OPTICS FOR PHASE IONIZATION COOLING OF MUON BEAMS*

Y. Derbenev, S. A. Bogacz, Jefferson Lab, Newport News, VA, USA
R. P. Johnson#, Muons, Inc., Batavia, IL, USA

Abstract

The realization of a muon collider requires a reduction of the 6D normalized emittance of an initially generated muon beam by a factor of more than 10^6. Analytical and simulation studies of 6D muon beam ionization cooling in a helical channel filled with pressurized gas or liquid hydrogen absorber indicate that a factor of 10^6 is possible. Further reduction of the normalized 4D transverse emittance by an additional two orders of magnitude is envisioned using Parametric-resonance or Phase Ionization Cooling (PIC). To realize the phase shrinkage effect in the parametric resonance method, one needs to design a focusing channel free of chromatic and spherical aberrations. We report results of our study of a concept of an aberration-free wiggler transport line with an alternating dispersion function. Resonant beam focusing at thin beryllium wedge absorber plates positioned near zero dispersion points then provides the predicted PIC effect.

INTRODUCTION

Muon collider luminosity depends on the number of muons in the storage ring and on the transverse size of the beams in collision. Ionization cooling (IC) as it is presently envisioned [1] will not cool transverse beam sizes sufficiently well to provide adequate luminosity without large muon intensities. A new idea to combine ½ -integer parametric resonances and IC (PIC) [2] in a linear focusing channel has been proposed that will lead to much smaller transverse beam emittances so that high luminosity in a muon collider can be achieved with fewer muons. In this report we describe basic principles and potentials of PIC and address the main constraints associated with tune spreads and energy straggling in the muon beam.

OVERVIEW OF A PIC CHANNEL

PIC takes place in the sections of a cooling channel where the beam is strongly focused onto beryllium wedges and where a special dispersion function is used to create the conditions for aberration correction and emittance exchange (EMEX). Since the dispersion oscillates in these sections, we call them snake phase cooling (SPC) sections. A basic constraint of the PIC channel is that there must be regular replenishment by RF cavities of the energy lost in the beryllium wedge absorbers. Since the beam must pass through several cavity apertures, the focusing must be relatively weak in each RF section. So there must be a compressor transition section between each strong focusing SPC section and the next RF section and an expander transition section between each RF section and the next SPC section. Another feature is a skew quad section to exchange the x and y planes since we have in mind that the whole channel will have only horizontal bends in the PIC region where a very special dispersion function is required. For optimal EMEX, the three cooling decrements should be equal on average along the cooling channel; this requires a relationship between dispersion *D* and the absorber wedge height *h* [3]:

$$(D/h) = 2-(4\beta^2/3) \tag{1}$$

To take advantage of the magnet apertures, the maximum beam size should be more or less constant. That is, as the beam cools, the focusing should increase. The maximum beta should increase and the minimum should decrease until it approaches the thickness of the absorber plate. This process is described by the usual relationships: $\sigma_{\max} \approx \sigma_0$; $\sigma_{\max}\cdot\sigma_{abs} = f\varepsilon$; $\beta_{\max}\beta_{abs} = f^2$; $\beta_{abs} \ge w$; and $\beta_{abs} \Rightarrow w_{final}$.

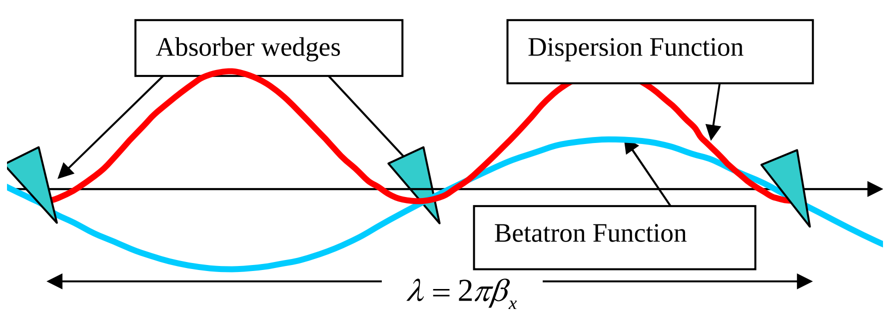

Figure 1: Schematic of a snake phase cooling (SPC) section, where wedge absorbers are placed at symmetric locations relative to the dispersion function, which has a period half that of the betatron function. Chromatic aberration correction sextupoles are placed at the maxima of the dispersion functions. The wedge absorbers require smaller dispersion for EMEX to control the momentum spread during the cooling process.

