A NEW ALGORITHM FOR THE CORRECTION OF THE LINEAR COUPLING AT TEVATRON*

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Abstract

The Fourier analysis of Turn by Turn (TBT) data provides valuable information about the machine linear and non-linear optics. A program for the measurement and correction of the linear coupling based on the TBT data analysis has been integrated in the TEVATRON control system. The new method is fast, allows the measurement of the coupling during acceleration and offers information about the sum coupling coefficient and the location of the sources of coupling.

BASIC RELATIONS

Here we limit ourselves to outline the theory behind the method, referring the reader to [1] for more details. We assume the unperturbed transverse motion to be uncoupled and will ignore coupling between longitudinal and transverse motion. We will also assume the coupling between the horizontal and vertical motion to be weak.
After exciting beam oscillations by a *single* kick, the beam position at the j-th BPM is

$$z_n^j = \sqrt{\beta_z^j} e^{i\Phi_z^j} A_z e^{iQ_z(\theta_j + 2\pi n)} + c.c. \qquad (z = x, y)$$

with n turn number, $\Phi_z \equiv \int_0^\theta d\theta' R/\beta_z - Q_z\theta$ (periodic phase function) and $A_z = |A_z| e^{i\delta_z}$ constant of motion. Fourier analysis of the TBT data has been used to investigate the linear uncoupled optics of ring accelerators (see for instance [2]). The relationships are

$$\beta_z^j = |Z_j(Q_z)|^2 / A_z^2 \quad \text{and} \quad \mu_z^j = \arg\,(Z_j) - \delta_z$$

$Z_j(Q_z)$ being the Fourier component of z_j at the betatron frequency Q_z evaluated at the j-th BPM. While δ_z is an unessential additive constant, the phase advance being defined anyway up to an additive constant, the oscillation amplitude $|A_z|$, which depends upon the β_z function value at the kicker location as well as the kicker strength, must be determined by some kind of averaging over the BPM's relying on a machine model. Owing to the fact that the quadrupole circuits are in practice adjusted to match the design tunes, and therefore $< 1/\beta_z > = < 1/\beta_{0z} >$, it is convenient to compute the amplitude as

$$|A_z|^2 = \frac{\sum_j 1/\beta_{0z}}{\sum_j 1/|Z_j(Q_z))|^2}$$

and to sum only over the BPM's in the regular cells where the phase advance between BPM's is in general much smaller than π.

In the presence of coupling, the excitation of one of the two modes will excite an oscillation in the other mode too; if for instance the beam is kicked in the horizontal plane, the resulting vertical motion is in first order approximation described by

$$y_n^j = \Big[\sqrt{\beta_y^j}\Big(e^{-i\Phi_y^j} w_+^j - e^{i\Phi_y^j} w_-^j\Big) - \sqrt{\beta_x^j} e^{i\Phi_x^j} \sin\chi_j\Big] A_x e^{iQ_x(\theta_j+2\pi n)} + c.c.$$

where χ_j is the tilt of the j-th BPM. When instead the vertical mode is excited, the resulting horizontal motion is

$$x_n^j = \Big[\sqrt{\beta_x^j}\Big(e^{-i\Phi_x^j} w_+^j + e^{i\Phi_x^j} w_-^{*j}\Big) + \sqrt{\beta_y^j} e^{i\Phi_y^j} \sin\chi_j\Big] A_y e^{iQ_y(\theta_j+2\pi n)} + c.c.$$

The functions $w_\pm$ are related to the distribution of coupling elements by

$$w_\pm(\theta) = -\int_0^{2\pi} d\theta' \frac{C_\pm(\theta')}{4\sin\pi Q_\pm} e^{-iQ_\pm[\theta-\theta'-\pi \text{sign}(\theta-\theta')]}$$

with $Q_\pm \equiv Q_x \pm Q_y$ and

$$C_\pm(\theta) \equiv \frac{R\sqrt{\beta_x\beta_y}}{2B\rho}\Big\{\Big(\frac{\partial B_x}{\partial x} - \frac{\partial B_y}{\partial y}\Big) + B_\theta\Big[\Big(\frac{\alpha_x}{\beta_x} - \frac{\alpha_y}{\beta_y}\Big) - i\Big(\frac{1}{\beta_x} \mp \frac{1}{\beta_y}\Big)\Big]\Big\} e^{i(\Phi_x \pm \Phi_y)}$$

