

RACETRACK NON-SCALING FFAG FOR MUON ACCELERATION

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Abstract

The non-scaling Fixed Field Alternating Gradient (FFAG) machines have very strong focusing, large momentum acceptance, small both dispersion and betatron functions. This report is a study of using a compact non-scaling FFAG in combination with the superconducting linac to accelerate the muons. The drift space between two kinds of combined function magnets in the previous non-scaling FFAG is removed. The time of flight in the non-scaling FFAG has a parabolic dependence on momentum.

The large energy acceptance of the machine requires matching between the linac and the non-scaling FFAG arcs for both the betatron and dispersion functions over the entire energy range.

INTRODUCTION

The Neutrino Factory and the Muon Collider Collaboration [1] were initiated and non-scaling FFAG development. The non-scaling FFAG muon acceleration was proposed 1999 at the Montauk muon Collider collaboration meeting [2]. The muons have very short lifetime and have to be accelerated very quickly. The superconducting linac would be a very good but expensive solution. The non-scaling FFAG has been seriously considered due to the RF saving cost. Optimization of the cost expenses [3] came to a conclusion that the present non-scaling FFAG design needs improvements. This report is trying to reduce the size of circumference, orbit offsets and the time of flight. It is the first known report of a racetrack type of the fixed field accelerator with momentum range of $\delta p/p = \pm 40\%$.

The non-scaling FFAG have an advantage with respect to the required aperture size with respect to any other fixed field accelerator. In the history of accelerator development the strong focusing alternating gradient synchrotron dramatically reduced the aperture from the other accelerating structures at the time. The non-scaling FFAG is also a very strong focusing machine with alternating gradient but with the fixed field. Major limitations of the non-scaling accelerators are relatively small energy-momentum range compared to the synchrotron or the scaling FFAG and large variation of tunes. This limits the number of turns during acceleration due to "resonance" crossings. Maximum reported ranges are $\delta p/p \sim 50-52\%$ while in the synchrotrons and scaling FFAG's could reach 80% and higher. The larger the number of basic small strong focusing cells in the non-scaling FFAG's the smaller the orbit offsets and the

smaller the path length is. There are also limitations in the magnetic field to be considered especially for the muon acceleration the superconducting combined function magnets are seriously being considered. The larger radius, and with that the circumference, contradicts the request for the shortest possible muon acceleration time need due their short lifetime. The bending magnets with the opposite field are required in the FFAG fixed field accelerators. This enlarges the circumference. The scaling FFAG by definition requires the opposite bend size to be 1/3 of the size major bending angle.

DESIGN OF THE "RACETRACK"

A "racetrack" solution is searched to allow acceleration of muons to energy of 20 GeV. A constraint of $\delta p/p = \pm 40\%$ sets the lowest energy to 8.57 GeV.

Three rings construct the "racetrack": two equal rings with smaller radii are connected at two sides with arcs of the second ring with a larger radius. The connection is established at the same minimum value of the dispersion function at the middle of the cell. The matching gets easier as slopes of dispersion and betatron functions are equal to zero at this point.

The two rings could be put together only if the geometrical conditions are fulfilled: slopes of a tangent to both rings at the point of connection have to be equal. This makes a connection between the two angles as:

$$\theta_1 + \theta_2 = \frac{\pi}{2}. \quad (1)$$

Two rings with quite different radii are designed separately. This is very similar to the previous matching conditions applied in the non-scaling FFAG gantry design [4] presented at this conference. They have very different radii – (in the example presented they are $r_1=51.649$ m and $r_2=92.718$ m). The length of the arcs, defined by the θ_1 and θ_2 angles, determines a number of cells participating from the ring with a larger radius or a number of cells missing from the arc of the ring with a smaller radius. There are 122 cells in each ring. The "racetrack" will be formed, applying the above conditions, by connecting arcs made of five cells from the larger ring and on both sides there are $56=2*28$ cells, of the two smaller rings.

Design of the Non-scaling FFAG with the RF

The number of cells in previous muon non-scaling FFAG rings was optimized from the performance and cost point of view. The most important performance parameters include:

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- The circumference length (the larger the length more muons are lost due to short lifetime).
- The time of flight (or path length): due to acceleration on the top of the RF and restrictions by the superconducting cavities.
- The orbit offsets.
- The tunes within the required $\delta p/p$.

A relatively large number of cells ($N_{\text{cells}}=122$) made orbit offsets significantly smaller than previous design. The ring is presented in Fig.1. Orbits of tracked particles for the momentum range of $\delta p/p = \pm 40\%$ are magnified 150 times.