*Supported by U.S. DOE Contract No. DE-AC05-84ER40150 and SBIR grant DE-FG02-04ER84016
rol@muonsinc.com

Here ε_0 and σ_0 are, respectively, the initial beam emittance and size (after 6D cooling) in a channel with characteristic focal parameter *f*. $\sigma_{\max}$ and $\beta_{\max}$ are the beam size and beta function at the entrance to a SPC section, while β_{abs} and σ_{abs} are the beta function and beam size at the absorber plates of the SPC. These last two parameters together with the beam emittance progressively decrease cell by cell of the cooling channel, while $\beta_{\max}$ (i.e. beam expansion after RF to next SPC) is designed correspondently to grow. The minimal β_{abs} is limited by the plate thickness w, which (together with achievable emittance and cooling channel length), in turn, is limited by beam loss due to muon decay (and also by the tuning demands). With this design concept, we avoid a large increase of beam size at the entrance to the SPC compared to its characteristic magnitude after the 6D cooling, while achieving a maximally strong beam focus at the absorber plates at the end of cooling channel. For optimum beam extension:

$$(\beta_{\max} / f) = (\varepsilon_0 / \varepsilon) \Rightarrow (f / w_{final})$$

BASIC BEAM TRANSPORT AND LINEAR OPTICS DESIGN PRINCIPLES

The main constraint of the parametric resonance ionization cooling design is to combine low orbit dispersion at the wedge absorber plates with the necessary large dispersion in the space between plates required to compensate for chromatic aberration. These conflicting requirements can be resolved in a channel with plane wiggling beam orbit created by an alternating dipole field (see Figure 1).

In such a channel, the dispersion alternates along with the wiggling orbit. The absorber plates then are positioned near zero dispersion points. Obviously, the betatron (i.e. focusing) wave length must not be same as the bend and dispersion period, but it can be two or four times longer then that. Figure 2 shows an example of such a cooling channel lattice cell with additional components to provide space for RF to replenish the energy lost in the absorbers.

TUNING DEMANDS

Strong focusing of the muon beam at the absorber plates (especially in the final cells) requires a sufficiently small spread of the focal parameter *f* caused by various aberrations (chromatic, spherical, third order non-linear fields, and space charge detuning). The general requirement for this condition is

$$\Delta f \cdot \pi q << \beta_a \approx f \cdot (\varepsilon / \varepsilon_0) \Rightarrow w_{final} .$$

Here *q* is the number of absorber plates in a cell (which may vary from a maximum at start to a minimum in th final cell). The first three aberrations can be compensated by sextupoles and octupoles. Detuning due to space charge cannot be compensated.

Calculation of the space charge defocusing impact on phase ionization cooling leads to the relation

$$\frac{\Delta f}{f} = \frac{f}{\sigma_z \sqrt{2\pi}} \frac{N r_\mu}{\beta \gamma^2 \varepsilon_0} .$$

Here *N* is the number of muons in a bunch, σ_z is the rms bunch length, and r_μ is the classical radius of the muon. This relation shows that that the space charge detuning in the SPC cells is determined by the initial beam emittance ε_0 and not the cooled emittance ε.

Table I shows the expected PIC performance for a muon collider cooling channel.

Table I: Potential PIC effect

Parameter	*Unit*	*Initial*	*Final*
Beam momentum, p (average)	MeV/c	100	100
Distance between plates, $\lambda / 2$	cm	40	40
Plate thickness, w	mm	6.4	1.6
Intrinsic energy loss rate (Be), $E_{\mathrm{int}\,r}$	MeV/m	600	600
Average energy loss in cooling section	MeV/m	20	5
Transverse emittance, norm.	µm	600	25
Beam transverse size at plates	mm	6.0	0.15
Angle spread at plates, $\theta_x = \theta_y$	mrad	200	200
PIC channel length	m	150	
Integrated energy loss	GeV	0.7	
Beam loss due to muon decay	%	20	
Number of particles/bunch		10^{11}	
Space charge tune spread, $\Delta f / f$		10^{-3}	

