# THE ISOCHRONOUS MODE OF THE COLLECTOR RING

S. Litvinov,\* A. Dolinskii, H. Weick, H. Geissel, F. Nolden, M. Steck GSI, Darmstadt, Germany

#### Abstract

The isochronous mode of a storage ring is a special ionoptical setting in which the revolution frequency of circulating ions of one species does not depend on their velocity spread. In this mode the ring can be used for mass measurement of exotic nuclei. The Collector Ring (CR) [1, 2] of the FAIR project [3] will operate in such mode as time-of-flight spectrometer for short-lived exotic nuclei ( $T_{1/2} > 20\mu s$ ) produced and selected in flight with the Super-FRS fragment separator [4]. This technique for mass measurements has been developed at the ESR at GSI [5].

#### **INTRODUCTION**

The Collector Ring (CR) is symmetric with two arcs and two straight sections and a total circumference of 210 meters. It is designed for operation at a maximum magnetic rigidity of 13 Tm. The momentum acceptance in the isochronous mode is  $\pm 0.5\%$  and transverse acceptance is 100 mm mrad in both planes [1, 2]. The mass-to-charge ratio m/q of the stored ions circulating in the ring can be measured from the revolution frequency (f) (or revolution time) and the velocity (v) of the ions.

$$\frac{\Delta f}{f} = -\frac{1}{\gamma_t^2} \cdot \frac{\Delta(m/q)}{(m/q)} + \left(1 - \frac{\gamma^2}{\gamma_t^2}\right) \frac{\Delta v}{v},\qquad(1)$$

where  $\gamma$  is the relativistic Lorentz factor and  $\gamma_t$  the transition point of the ring. The isochronous condition is reached when  $\gamma = \gamma_t$ . In the isochronous mode of the CR  $\gamma_t$  was adjusted by a special quadrupole setting to the value of 1.84 [6] which would allow to look at mass-to-charge ratios up to m/q = 2.71. The revolution time of the circulating ions is measured with a time-of-flight (TOF) detector installed in the straight section of the ring [7]. To achieve the frequency resolution of  $10^{-6}$  we have to measure the revolution time over many turns. There are many factors which influence the revolution frequency of particles in the ring. Here we consider the influence of the field errors in the magnets, the fringe fields of the quadrupoles, the closed orbit distortion (COD), and of the transverse acceptance.

## ANALYSIS OF THE REVOLUTION FREQUENCY

An accurate determination of the m/q ratio of an ion depends on how well one knows references m/q for calibration and how well one can measure the centroid of the frequency signal. The ability to recognize the frequency signals depends on its width  $(\delta f)$ . Ions with different m/q can be separated in frequency if their mean frequency separation  $\Delta f$  is larger than the full frequency width of the beam

$$\Delta f > \delta f. \tag{2}$$

For the particles of one species we can write:

$$\frac{\delta f}{f} = \frac{\Delta v}{v} - \frac{\Delta L}{L},\tag{3}$$

where L is the path length of the particles.

To understand what effects are more critical for the relative frequency spread  $\delta f/f$  we made simulations with a special Monte-Carlo particle tracking code. In this code all dipoles and quadrupoles are represented by first order matrices and all multipoles are introduced by thin nonlinear lenses. This approximation is tolerable since multipoles are usually weak compared to the normal elements. Such a thin lens keeps the transverse spatial coordinates of a particle unchanged, while their effect on the transverse momentum of an ion is considered (kick-approximation). Thus phase space is conserved when passing a thin multipole. The beam is generated with a homogenous or Gaussian distribution according to longitudinal momentum spread and rings' initial transversal acceptance. The frequency of each particle is calculated turn by turn. The frequency width can be written as a convolution of:

$$f = f_{accept} * f_{COD} * f_{field\ error},\tag{4}$$

where  $f_{accept}$ ,  $f_{COD}$ ,  $f_{fielderror}$  are the contributions of the transverse acceptance, closed orbit distortions and nonlinear field errors to the frequency width. The investigations of these effects have been studied separately in the following steps:

- Influence of the rings transverse acceptance on the frequency width. The ring is assumed to be ideally isochronous e.g., whithout closed orbit distortions and without field errors ( $\delta f_{field\ error} = \delta f_{COD} = 0$ );
- Influence of the closed orbit distortion for a given emittance of the beam in the ring but without field errors ( $\delta f_{field\ error} = 0$ );
- Contribution of nonlinear fields to the frequency spread. This effect appears due to field imperfections of the quadrupole and dipole magnets and additional sextupole and octupole correctors;
- The influence of the fringing fields resulting from the wide aperture quadrupole magnets.

#### Influence of transverse acceptance

Particles away from the central orbit perform betatron oscilations which cause the path length to increase. Thus

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Figure 1: Calculated frequency spectra for two beams with emittance of 10 and 100 mm mrad.