R being the machine radius. The functions $\tilde{w}_\pm \equiv w_\pm e^{iQ_\pm\theta}$ are constant in coupler free regions and experience a discontinuity $-iC_\pm\ell/2R$ at coupler locations, with ℓ coupler length. On the resonances $Q_x \pm Q_y = int$ the functions $\tilde{w}_\pm$ are constant. The minimum reachable tune distance, Δ, is given by $\Delta \equiv |\bar{C}_-|$ with

$$\bar{C}_\pm = \frac{n_\pm - Q_\pm}{\pi} \int_0^{2\pi} d\theta\; w_\pm e^{in_\pm\theta}$$

with $n_\pm \equiv \text{Round}(Q_x \pm Q_y)$. If the kick occurs in the horizontal plane (x and y must be exchanged otherwise) the Fourier component $Y_j(Q_x)$ of $y_j(\theta)$ is related through the twiss functions to the values of $w_\pm$ at the j-th BPM. When the BPM tilt is negligible or already known, the number of unknown quantities per BPM reduces to two and, under the assumption that between two consecutive BPM's there are no strong sources of coupling, we can retrieve the (constant) value of $\tilde{w}_\pm(\theta)$ in the region between them by knowing $Y_j(Q_x)$ and $Y_{j+1}(Q_x)$. At TEVATRON the BPM tilt

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is one of the parameters fitted by the optics measurement method based on difference orbits[3]. Non systematic tilt errors can be evaluated also through the here presented approach by imposing $w_\pm$ to vary smoothly along the ring. Both methods agree in finding the "worse cases".

APPLICATION TO TEVATRON

TEVATRON has 118 horizontal and 118 vertical BPM's. They can store 8192 positions data per BPM. The recent upgrade of their electronics allows a precise measurement of the TBT beam position (resolution $\simeq$ 50 μm) making possible the use of the technique here outlined for optics diagnostic and coupling correction. In particular, TEVATRON being a fast ramping machine (83 seconds from 150 to 980 GeV), the TBT analysis is the only practical method for measuring optics and coupling also during acceleration. Under ideal conditions ("un-coalesced" beam, small chromaticities and octupoles turned off) the oscillations last some thousands turns. The reconstructed β functions for the injection optics are shown in Figs. 1 and 2 together with the model ones which are obtained by the difference orbit method[3]. The coupling functions $\tilde{w}_+$ and

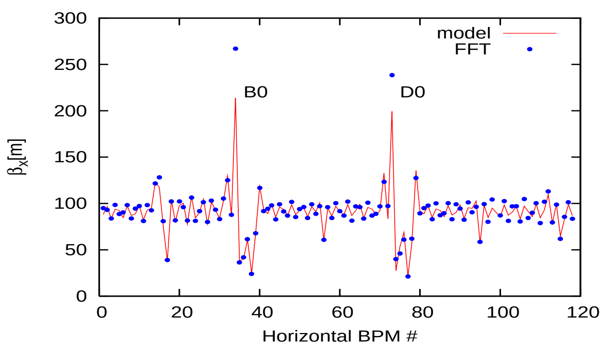

Figure 1: β_x at horizontal BPM's .

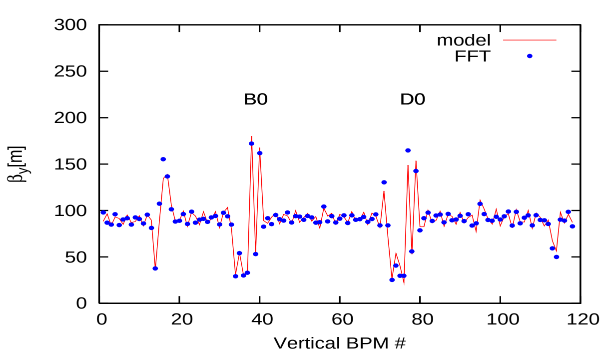

Figure 2: β_y at vertical BPM's .

