increased emittance from intrabeam scattering.



At this point we can examine various parameters as a function of the emittance for a particular choice of ring, i.e. fixed G, R . We find that N_e^* increases very rapidly with ϵ^* . The value of B^* increases monotonically with the emittance. Thus

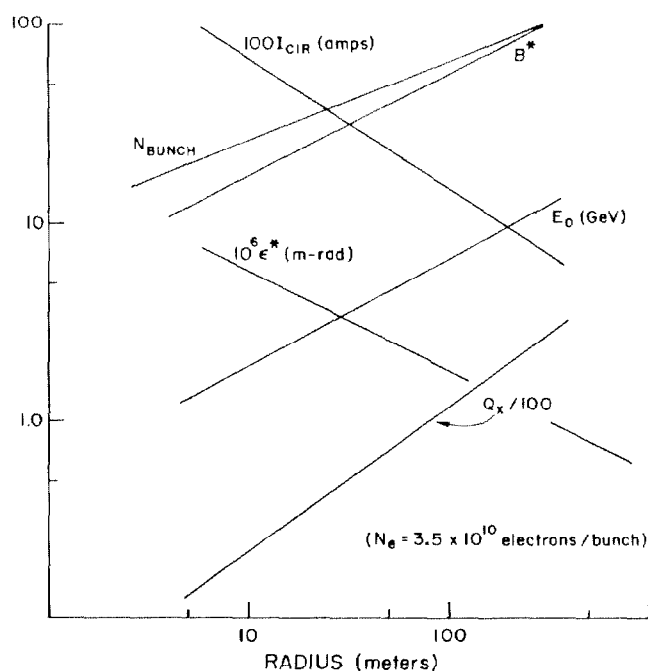
the maximum brightness will be limited by the maximum possible number of electrons which can be contained in a bunch. The actual determination of the maximum possible N_e^* depends on a detailed study of multi-bunch and single-bunch beam instabilities, bunch lengthening effects, etc. For the linear collider application, we have taken the rather short bunch length of $\sigma_L = 0.7$ mm to be a fixed parameter in the calculations. If a different bunch length were assumed, the intrabeam scattering scales as $N_e^* \propto \sigma_L$. To avoid getting too involved in details of the transverse and longitudinal impedances of possible rf systems at this point, we have just constrained $N_e^* = 3.5 \times 10^{10}$ e/bunch. This is not too different from the design value for the SLAC SLC damping ring.

The resulting rings have B^* up to two orders of magnitude higher than existing storage ring beams. The tune values are significantly higher than are normally encountered.

The scaling of selected parameters for conventional rings is shown in figure 3 as a function of ring radius.

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SCALING OF VARIOUS QUANTITIES
WITH RADIUS
FOR CONVENTIONAL QUADS



To complete the parameters for a linear collider damping ring, we assume that we must extract 6×10^{12} electrons/second. (This is one megawatt of single beam power at an energy of 1 TeV.) To damp the emittance completely, we assume each electron remains 20 damping times in the ring. These two constraints determine the number of bunches, hence also the bunch spacing. For the choice of conventional quadrupoles with $G = 60$ Tesla/meter, the number of bunches ranges from 86 if $R = 300$ meters to 21 if $R = 6$ meters.

Initially it was thought the higher gradient possible with superconducting quadrupoles would be a significant advantage. However all of these rings have a synchrotron radiation power density of more than hundreds of watts/meter. This appears to be difficult to handle in a cryogenic ring with a necessarily small aperture.

Ignoring this difficulty, we considered superconducting rings with five times the gradient of the normal conductor rings. Ring 3 is an example of this case, with the same radius of 100 meters as Ring 2, the normal ring. We see that the superconducting case gains a factor of 3 in B^* , but at the price of a considerably higher tune, thus a correspondingly larger number of cells. For Ring 3 the number of cells is 614 with a $Q_x = 230$, while for Ring 2 the number of cells is 318 and $Q_x = 119$, about a factor of two difference. Sextupole strengths to produce zero chromaticity would increase by a factor of 20 if we compare Ring 3 with Ring 2. This may be prohibitively large. We conclude that the disadvantages of using superconducting magnets outweigh the advantages in this application.

Ring 4: A Small Machine With A 6 Meter Radius

We can do surprisingly well with a small machine. We list here the parameters of Ring 4:

Table I
Ring 4 Parameters

Radius	6	meters
E_0	1.403	GeV
Q_x	15.75	
Q_z	10.91	
N_{cell}	42	
L_{cell}	.898	meters
$\bar{B}$	.78	Tesla
G	60	Tesla/meter
sextupole fields:		
F	.068	Tesla (at 1 cm.)
D	-.131	Tesla (")
N_e^*	3.5×10^{10}	electrons/bunch
I_{cir}	.93	total current in amps
radiated power	53	kilowatts
power density	1420	watts/meter
ϵ^*	6×10^{-6}	meter-radians
B^*	13	
N_{bunch}	21	bunches
$f_{extraction}$	173	Hz.
energy spread	.049	%
σ_L	.7	mm
synchrotron tune	.017	

Dynamic Aperture For High Tune Machines

Based on tracking calculations for a particular high tune damping ring, Talman ⁵ has pointed out that the sextupole strength necessary to produce zero chromaticity in such rings is very large, and may have the undesirable side effect of reducing the dynamic aperture below tolerable limits. A crude estimate shows that the sextupole strength S scales like

$$S \propto \frac{G}{R} Q_x^2 \quad (7)$$

However, this does not seem to be a problem for the particular ring we have considered. The dynamic aperture was estimated by first calculating the Lie coefficients ⁶ and then employing them in a second order symplectic tracking program, using a method invented by Dragt ⁷. A particle was considered to be

on a stable orbit if it remained in the machine for at least 10^4 turns. Satisfactory horizontal and vertical dynamic apertures are obtained for the 6 meter Ring 4 machine. The stable limits of emittance are more than 500 times the emittance produced by the ring.

In addition, the dynamic apertures can further be increased by appropriate small perturbations on the individual sextupoles. We are currently developing a method for optimizing the apertures for a given configuration.

After this optimization procedure is implemented, it will be possible to see whether there is a practical upper limit to the size of a damping ring from the dynamic aperture requirement.

Conclusions

We have shown that it is possible to improve the value of the invariant brightness B^* over its value for existing beams if we use conventional quadrupoles in a high tune damping ring. The choice of design parameters is strongly influenced by intrabeam scattering. In a ring with a radius of 300 meters and an energy of 12.3 GeV, it should be possible to obtain $\epsilon^* \sim 1 \times 10^{-6}$ meter-radians and a B^* value of ~ 98 . These are, respectively, one and two orders of magnitude better than with the present SLC damping ring. At the other extreme, a ring with a radius of 6 meters and an energy of 1.4 GeV will have $\epsilon^* \sim 6 \times 10^{-6}$ meter-radians and $B^* \sim 13$. For comparison, the SLC damping ring has $\epsilon^* \sim 10^{-5}$ and $B^* \sim 1$.

We find there are no serious problems with dynamic aperture for the choice of parameters proposed here.

Superconducting quadrupoles do not appear to offer sufficient advantages in this application to outweigh the difficult engineering problems expected if they were to be used in electron machines.

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