Figure 2: Relative spread in frequency as a function of emittance. The change of the slope for very large emittance is due to particle loss.

these ions will have a longer time-of-flight. This can be also seen in Fig.1. The calculated frequency spectra of two beams only differing by their emittance are shown. The spread of the frequencies increases but also the mean value changes. The latter fact is of even larger importance for a mass measurement based on the centroids of the frequency distribution as it can cause systematic errors that cannot be improved by better statistics. The frequency width increases linearly with the beam emittance, which can be treated as acceptance. The dependence on the emittance can be seen in Fig.2. In the calculations we use a beam of 100 particles of one species are tracked over 100 turns. The linear dependence of the frequency spread on the emittance can be understood. In a simplified case, there is no path length variation with energy ( $\gamma = \gamma_t$ ), no chromatic effects and no closed orbit distortions. This means the path length and consequently the revolution frequency depends only on the betatron oscillations. In a leading order expansion the

path length differences are caused by the angle of an ion versus optical axis and can be expressed by the emittance via the Twiss parameter  $\gamma_{x,y}^{Twiss}$ :

$$\frac{\Delta L}{L} \sim \frac{1}{2} (\gamma_x^{Twiss} \varepsilon_x) + \frac{1}{2} (\gamma_y^{Twiss} \varepsilon_y)$$
(5)

### Influence of misalignments

A deviation from the the ideal path can also occur for small emittances due to a distortion of the closed orbit. Thus misalignments may have a strong influence on the resolution. In Fig.3 we present the influence of the closed orbit deviation on the frequency spread. In the simulation we assumed that the closed orbit is shifted simultaneously in both horizontal and vertical plane. 100 particles of one species filling an emittance of 40 mm mrad were tracked in the ring over 50 turns. One can see that to reach practically the time resolution around  $2 \cdot 10^{-6}$  the initial shift leading to a closed orbit distortion should not exceed the value of 1 mm.



Figure 3: Dependence of the frequency spread on the initial shift of the orbit in the middle of the straight section of the ring.

## Influence of nonlinear fields

In a storage ring with large acceptance the nonlinearities associated with the motion of particles at large betatron amplitudes and with large dispersion play an important role. Chromatic effects change the betatron tune and nonlinearities can alter the path length significantly. The variation of the dispersion in higher orders is a critical parameter in a storage ring for TOF mass measurements since it directly leads to a path length variation and therefore reduces the frequency resolution. The main contribution to nonlinear fields is given by the field imperfections of the dipole and quadrupole magnets. Here components of a multipole expansion up to order 9 were considered from calculated fields based on a first magnet design. The nonlinear field errors affect the focusing of the ring magnets.

The second most important contribution in dispersion variation comes from the quadrupole fringe fields. Since

in the CR large aperture quadrupole magnets (diameter = 40 cm) will be installed, the fringe fields become important and will have a considerable influence on orbits of ions passing them far from the optical axis. The influence of these effects is shown in Fig.4.



Figure 4:  $\gamma_t$  as a function of momentum spread in the isochronous ring. The chromaticity of the quadrupoles is always included.

curve (1): without field errors and without higher order corrections. (2): with the effect of field errors but without fringe fields. (3): Only the influence of the fringing fields without field errors. (4): With all field errors and fringe fields and a correction by sextupoles and octupoles.

As one can see that the field imperfections lead to a strong sextupole component changing the value of  $\gamma_t$  for different momenta. At very large relative momentum deviation also octupole and higher orders become visible. The fringe fields of the quadrupoles include octupole components which are also clearly visible in the graph. In both cases the isochronicity would be not sufficient to perform a good mass measurement.

To control the nonlinear dispersion function a special correction scheme is needed. The effect of the field imperfections can be compensated by using three independent families of sextupoles installed in the arcs of the ring. Fortunately, the fringing field effects act to the opposite direction compared to the nonlinear field errors of the magnets. To correct them one has to use two families of octupole correctors. From Fig.4 one can see that octupole correction can reduce the variation of  $\gamma_t$  over a wide range of the momentum aceptance of the ring. Within the span of  $\Delta p/p = \pm 0.4\%$  the standard deviation of  $\gamma_t$  is only  $\sigma_{\gamma_t} = 0.001$ . Beyond no isochronicity can be achieved with sextupole and octupole correction only. This shows again the importance of the higher order terms.

To improve this shortcoming a new development is to apply two TOF detctors placed on a straight section of the ring [8]. With them one can measure the velocity of an ion in addition to the revolution frequency. Since the mass and charge are known sufficiently well, this leads to momentum or magnetic rigidity, which can be used for corrections once the shape of the isochronicity curve (like the one shown in Fig.4) is known. Such a curve could be measured with test beams of well defined momentum.

### CONCLUSION

The influence of different errors on the frequency resolution in the isochronous storage ring has been studied. The numerical simulations show that the frequency resolution strongly depends on the ring acceptance. For higher emittance the resolution gets worse but also the centroid is shifted. Limits for closed orbit distortions were defined. To correct effects of nonlinear field errors or the influence of fringe fields one has to use higher order corrector magnets such as sextupoles and octupoles. Still even higher order terms limit the isochronous range, a solution for this could be the additional measurement of the velocity of the ions.

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