$\tilde{w}_-$, measured at the vertical BPM's when kicking in the horizontal plane and at the horizontal BPM's when kicking in the vertical one, are shown in Figs. 3 and 4 respectively.

BPM's calibration errors do not affect the reconstruction of the phase advance, but do affect the value of β_z^j ($z = x, y$)

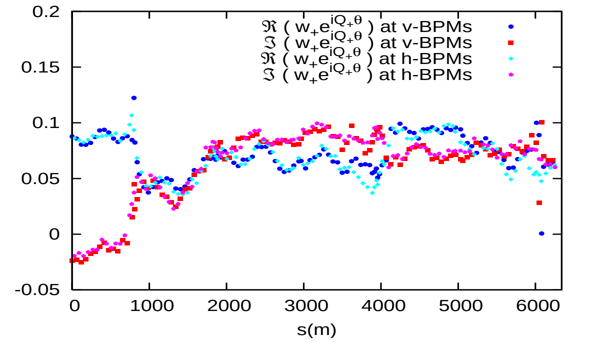

Figure 3: Measured $\tilde{w}_+ \equiv w_+ e^{iQ_+\theta}$ at the vertical BPM's (blue and red) after a horizontal kick and at the horizontal BPM's (cyan and magenta) after a vertical kick.

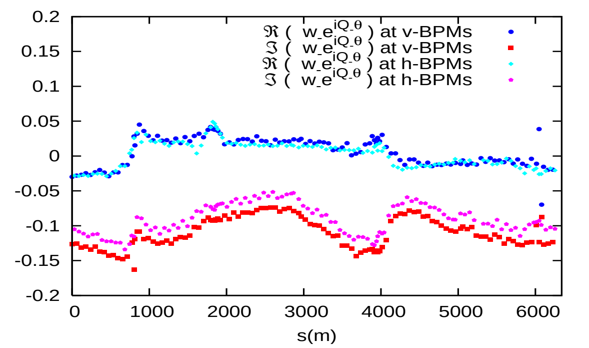

Figure 4: Measured $\tilde{w}_- \equiv w_- e^{iQ_-\theta}$ at the vertical BPM's (blue and red) after a horizontal kick and at the horizontal BPM's (cyan and magenta) after a vertical kick.

computed through the Fourier analysis. The effect of the random (that is a monitor dependent constant) part of such calibration errors results in a unphysical beta-beating, but it is likely to average away when computing the oscillation amplitude. A systematic (that is common to all BPM's) calibration error instead has almost no effect on the evaluation of the β functions, but results in a wrong estimate of the oscillation amplitude and therefore of $w_\pm^j$, unless the error is the same, for both horizontal and vertical BPM's. By equating the element M_{12} of the measured transfer matrix and the design one and solving for β_z^j one can compute β_z^j resorting on the measured phase advance[2]. This requires (at least) three (consecutive) BPM's. The scatter of the triad of values, β_z^{ji} (i=1,2,3), obtained for each BPM give informations about the presence of focusing perturbations in the region between the involved BPM's and/or about their possible diseases; comparison with the value computed through the Fourier analysis may be used to calibrate the BPM's involved. Fig. 5 shows for instance typical ratios of the β_x values computed by the Fourier analysis to the values computed through the transfer matrix. The average ratio is 0.9987, indicating that the estimate of the oscillation amplitude is quite good. The BPM's where each of the three values β_x^{ji} differs less than 1% from their aver-

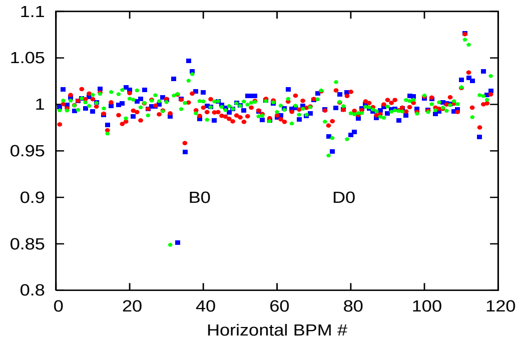

Figure 5: h-BPM's relative calibration.