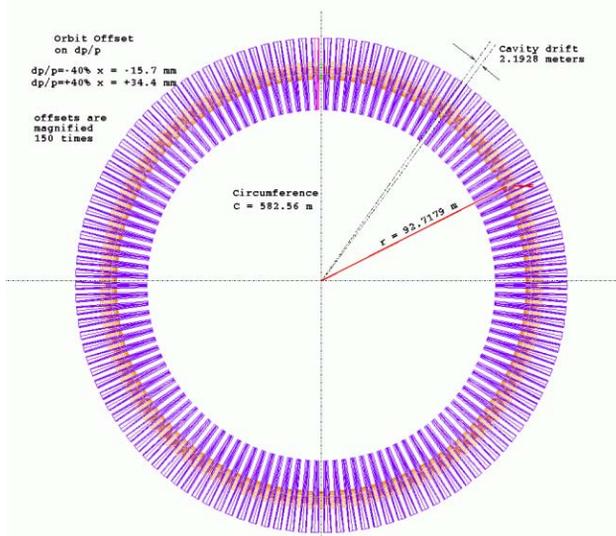


Figure1: The available drift space for the cavity is 2.19 meters, the radius is $r_2=92.72$ m, and the cell length 4.7751 m. Orbits are presented by red color with 150 magnification.

The orbit offsets in the lattice vary between $x_{\text{off}}=-15.7$ mm at the lowest energy $\delta p/p=-40\%$ and of $x_{\text{off}}=+34.4$ mm at the largest momentum $\delta p/p=+40\%$.

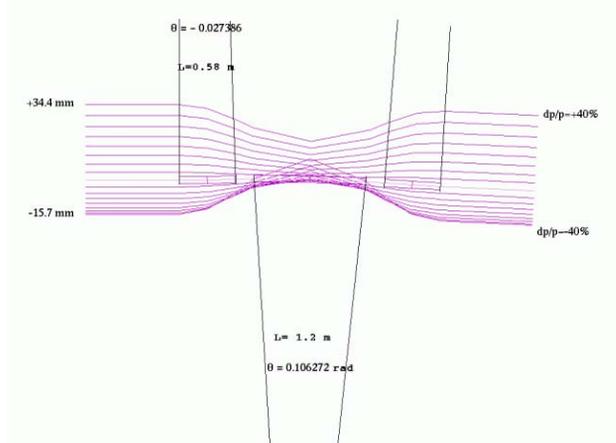


Figure 2: Orbit offsets in one RF ring with bending angles.

The small orbit offsets reduce the size-price of the magnets. This is a consequence of the larger number of

cells. There are only five 23.876 m long cells included in the “racetrack”. The drift larger than 2 meters is due to a size of the 200 MHz superconducting cavities from the Cornell University. The betatron functions and magnet blocks are presented in Fig. 3.

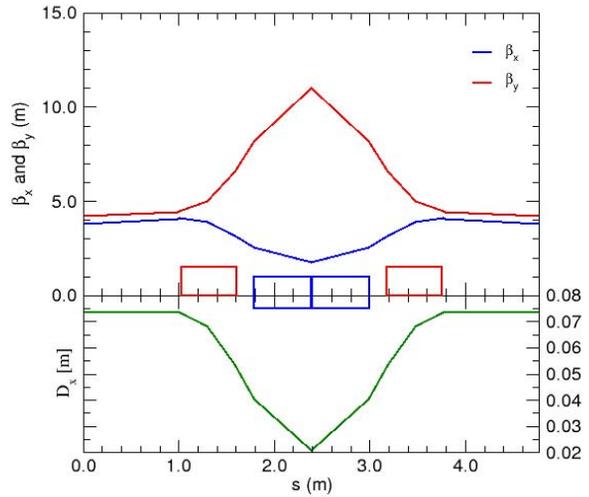


Figure 3: Betatron functions in the cell of the RF ring.

The magnet sizes and fields are presented in Table 1.

Arcs Made of Packed Non-scaling FFAG

The 2-meter limitation for the minimum value of a drift size makes a design of the non-scaling FFAG difficult in many respects: orbit offsets are large for smaller number of cells the time of flight makes limiting number of turns etc. The ring design without this limitation is presented in Fig. 4.

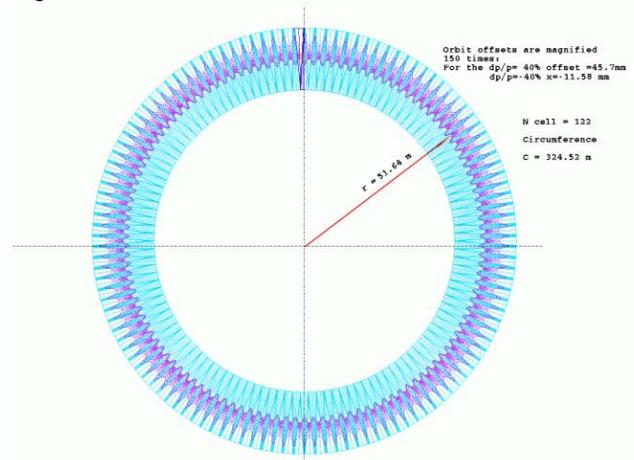


Figure 4: FODO like design of the ring made of packed combined function magnets. The radius is 51.65 m. Beam orbits for momentum range of $\delta p/p = \pm 40\%$ are presented by darker red color with 150 magnifications.