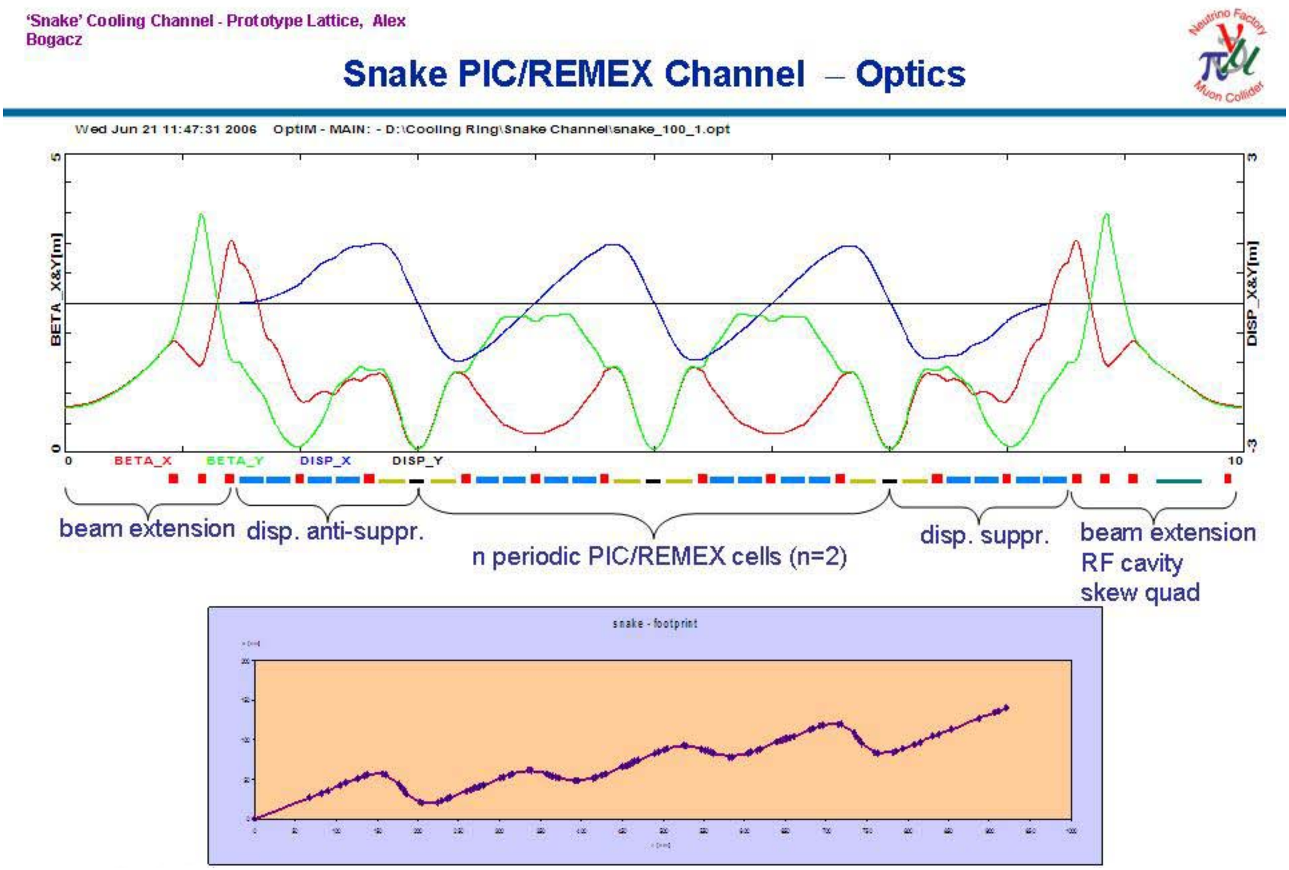

Figure 2: OptiM displays of a PIC (or REMEX) cooling lattice cell. The upper graph has the lattice functions for a 10 m long cell with sequential sections for beam extension, dispersion creation, SPC, dispersion suppression, beam compression, RF, and x-y plane interchange. The dispersion is created by the blue dipoles which have significant edge focusing to complement the red quadrupoles and yellow solenoids to achieve the required tight focusing at the black wedge absorbers. Sextupoles and octupoles in the high dispersion regions will correct chromatic and spherical aberrations. The lower graph is a plan view of the "snake" channel.

PIC POTENTIAL

The PIC effect is related to the absorber material and thickness by

$$\varepsilon \Rightarrow \approx \frac{\sqrt{3}}{4\beta}(Z+1)\frac{m_e}{m_\mu}w;$$

$$wE'_{\mathrm{int}\,r} = < E' > \pi f$$

Where $E'_{\mathrm{int}\,r}$ is the intrinsic energy loss rate of the absorber. Beryllium wedges are favorable since they have low Z, have sufficient density to allow a small *w*, and are technically appropriate in that they can be easily shaped into wedges and refrigerated.

CONCLUSIONS AND OUTLOOK

The transverse emittances of muon beams can be reduced to those normally associated with conventional electron or hadron colliders by implementing Phase Ionization Cooling. This has extremely important consequences for any muon collider design, primarily in the number of muons required for high luminosity.

Some concepts of beam focusing and tune spread compensation for the best parametric resonance cooling and reverse emittance exchange have been proposed. The compatibility of beam resonance focusing with effective exchange is understood. The analytic expressions shown here in the preliminary OptiM design will be used for guidance of simulations.

REFERENCES

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