age are retained to re-scale the oscillation amplitude. However this will *not* correct for a possible systematic calibration error. Through simulations we have estimated that for TEVATRON the error on the evaluation of $|\bar{C}_-|$, is about 2.5% for 5% systematic calibration error of either horizontal or vertical BPM's (they cancel out when the error has the *same* value) and 0.5% for 5% random calibration errors. A systematic tilt by 1^0 of all BPM's results in a error $|\delta\bar{C}_-| \lesssim 0.0002$; the error due to random tilts is negligible. An application program for the TBT analysis has been integrated in the TEVATRON control system. The program

- fires the horizontal or vertical kicker
- computes the linear twiss and coupling functions
- computes and applies the needed corrections to the skew quadrupole circuits SQA0 and SQ.

The application has been thought mainly as a fast tool for correcting the coupling during shot set up, when octupoles are powered on and the chromaticity amounts to about 4 units in both planes. Although these are not ideal conditions for the measurement, the TBT based coupling correction proved to work well.
Currently, the time needed to retrieve the data is too large (for instance the reading of 256 turns for all 236 BPM's takes 7 minutes) to make use of all BPM's during routine operation. This is not a too serious limitation.

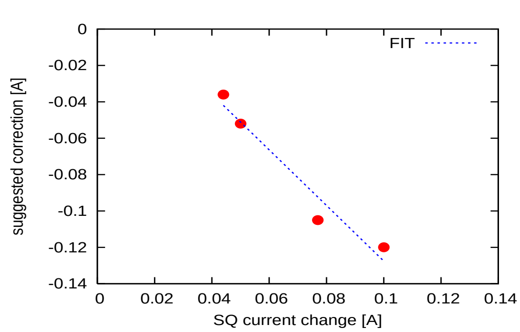

Figure 6: Correcting SQ current vs. SQ excitation.

Resorting to the fact that $\tilde{w}_\pm \simeq const$ near the resonances $Q_x \pm Q_y = int$ (the TEVATRON working point Q_x=20.584 and Q_y=20.574 lies reasonably close to the

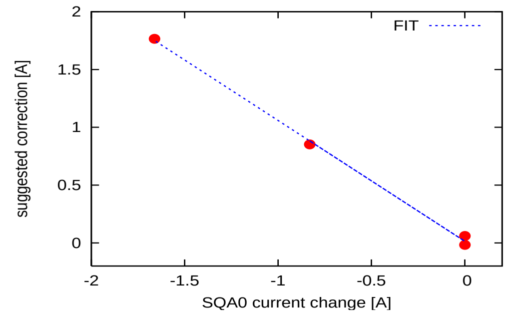

Figure 7: Correcting SQA0 current vs. SQA0 excitation.

sum resonance too), we use only few monitors (typically 5 horizontal and 5 vertical BPM's) to evaluate the tune Q_z and the amplitude invariant $|A_z|$ of the excited mode and the functions $w_\pm$ at the orthogonal mode BPM's. Subsequent off-line analysis using all BPM's has shown only little differences. Figs. 6 and 7 show the correction suggested by the program when, after having well corrected the starting coupling, the SQ and SQA0 circuits were powered so to introduce a "known" coupling (Δ between .004 and 0.010 units). Ideally, the points should lye on the straight line $y = -x$. Finally, Fig. 8 shows how the value of Δ com-

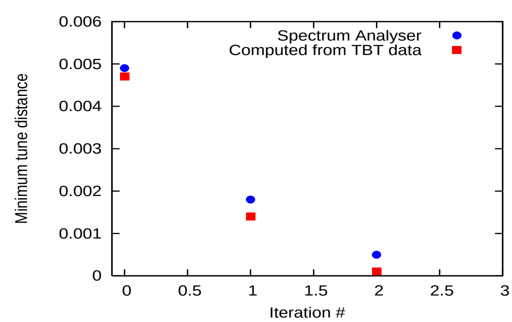

Figure 8: Minimum tune distance measured by the spectrum analyser (blue) and computed through the TBT analysis (red).

puted by the program through the TBT analysis compares to the minimum reachable tune distance measured through the spectrum analyser. The value of Δ was reduced in two iterations. Both measurements agree that the coupling was reduced by about an order of magnitude.

REFERENCES

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