The circumference of this packed ring is 324.52 meters although there are 122 cells, 2.66 m long.

The orbit offsets and the magnet bending angles are presented in Fig. 5 and the betatron functions in Fig. 6.

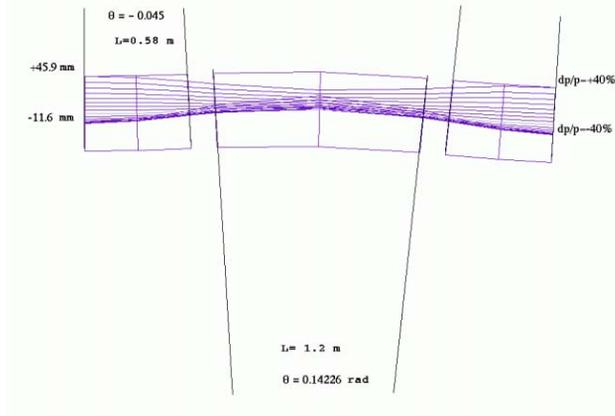


Figure 5: Maximum orbit offset of $x_{\text{off}}=+45.86$ is at momentum $\delta p/p = +40\%$, and it is $x_{\text{off}}=-10.6$ mm at $\delta p/p = -40\%$.

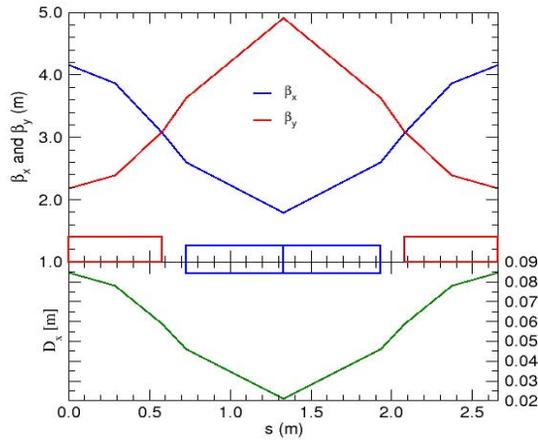


Figure 6: Betatron Functions in the cell of the packed ring.

“*Racetrack*” Properties

The combined “*racetrack*” ring is presented in Fig. 7. The geometrical conditions are also added to the tracked (Polymorphic

Table 1: Magnet Specifications

Mag.	L(m)	B(T)	G(T/m)	A _x (mm)
BD	1.2	4.22	-37.8	± 11
BF	0.58	-2.2	47.5	-15 ↔ 34
BD2	1.2	5.65	-40.2	± 11
BF2	0.58	3.78	43.4	-11 ↔ 44

Tracking Code-PTC) orbits of the muons for the momentum range of $\delta p/p = \pm 40\%$. The orbits had shown stable motion with maximum orbit offsets $x_{\text{off}}=+49.99$ mm at momentum of $\delta p/p = +40\%$. An orbit was observed

at momentum offset of $\delta p/p = +45\%$ with the maximum orbit offset $x_{\text{off}}=+54.93$ mm.

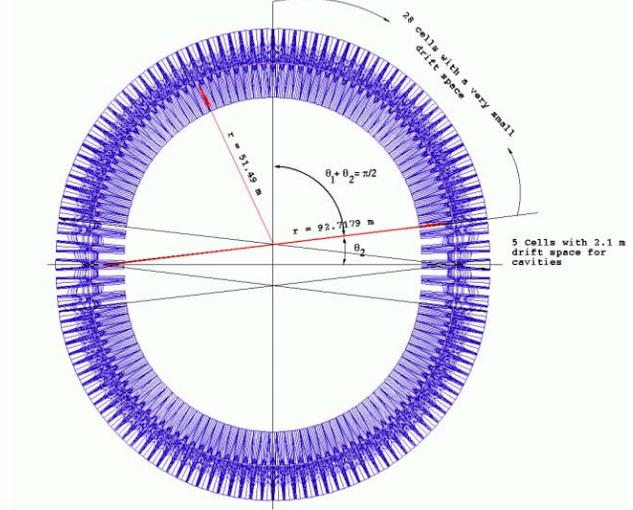


Figure 7: The “*racetrack*” non-scaling FFAG lattice for $\delta p/p = +40\%$ with a circumference of $C=345.67$ m.

SUMMARY

An attempt for a design of the racetrack type non-scaling FFAG is presented. The results are very encouraging. The muon acceleration from an energy of 8.57 to 20 GeV is provided within a circumference of $C \sim 345$ m, with the largest orbit offsets of $x_{\text{max}} < +44$ mm and $x_{\text{max}} > -15$ mm. There are only ten cells, in this example, to be used for cavities and an extraction/injection purposes. The additional matching of the vertical betatron function will be added. The present solution clearly shows stable orbits from -40% to $+40\%$ in momentum and stable horizontal betatron function. It is important to note that magnets have significantly smaller gradients and magnetic fields.

REFERENCES

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